The Commonwealth of Massachusetts

Survey Manual

Massachusetts Highway Department

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FORWARD

This manual describes survey theory and methods as practiced by the Massachusetts Highway Department and is a guide for all surveyors working on Department projects. A major goal of this Manual update is to introduce the implementation of the metric system of units to Department projects. Although these procedures are recommended as standards in most cases, we recognize that surveyors will deviate from them as conditions warrant. Addenda may be issued and incorporated into the Manual as technology and procedures evolve.

SECTION ONE

GENERAL INFORMATION

1.1 PUBLIC RELATIONS AND THE SURVEYOR'S RESPONSIBILITIES

Surveyors working on Massachusetts Highway Department (MHD) projects are professionals representing a public service agency. Whether a surveyor is an MHD employee or a member of a crew under contract, it is important to the Department that these representatives maintain good public relations in their work.

Surveyors are required to be courteous and tactful with citizens seeking information on a project. The crewchief should be the only spokesperson for the survey party. He/she may answer reasonable questions relating to the project, but should not speculate about possible alignment changes, easements or takings, or any aspect of a proposed design. He/she should not advise property owners or make statements that could cause misunderstandings. Any requests for information or questions of more than a basic nature should be referred to the District Survey Engineer or District Highway Director.

Conduct of survey crew members must be professional and proper at all times. Care should be used when marking survey points. Paint, flagging, and other marking techniques should be used sparingly. Crewchiefs should use good judgment in laying out surveys to minimize property damage and avoid creating hazardous conditions. When line cutting is required, the brush should be removed from the line and disposed of properly.

Where pedestrian traffic is likely, stakes should be hubbed (set flush), or other surface marks should be used. On those occasions when the crewchief must perform calculations in the vehicle, he/she should find duties for the rest of the crew.

1.2 RIGHT TO ENTER PROPERTY

Surveyors should not perform work on private property, or property of any entity other than MHD, without obtaining legal right to do so. District Survey Engineers and crewchiefs should make certain that property owners have given verbal or written permission, or have been notified by the District Highway Director as set forth in MGL, Ch. 81, Sect. 7F, as amended by Ch. 582 of the Acts of 1958 and Ch. 30 of the Acts of 1979:

Entry on private land for purpose of surveys, soundings and drillings

Whenever the department deems it necessary to make surveys, drillings or examinations to obtain information for or to expedite the construction of state highways or other projects under its jurisdiction, the department, its authorized agents or employees may, after due notice by registered or certified mail, enter upon any lands, waters or premises, not including buildings, in the commonwealth for the purpose of making surveys, soundings, drillings and examinations as they may deem necessary or convenient for the purposes of this act, and such entry shall not be deemed a trespass nor shall an entry for such purposes be deemed an entry under any condemnation proceedings which may be then pending. The department shall make reimbursement for any injury or actual damage resulting to such lands, waters and premises caused by any act of its authorized agents or employees and shall so far as possible restore such lands to the same condition as prior to the making of such surveys, soundings, drillings or examinations.

It is standard practice on large projects for the MHD District Office to make a certified mailing to all abutters. This notice should be prepared by District Survey personnel for the signature of the District Highway Director. Property owners generally cooperate, but some may be hostile. An explanation of the need for entry and a request for the owner's permission encourages cooperation. Diplomacy and tact on the part of all survey crew members is required in all cases.

1.3 UNIT OF LENGTH

The standard unit of length for MHD land surveying and mapping is the meter. The 100-foot survey station is replaced by the 100-meter survey station. The standard interval for marking baseline, taking detail and/or cross sectioning shall be 20 meters. MHD conforms to the National Geodetic Survey (NGS) policy of providing state plane coordinates (SPCs) in meters when dealing with the North American Datum of 1983 (NAD 83).

When the previous datum (NAD 27) became law, the "U.S. survey foot" was the standard unit of length. The distinction between the "U.S. Survey Foot" and the "international foot" is a subtle but important one. The "international foot" is defined as 0.3048 m exactly and the U. S. Survey Foot is defined as 1200/3937 m, or 0.30480061 m. The "international foot" is shorter than the "U.S. survey foot" by 2 parts per million. This difference may not be a factor in distance measuring, but can introduce major errors in coordinate conversions if not considered.

By statute and regulation, for conversion of meters to U.S. survey feet "...the meters shall be multiplied by 39.37 and divided by 12 which results in a constant multiplier having a value of 3.280833333333 to 12 significant figures." Surveyors using calculators or computer programs to make unit conversions should be certain to use this factor.

1.4 HORIZONTAL AND VERTICAL DATUMS

The horizontal datum in use by MHD is the North American Datum of 1983, following a 1991 amendment to Chapter 97 of the Mass. General Laws. (See Appendix 1, Excerpts from Mass. General Laws). The ellipsoid that forms the basis for NAD 83 is the Geodetic Reference System of 1980 (GRS 80) Ellipsoid. The previous system, the North American Datum of 1927 (NAD 27), used the Clarke Spheroid of 1866 and was the basis for the State's Lambert Conformal Conic Projection. The difference between the two ellipsoids is the major reason for the coordinate shift from NAD 27 to NAD 83. The GRS 80 Ellipsoid provides improved values for the earth's size and shape and removed previous distortions. Surveys performed for MHD should be in NAD 83 state plane coordinate values.

The vertical datum in use by MHD for many years is the National Geodetic Vertical Datum of 1929 (NGVD 29), formerly known as Mean Sea Level datum. NGS has developed an improved vertical datum, the North American Vertical Datum of 1988 (NAVD 88). MHD is now using the NAVD 88 datum on all recently established bench marks so there will soon be sufficient control to officially adopt the new datum. The Federal Geodetic Control Subcommittee has affirmed:

> ...the NAVD 88 datum as the official civilian vertical datum for surveying and mapping activities in the United States performed or financed by the Federal Government, and to the extent practicable, legally allowable, and feasible, require that all Federal agencies using or producing vertical height information undertake an orderly transition to NAVD 88.

Any questions on local control or the proper datum to be used, or for conversion from one datum to another, should be referred to the MHD Boston Survey Office.

1.5 SAFETY

Survey personnel, whether MHD employees or employees of survey contractors or sub-consultants, are required to conform to MHD safety requirements, applicable safety laws and regulations, and the latest version of the following publications:

Federal Highway Administration's Manual on Uniform Traffic Control Devices (MUTCD).

U.S. Department of Transportation's Traffic Control Devices Handbook, Part VI - Work Zone Traffic Control.

MHD's WORK ZONE SAFETY - Guidelines for Massachusetts Municipalities and Contractors.

U.S. Occupational Safety and Health Administration's Safety and Health Regulations for Construction, Part 1926.

1.5.1 Personal Safety

Survey personnel must wear safety glasses while using any hand tools, including line cutting tools. One of the most frequent tasks crew members do is setting a mark to occupy or to use as a tie. A sledge hammer, the common means of driving stakes, rods or pipes, is also the major source of eye injury for surveyors. Use of safety glasses is extremely important when driving stakes, especially in cold weather when a frost pin is needed to set most marks. Hard hats are also a requirement at all construction sites.

Warm weather attire is also a safety concern. MHD Policy Notice N-90-035 states:

> All employees must report for work suitably dressed for protection against sunburn, various insect bites, and contact with poisonous shrubs/vines. Tube and tank tops, shorts, sneakers, thongs, sandals, or open toe shoes **will not be allowed**. If any employee is not suitably dressed for work, he/she will not be permitted to work until suitably attired.

1.5.2 Roadway Safety

Survey crews spend a good part of their time on roadways of many different types from country roads to city streets to high speed expressways, under a variety of climatic conditions. With electronic instruments becoming more common, the need for survey crew members to be in the traveled way is minimized. However, the crewchief is expected to act responsibly by analyzing every situation and to take any action necessary to ensure the safety of the crew members as well as the motoring public. The roadway shall be properly signed and coned, crew members shall wear proper safety gear and shall stay alert.

SECTION TWO

SURVEY OPERATIONS

2.1 CONTROL SURVEYS

All horizontal data for MHD highway and bridge projects are required to be in the Massachusetts State Plane Coordinate System (SPCS) on the North American Datum of 1983 (NAD 83), except for those excluded by the Department. The SPCS in Massachusetts is a Lambert Conformal (conic) Projection, which is divided into two zones. The Mainland Zone encompasses all counties from Berkshire to Barnstable, the Island Zone is comprised of Dukes and Nantucket Counties.

All vertical data on MHD highway and bridge projects are required to be on the National Geodetic Vertical Datum of 1929 (NGVD 29), or on the North American Vertical Datum of 1988 (NAVD 88).

Some projects, usually small isolated ones, may not be within reasonable distance of geodetic control. The Department reserves the right to exclude such projects from standard control requirements.

The latest available information on the location, suitability, and known condition of Massachusetts Geodetic Survey (MGS) and National Geodetic Survey (NGS) control is available at the MHD Survey Office at 10 Park Plaza, Boston. That office should be consulted before survey work requiring geodetic control is undertaken.

2.1.1 Surveys by Consultants

All surveys performed for the Department by consultants (design engineers, survey contractors, survey sub-consultants, etc.) are subject to the Department's inspection at all stages of a project.

Such consultants are directed to seek advice and information from the MHD Survey Office for the best available control in the area of a project before they estimate survey costs or begin reconnaissance. Control anticipated to be good may be destroyed or inadequate, and new control may have been added that will simplify the survey operation. Circumstances may warrant the Department to expand or readjust control. Survey consultants working on MHD projects will run control only with prior MHD approval and based on MGS or NGS control approved as sufficient.

2.1.2 Horizontal Control

MHD will use Global Positioning System (GPS) procedures to establish control at most MHD projects as a base for all survey work to be performed.

On most large projects, basic horizontal control consists of a random-type traverse connected and adjusted to MGS or NGS control. Such traverses are usually run out using electronic total stations, or theodolites with electronic distance measuring devices (EDMs), of suitable quality to obtain the desired precision.

Any reproduced highway baseline included in the control must be measured and tied in to meet the required precision.

Primary control should be tied in strongly to MGS or NGS control unless other control has been stipulated. Secondary control should be connected to control of equal or better precision.

Unless a closed survey is made, the use of control that is not part of the project's particular net is not recommended since other points may not be compatible.

2.1.3 Horizontal Order of Accuracy

MHD follows the FGCC Standards and Specifications (Appendix A-11) for survey work. If a surveyor deems it necessary to vary from these standards, prior approval should be obtained from the District Survey Engineer or from the MHD Boston Survey Office.

First-Order: applies to control established by static GPS.

Second-Order, Class 1: applies to major bridges, tunnels, and structures of such size and importance on controlled access state highways as to justify geodetic precision.

Second-Order, Class 2: applies to minor bridges and State highways not covered under Second-Order, Class 1, as well as to all other secondary roads except those of minor nature.

Third-Order, Class 1 is a minimum requirement, for minor projects where higher accuracy is not essential.

MAXIMUM ERROR OF CLOSURE

Order of Accuracy:	First-Order	Second-Order Class 1	Second-Order Class 2	Third-Order Class 1
Allowable Error:	1:100,000	1:50,000	1:20,000	1:10,000

Error of closure is only one of the many factors, which determine the accuracy of a survey. Proper procedures and instrumentation as detailed in Appendix A-11, FGCC Standards and Specifications for Geodetic Control Networks, should be used.

2.1.4 Measurement of Angles

Use FGCC specifications, Appendix 11, for angle work. Settings for instrument positions, or sets, should be evenly spaced around the circle according to the formula:

$$I = \frac{180}{N}$$

where I = Increment
N = Number of sets or positions

With the use of digital reading theodolites and total stations in conjunction with electronic data collectors, the initial setting has become less important. While many data collectors or post processing software "mean" angles automatically, it is good practice to set zero degrees, minutes and seconds on the initial sight in the first set of angles and begin any subsequent set with an increment of degrees. The raw data can then be reviewed to readily differentiate between sets of angles.

In turning angles, considering the direction of progress of the baseline or the traverse, the angle should be turned to the right, with the initial station to the rear.

In closing the horizon with a transit, the initial will be ahead for the second angle. If there are more than two rays, each angle will be turned separately, the last angle closing the horizon.

In using a theodolite/total station, all of the rays may be picked up in each position, the initial being the station to the rear. If there are more than five rays, it may be best to pick up the main rays, using the same initial. Do not reverse the scope on a poor ray. In the case of highly unbalanced rays, use the longest sight as the initial.

If the surveyor finds it necessary to deviate from the rule of turning from behind and to the right, it should be clearly detailed in the notebook.

2.1.5 Measurement of Distances

See Appendix A-11, FGCC specifications, for various classes.

Most distances are now measured electronically. Surveyors should be aware of the limitations of the EDM device and should build sufficient redundancy into their procedures to maintain allowable accuracy. Since a common accuracy requirement for an EDM is +/-5mm + 5ppm, critical distance measurements of less than 30 m will be performed with a standardized (highway) baseline-grade tape.

EDMs and tapes should be regularly checked on one of the five calibration baselines established throughout Massachusetts for the use of the surveying community. (See Addendum A-8, Massachusetts Calibration Baselines).

A) Taping

If measured with a tape, each course should be measured twice, once in the forward and once in the backward direction. Two such measurements must agree within 1:20,000, or they must be repeated until the desired result is secured. Taping should be done with a 30 or 50-meter heavy-duty baseline tape, carefully standardized and equipped with a thermometer and spring balance.

Tapes must be used under the same conditions as when standardized or under such conditions for which a correction can be applied. Sufficient data for determination of corrections for alignment, tension, sag, grade and temperature must be recorded with tape measurements.

1. Alignment

No part of the measured line should be more than .03 m from the straight line between angle points. This aligning should be done with a theodolite.

2. Tension

For tape supported throughout (laid flat), tension should be 4.5 kg. For tape supported on two supports 15 m or more apart, tension should be 9.0 kg. With two supports and less than 15 m apart, tension should be 4.5 kg.

3. Support

Method of support for each tape measurement must be clearly recorded in the notebook. "T-4.5" indicates tape supported throughout, 4.5 kg tension; "2-9" indicates two supports, 9 kg tension.

4. Grade

Difference of elevation between points of contact of measurements must be recorded in the survey notebook under the heading of "Inclination" or "Elevation Difference" opposite each measurement. Grade corrections are to be calculated or interpolated from grade correction tables, and applied to each measurement.

5. Temperature

Temperature readings should be recorded for each tape length and an average temperature over the section determined. From the temperature correction table, the correction per 30 m can be selected and applied to the entire length of the section.

6. Tape Correction

A correction should be applied to each measurement according to figures derived from standardization.

B) Taping Techniques

Accurate taping can be readily accomplished by measuring with the tape laid flat on a smooth surface such as on railroad tracks or on bituminous concrete. The points may be marked with a pencil on masking tape. Care should be taken to keep the tape straight when marking.

This type of taping is best done in dry, cloudy weather. If the surface follows a vertical curve it will be necessary to determine elevations every 10 m, in addition to the 0 and 30 m points.

Accurate taping may also be accomplished on low taping bucks. Higher stools or tripods may be used with good results where low bucks cannot be utilized.

2.1.6 Methodology for Horizontal Control Surveys

A) Reconnaissance

Control information should be obtained from the MHD Boston Survey Office. Recovered monuments should be inspected for damage or movement, and the physical condition, description, etc., checked. A Recovery Report (see Addendum A-13) should be completed and returned to MHD.

Some requirements for laying out the traverse:

1. Position and azimuth should be connected to known control;
2. Courses should be long and balanced;
3. There should be few sharp angles in traverse alignment;
4. Sight clearance should be about 1 m;
5. Traverse point locations should be sound and practical, with marks not likely to be disturbed or destroyed and well tied down;
6. All main monuments should have azimuth marks. Azimuths may be turned to spires or other prominent features, or azimuths and distances taken to local bounds, e-pins set in boulders, etc., 60 m or more from station.

B) Monumentation

Disks for permanent points should be set in 1.2 m concrete monuments, or in concrete structures, ledge, etc. For semi-permanent points, punch marks should be placed in 48" deformed bars or similar material driven flush with the ground, or countersunk in areas subject to traffic. E-pins should be placed in concrete or stone, spikes in roots or stumps.

Important points should be tied in with three hook ties or two sets of range ties, with survey book sketches and descriptions showing local topo such as buildings, curbs, trees. Station and offset should be used if baseline is available.

C) Field Checks

Standard field procedure is to close an angle traverse by azimuths or bearings, and to check forward and back measurement runs, and all notes and figures. Notebooks must be indexed and cross-indexed, and should always contain a plan or sketch of each project.

The crewchief does not normally adjust the traverse or perform computations unnecessary for field work. However, if a data collector has closure capabilities, it is prudent to check the closure in the field to avoid a return trip. If recording is done manually, enough checking should be done to ensure that the traverse is complete and will close.

2.1.7 Vertical Control

Most existing vertical control is based upon NGVD 29, with new vertical control being established on NAVD 88. MHD may specify NAVD 88 datum for a particular project or purpose, or may allow use of existing local benches, or even an "assumed" bench for small isolated projects.

Vertical control for a project shall be connected and adjusted to MGS or NGS bench marks of an equal or higher order. At least two MGS or NGS benches shall be used and these should be checked for relative difference of elevation.

2.1.8 Vertical Order of Accuracy

See Appendix A-11, FGCC Specifications. Control bench marks should meet First-Order, Class 2 standards. Levels should be run in sections and should seldom be more than 2 km in length. Each section shall be run forward and backward, and the two runnings shall not differ more than 4 mm times the square root of distance, one-way, in km. This work requires the use of one-piece precise (invar) rods and First-Order bench marks.

Second-Order, Class 1 is normally required for major bridges, tunnels and all structures of such size and importance as to justify geodetic precision. Levels shall be run in sections seldom more than 2 km in length. Each section shall be run forward and backward, and the two runnings shall not differ more than 6 mm times the square root of distance, one-way, in km. This work requires the use of one-piece precise (invar) rods and bench marks of equal or higher precision.

Second-Order, Class 2 applies to the majority of MHD projects, most highways and roads except minor ways. Levels shall be run in sections seldom more than 2 km in length. Each section shall be run forward and backward, and the two runnings shall not differ more than 8 mm times the square root of distance, one-way, in km.

Third-Order accuracy is a minimum requirement, to be used where higher accuracy is not needed. Two runnings should not exceed 12 times the square root of the distance one-way, in km.

2.1.9 Methodology for Vertical Control

Although some general guidelines are detailed here, Appendix A-11 should be reviewed for FGCC standards for differing instrumentation and field procedures by order and class.

Level lines shall originate at and close upon First-Order bench marks when available or be run in closed circuits. At least two bench marks, shown by new leveling not to have changed their relative elevations, must be used as the starting point.

Third-Order levels should be run by three wire leveling and be run in sections between permanently marked bench marks not more than 2 km in length.

All lines are to be leveled independently in both the forward and backward directions, preferably under different atmospheric conditions (such as forward in the morning and back in the afternoon).

Portable turning points such as 0.3 - 0.5 m iron pins may be used. Turning plates should be used if pins are inappropriate.

Any permanent or temporary bench mark that is established on the forward run must be included on the back run.

The thermometer should be read to the nearest degree at the beginning and end of each section.

A daily observation of the collimation error of the level should be made and recorded on a separate page of the level book, using the C-Factor method. If "C" is greater than .05 mm/m, the level must be adjusted.

The difference between lengths of foresight and of backsight shall not exceed 10 m.

The length of sight should be great enough so that 5% to 15% rerunning may be expected. Short sights and a small amount of rerunning may indicate lack of production and excessive caution. Longer sights of not exceeding 60 m generally indicate satisfactory production. Experience will show ideal lengths of sight for a survey party.

A temporary bench mark should never be used to begin or end the day's work unless it is properly recorded in the notebook. It must be permanent in nature such as the top of a bolt or pin in a concrete base, a chiseled square on solid concrete, etc.

Setups should be "broken", i.e., the instrument height (HI) at the end of the forward run should be changed before the back run is begun.

Some common leveling errors include:

*Unbalanced backsights and foresights: errors are mostly systematic (cumulative) due to instrument adjustment error, curvature of the earth and other factors.

* Settlement of the instrument: systematic errors.

* Leveling in windy weather: the instrument and the rod are affected and accurate readings are difficult to obtain.

* Careless instrument work: improper focus (parallax); reading rod with bubble not centered; faulty rod readings due to factors such as obscured rod, sights too long, etc.

* Careless rod work: poor turns; careless rod plumbing; turns not marked; wrong turn used.

* Errors in recording and communications: notekeeper failing to repeat readings for check; failure to write legibly.

While survey crews may not be able to follow geodetic procedures exactly because of practical considerations, they should follow good field practice and strive for precision and accuracy.

2.2 PRELIMINARY SURVEYS

"Preliminary survey" generally means the collection of "as-is" field data for use in design or construction so that accurate project plans, specifications and estimates can be prepared. Needed information includes baseline data, topography, property lines, etc.

The Department now does most preliminary survey electronically using a total station and electronic data collector, often called an electronic fieldbook (EFB). Use of a traditional fieldbook is still required for all MHD survey operations, however, so that supplemental information and sketches can be recorded. EFBs allow substantial time and labor savings in processing and plotting field data to create an "as-built" plan. Much design work is rehabilitation and reconstruction of existing structures so that the as-built information becomes the base plan.

2.2.1 Field Survey and Photogrammetry

On many projects, aerial surveys by photogrammetric methods furnish a major part of the information needed. Traditional field survey work is considerably reduced, but still needed. Field survey is used to establish ground control as a means of measuring and checking the accuracy of photogrammetric mapping, to gather missing data or to provide data of higher precision. For example, field crews will run out location and baseline information, tying into horizontal control. They will also obtain critical elevations, details of utilities, property lines, drainage, etc. If needed, coverage of areas outside the flight lines or areas obscured by natural features and structures will also be provided.

2.2.2 Preliminary Survey

The Department obtains field survey data for preliminary engineering purposes (whether the design is performed by MHD, or by a design consultant) by using survey parties:

* comprised of MHD personnel,

* provided by survey firms working under contract to MHD,

* provided by a design consultant under the terms of its contract, or by survey firms working as sub-consultants to a design consultant.

Survey work is initiated by written survey request originating with an MHD project manager or an MHD engineer performing in-house design. Survey requests are routed through the office of the Deputy Chief Engineer for Highway Engineering and the Boston Survey Office to the appropriate District.

Before authorizing the start of survey work, the District Survey Engineer should obtain a copy of the applicable Engineering Work Order (ADM 720), which provides the activity code, job number, expenditure accounts and other project data.

The District Survey Engineer will furnish survey parties with instructions and data needed such as plans, survey books, administrative information, etc. As the survey progresses, the crewchief may acquire additional materials such as property plans, deeds, record layouts, utility information, etc.

MHD survey notebooks are required to be used for all surveys performed for the Department whether or not data is gathered electronically. Horizontal data (detail, baseline and traverse information) is kept in even-numbered notebooks, with odd-numbered notebooks used for vertical data (levels, profiles and cross sections).

The District Survey Engineer will supervise and periodically check on the conduct of the work.

Preliminary Survey will include the following operations, as needed:

* Horizontal control should be recovered (i.e., located in the field) and extended as needed.

* Existing baselines and other important lines should be run out and connected. This work should be checked in the field with subsequent review of computations and adjustment. Final or proposed final location lines should be run out and connected. All important points should be tied in.

* Existing vertical control should be recovered and bench runs made, checked, computed, and adjusted.

Detail and cross sections should be taken from proper baselines. On some projects, this data may have to be located from random or supplementary lines. EFBs allow most of this data to be obtained from random traverse lines, which reduce roadway traffic hazard as well as transcribing errors.

Property corners should be recovered and tied into survey lines. Angles and distances to physically evidenced, or approximated, property sidelines should be observed and recorded.

Bridge and stream surveys are made as directed. (See Section 2.2.8, Bridge Surveys).

Other work, under general classifications of cross sections and detail, includes punchings, soundings, ledge sections, grid sections, drainage, location of utilities, etc. (See Section 2.2.7, Cross Sections and Related Work).

In addition to running out final location baselines, the survey crew may make surveys for right-of-way locations, ramp locations, structures, etc., including some work which is not strictly "as is" but which will prove useful for design or other purpose of the work.

Appendix A-3, Abbreviations, contains MHD field work and design standards for illustrations, abbreviations and plan symbols. The MHD Highway Design Manual also provides much useful information.

2.2.3 Base Plans

A base plan, also known as an "existing conditions" plan, is a graphical representation of the preliminary "as is" survey data. MHD requirements are detailed here to help the survey crewchief understand what field data is needed for base plan preparation. While field personnel will not usually be involved in the actual plotting of the data, they should fully understand how field data is utilized in the preparation of an acceptable base plan.

A) Survey Field Notes

When the field survey work is completed, whether performed manually or with EFB, the notebooks containing alignment data, details, sketches, and bench level and cross section notes should be checked for completeness. For major projects, survey baselines and traverse lines are computed and adjusted by the MHD Boston Survey Office. The surveyor shall use MHD methods to adjust all other baselines, traverses and levels, to the closure limits in Appendix A-11, FGCC Standards and Specifications for Geodetic Control Networks, or as directed by the District Survey Engineer. Any discrepancies in field notes shall be carefully checked and remeasured, if necessary. Adjusted data should be checked in the field.

B) Data Obtained Manually

The base plan should show all cultural and natural details within the area of the proposed project. In general, base plans are to show all structures with the type of each noted, as well as water lines, sewer lines, drains, underground utilities, poles, manholes, hydrants, catch basins, wells, curbing, roadway edges, street signs, streams, bridges, etc. The plans shall show the location of state, county and town layouts, approximate property lines, owners and deed and Land Court numbers.

C) Data obtained by Electronic Fieldbook (EFB)

A base plan by EFB is a two-dimensional (2-D) (planimetric) drawing of the existing detail within the project limits. This is most commonly generated from three dimensional (3-D) electronic data collection, but is prepared as 2-D lines for ease of editing and adding design data.

The plan should contain plan symbols and line types in conformance with MHD Highway Engineering Division drawing standards. Sample abbreviations can be found in Appendix A-3. The Highway Design Manual shows a sample base plan and details additional requirements.

A survey reference line, established in the field by random traverse between geodetic control points, shall be shown on the plan. When required, a supplemental center line can be created by splitting some suitable feature (bridge structure, roadway surface, etc.).

A Digital Terrain Model (DTM) will be compiled from the 3-D field data and checked for correct location of break lines. Contouring of the DTM will be done at an interval between 0.2 and 0.5 m, with the concurrence of the District Survey Engineer. The DTM (with contours) will be submitted in electronic format along with the base plan.

The final submittal to MHD should include all relevant electronic files that the CAD software may create.

2.2.4 Baselines, Sidelines and Property Location

The term "baseline" has acquired a variety of meanings over the years. Its definition herein is: a reference line of known dimension and bearing, from which various data can be related through field measurements.

A) Some common baseline types are:

1. Survey Baseline - Until electronic data collection, the Survey baseline has historically been the reference line laid out on the ground by the survey party so that detail and cross sections could be tied by station and offset and recorded in a survey notebook. For a new roadway in virgin terrain, the survey baseline would often be a random traverse line located in the field by the survey party. In some cases a preliminary alignment would be given to the survey party to lay out. On an existing roadway this line would be established from monumentation and appear similar to the baseline of record. It would be an approximation, however, because distances would be apportioned both longitudinally and transversely to conform to record stationing. Electronic data collection allows the use of a random traverse line with angle points situated off of the traveled way so that the same data can be obtained with less safety risk and less traffic disruption.

2. Record Baseline - The baseline of record, or layout baseline, is the original line laid out and on which the Layout Plan, and the order of taking (or county decree) are based. It is rarely possible to exactly recreate the baseline of record because the original ties are usually destroyed. When this line is required, it has become standard practice to recreate the line from the best witnesses available: the sideline monuments that were set from the record baseline during construction.

3. Construction Baseline - The baseline provided on the construction drawings, also called the design baseline. This line is most often, but not always, the centerline of

construction. It is created by the designer to make estimating, design and construction operations easier. Construction plans should include detailed sketches of ties relating the construction baseline to the survey baseline at several locations, by geometry and by coordinates.

4. Historical Baseline - any documented line that can be recreated from said documentation. This category could include any of the baselines described above.

B) Baseline Layout

The first step in making the preliminary survey is to establish state plane coordinates at the site using geodetic control. Next, lay out and set a survey baseline (random traverse) and then the control points of a historical baseline if records exist: the PCs, PTs, PIs if accessible, and the angle points. The survey baseline is an important reference line on the project, since location, survey and right-of-way data is referenced to it. Since the baseline is a prime control line for the project, it must be accurately located. Standard baseline layout stationing shall be at 100 m intervals. If the survey is to be by EFB, it is recommended that the random traverse line be established to accuracy equal to a manual survey baseline. Baseline control points shall be located with particular care. Specifications for measuring may be found in Section 2.1, Control Surveys.

On major MHD projects, predetermined alignment data is furnished to the District Survey Office during the design phase. This information should be provided on plans and other documents on which the proposed alignment, existing control and connections, coordinates, angles, and distances are shown.

The survey crew should run out the preliminary baseline promptly to determine how well the lines will fit on the ground. Record data will normally be held despite minor discrepancies. Errors, large or small, should be called to the attention of the District Survey Engineer. Baseline and layout data should be adjusted before the layouts are recorded.

Some preliminary survey requests are for EFB surveys on reconstruction projects for which no record baseline information is available. A "survey baseline" may then be created by splitting some permanent feature, e.g. bridge roadway, stone wall, etc., and incorporating it into the traverse. The random traverse line is then the most important line for the project as all information will be referenced from it and any property or layout monumentation should be tied to it.

When there is an option, stationing for main baselines should run south to north, and west to east.

Baselines and traverse points must be well tied down with strong semi-permanent markers such as range ties extended beyond any contemplated construction. The baseline should extend 200-400 m beyond the limits of the project, or as needed for the preservation of such points.

Traverse line and baseline measurements and notes should be recorded in even numbered MHD notebooks (HED 750).

C) Reestablishing Baselines and Sidelines

Replacing baselines and sidelines is a common survey function and may range in difficulty from recovering a section of sideline to within 0.3 m to rerunning the baseline and recovering the sidelines of a major highway for alteration or reconstruction purposes.

For recent roadway layouts, record data such as layout plans and notebooks are usually readily available, making the survey a relatively routine process. However, the work is of some technical difficulty, especially on limited access highways, requiring survey personnel with skill in baseline work. The party must check all work possible, add ties where needed, and note any discrepancies.

In working on older roadway layouts, surveyors will generally run into problems. Bounds may be missing, damaged or out of position. The crewchief is responsible for accurately locating the original sidelines even if incorrectly set originally. The surveyor must also recover all physical evidence, including Massachusetts Highway Bounds (MHBs) and any other indications of ownership, such as property corners, fences, walls, etc. This information is a necessary part of both detail and baseline notes. The surveyor should also recreate the baseline, including a complete rerun for readjustment, if needed.

Small rural roads which have not been defined either by plan or by written description can usually be located by physical evidence, the most common being the location of stone walls which parallel the traveled way. Measurements should be taken between the faces of the walls, considering the walls themselves as belonging to the abutters. Often the approximate distance between them will be a commonly used number of rods (1 rod = 16.5 ft. or 5.029 m), such as 2, 2 1/2, 3, etc., although the distances can vary considerably. It should be noted that the records of many legally defined roadways have been lost. It is important that the surveyor locate all physical data such as fences, property corners and buildings, and should be particularly alert to evidence of metes and bounds descriptions such as inscribed granite markers. All of this data should be shown as detail.

Originally, many rural roads were simply rights-of-way, with abutters' properties meeting within the way and not necessarily in the middle. Such ways may later have been defined by town meeting action, but often were not recorded at the County Registry of Deeds. A Massachusetts Land Court document prepared for surveys performed under its jurisdiction, Manual of Instructions (1989), may be followed as a means of locating these roads. Section K, Streets and Ways, defines "way" as follows:

The word way as hereinafter used will include all highways, boulevards, avenues, roads, streets, paper streets, traveled ways, cart paths etc., whether public or private, constructed or not, in use or not, existing physically on the ground or legally of record.

Survey notes should indicate whether a way is public or private.

Monuments such as stone bounds "supposedly located at the termini of the curves and at the angle point of the ways.... must be surveyed as they exist on the ground." They should be identified by type (e.g., C.B., MHB, S.B.), mark (e.g., d.h., e.pin), and if "held", be so noted. (See Appendix A-6, Sample Survey Notes).

D) Property Location

Property surveys performed for the Department vary in scope from the taking of routine detail to a determination of complete property boundaries. Although thoroughness of property work should be limited to only what is needed, the surveyor may also be required to obtain additional data and establish new property lines, etc. Accurate boundary locations are required for takings and easements. All available evidence should be researched and weighed according to land surveying principles; the locus should be determined from the best evidence, and the property tied in accurately to the baseline or traverse line. All research data should be turned in with the survey. Surveyors are expected to understand and apply principles of property survey so that they provide reliable data.

Property surveys involving registered land, or land proposed to be registered, must be performed according to the Massachusetts Land Court's Manual of Instructions, available at its Boston office and at the Registry of Deeds in the various counties.

E) Specific Instructions and Order of Calls

The District Survey Engineer shall specify in detail to the crewchief any property work needed for MHD survey purposes. When a land taking is involved, a plan of the entire property and buildings may be required. Although it may not be necessary to run an entire field survey, the proposed taking and connections to the remainder must be measured. Plans of proposed private projects that abut a roadway like subdivisions, plazas, industrial parks, etc. should be obtained and tied into the baseline.

For a taking involving registered land, a complete resurvey need not be made. The Land Court requires that a subdivision plan be prepared whenever a portion of a registered parcel is taken. Therefore, it is MHD policy that only easements will be taken on registered land unless an entire parcel is taken, as it exists.

The corners and lines of the affected areas should be located in relation to the baseline and any Land Court monuments found should be located to aid in locating the parcel in relation to the highway layout. Angles should be turned connecting the remainder to the taking. The party should recheck any discrepancies found in old angles and distances. Bounds that are recovered should be noted as "fnd," with physical condition noted, and "quick-tied." Conflicting evidence and descriptions are commonly encountered in this type of work. The surveyor should locate the property from the evidence, not relocate it. The "order of calls" should be considered, in the following order of importance:

1. Natural monuments
2. Artificial monuments
3. Adjoiners' or abutters' lines
4. Courses and distances
5. Area

Layout evidence such as property corners and courses should be shown in the detail for every project whether or not it appears significant. Fences, walls, etc., along the layout should be located to 0.05 m. Property lines may be ranges from the baseline with two sets of angles turned to them, and measurements made to corners.

F) State and Town Lines and Corners

Town lines run from angle point to angle point (bound to bound). Road stones are often found on line between these bounds at all intersections with highways, in accordance with MGL Chapter 29, Section 4, as amended. Road stones do not carry the legal weight of town corners and should not be considered correct unless confirmed by survey measurements. Many road stones have been shown to be in error by 50 m or more. Road stones placed by MHD are surveyed from town corner to town corner or located by coordinates, with the work documented in survey notebooks.

Surveyors performing preliminary location surveys should locate town corners and road stones in the area.

G) Town Boundary Atlases

Town and city boundary lines, which sometimes are also county and state boundary lines, are thoroughly described and illustrated in the Town Boundary Atlas series, published by the Harbor and Land Commission, circa 1900. Copies of the Atlas may be found in city, town and county offices, although the surveyor is cautioned to check for any possible subsequent revision by the Legislature.

The MHD Boston Survey Office keeps an up-to-date Atlas set available for reference and can also provide approximate state plane coordinates (+/- 0.3 m) for town corners that have not been surveyed for many years.

Any surveyor who recovers a town corner should make recovery notes of the location and condition on HED 697, REPORT ON CONDITION OF SURVEY MARKER (See Appendix A-13), and should redescribe and tie these valuable monuments when necessary.

2.2.5 Detail

A) General Procedures

Detail may be called the "plan view" or top view of a project, topography taken on a horizontal plane, while contours and other vertical information are considered topography. Now that detail is obtained electronically, vertical data is captured at the same time.

The collection of detail generally begins after the baseline or traverse line is set. Detail includes all natural and cultural features on, above, and below the ground in the proximity of the baseline, and those more distant from the baseline, which may affect proper design of the project. The survey crew must be alert to picking up all necessary detail, including objects not specifically detailed herein.

Appendix A-6 shows typical examples of detail notes. Generally, the surveyor should follow the examples but should make changes to suit conditions. The main purpose is to take easily understandable notes conducive to recreating the survey, should it be necessary.

Traditionally, detail has been taken by station and offset (or by angle and distance) from the baseline. Features not within convenient distance from the baseline may be taken from auxiliary loops closing on the baseline. Detail such as the outline of swamp or wooded areas, meandering streams, ledge outcrops, etc., is often measured by stadia.

B) Procedures for Collecting Detail Manually

Two cloth tapes should be used, one for baseline and the other for offsets.

Stations should be made visible from a distance by use of tags, keel, etc.

A square-off (right-angle prism) should be used at stations and other points on the baseline where precision is needed.

Offsets should be taken at right angles to the baseline at whole stations, at 20 m intervals, or at 10 m intervals if required. For a baseline on a curve, detail should be taken at a right angle to the chord of a 20 meter arc interval. Between station intervals, normal is 90° to the tangent if the baseline is on a tangent, or if on a curve 90° to the chord of a 20 m arc, with measurements made to the chord.

Stationing for detail should be to the nearest 0.1 m, with offsets to the nearest 0.02 m. When using an instrument, stationing should be to the nearest 0.02 m. Offsets to property corners and other monuments should be taken by steel tape to 0.005 m. Other offsets such as wood lines, edges of fields and detail fairly distant, etc., may be taken by tape or stadia to 0.3 m.

A symbol such as a small arrowhead should be used in the notebook to show the distance measured to an object, as in measuring to center of a manhole, middle of face of curb or curb inlet, pole in concrete base, etc.

Detail features should be clearly but briefly identified, e.g., "balanced stone wall", "NYNEX #31", "0.15 m sq. SB up 0.3 m."

Standard MHD symbols and terminology should be used. Refer to Appendix A-3, Abbreviations, and MHD Highway Design Manual.

Notebook notes and sketches should not be crowded. A straight-edge or "fish" (whale) should be used. Sketches, numerals and words should be clear and neat. Detail can be stretched over double pages if needed for clarity, with cross-references for derivation of baseline, layout used, notebook number for cross sections, etc.

C) Procedures for Collecting Detail Electronically

The features to be located are virtually the same as in manual detail collection, but the surveyor must be aware that three dimensions are obtained with every shot.

The crewchief must ensure that the rodperson calls shots correctly. There are many cases where the elevation obtained will have no bearing on the DTMs, and must be noted in the EFB or notebook. Examples are shots of overhanging objects, top of hydrant, top of wall if not on level terrain, and any shot were the prism pole is not held vertical.

Feature coding (attributes), also referred to as string labels or point codes, must conform to the latest MHD feature code list. Since the list is being continually updated, it is not included herein. The latest list should be obtained from the MHD Boston Survey Office when needed.

Due to the convenience and speed of electronic data collection, many surveyors feel sketching detail is less important for EFB surveys. A notebook is required, however, and is to include sketches of the roadway and any major structures within the project limits. Most building dimensions and other data and comments which cannot be recorded in an EFB should be noted in a sketch in the same even numbered book as the traverse sketch. While the sketches will not require the minute dimensions and detail necessary for a manual survey, they should be explicit enough so that an office engineer can orient position on the electronic drawing and complete the base plan. Occasionally writing a point number on the sketch of a prominent detail shot is an aid to the office person working with the data. The importance of detail sketches is emphasized because often the designer working with electronic data does not have ready access to the surveyor who collected the data.

D) Features to be Located

Detail feature locations to be recorded at whole baseline stations and at 20 m interval stations shall include all routine topography that runs along the route such as edge of pavement, medians, traffic islands, curbing or berm, sidewalks, fences, walls, brooks, ditches, edges of swamps or lakes, tree and brush lines, etc. Objects normal to the line of the station, such as points on islands, drives, buildings, etc., should also be collected.

Detail to be picked up at odd stations (in between whole stations or interval stations) should include any changes or additions that occur to the running features mentioned above, such as the beginning or end of a feature, change of type, change of direction, or ending of walls, fences, walks, wood lines, etc. In order to correctly plot the detail in between stations, additional measurements for running features should also be collected, as several additional points are needed to plot curved or irregularly shaped pavement, curb, walks, drives, fences, etc.

Features must be described using brief, clear engineering language, e.g.: Type I, paved w.w., sloped edging, 12-inch R.C., brick retaining wall. If uncertain, use "?".

1. Buildings

Locate, measure and describe all buildings as needed for the particular project. Generally all buildings in the proximity of the baseline must be located for all construction projects.

Buildings within 30 m of the baseline may be located by station and offset to corners by range ties and angles. A combination of ties from different stations may also be used. At least one extra measurement should be obtained for a check.

A building may also be measured by angle and distance to more than two corners from one or more stations. This method is advantageous for locating buildings some distance away, and is most easily obtained by EDM.

The entire building including garages, entrances, walks, walls, etc., should be located so that measurements can be checked against one another. Steps should have the number of risers and treads noted.

The street number of building, owner (if known), type, construction, and use should be identified. Facts which may affect design such as "trailer truck terminal", or "car wash--busy" should be noted.

2. Trees, Shrubs and other Growth

   Shade trees, ornamental trees, shrubs, hedges, etc., inside the proposed layout should be located and measured. Orchards and crop areas in or near the layout should be located.

   Locate and measure individual trees where practicable. In wooded areas show wood line, describe types of trees and sizes: e.g., "hardwoods, 0.1 m to 0.3 m dia." or "pine grove, pines 0.2 m to 0.4 m dia."

   Measure single trunk trees one meter above ground level. Measure multiple-trunk trees (separating below the one meter level) where the multiple growth begins.

   In taking detail, the crewchief should keep in mind that MHD Standard Specifications contain a clearing and grubbing item for large areas, paid for by the hectare (2.471 acres), for removal of brush and trees up to medium size. There are separate payment items for removal of individual trees: from 0.25 m to 0.6 m in diameter, and over 0.6 m in diameter.

3. Public Utilities

   All utility features should be accurately located and recorded in the notebook, with sketches as needed. If utilities or related data are on different pages of the notebook from the regular detail, the respective pages should be cross-referenced.

   Telephone and electric towers, poles and other structures in or near the layout area must be located. It is necessary to know the name of the utility, and the direction and height of cable above the ground. The cable height should be measured by horizontal distance and vertical angle, but contact with cables or wires should be carefully avoided.

   Other utility detail features such as poles, hydrants, manholes, gates, etc. should be located and identified by type, company name, number, etc. Storm drains, water, sewer and gas lines, electrical and telephone conduits and other underground utilities should all be located, with the size, type and depth below surface noted.

   It may be necessary to obtain help from utility companies to get all pertinent data. If utility companies are reluctant or uncooperative in marking their equipment, lines, etc., the surveyor should locate whatever he/she can and note in the book: "Approximate locations only. Contact the utility company prior to construction."

4. Private Utilities

All wells in the vicinity of the layout should be located. The height of water from the top of well should be determined.

Private sewage disposal facilities should also be located and identified (e.g., cesspools, septic tanks or disposal fields).

5. Drainage

All drainage features found along an existing highway or road should be accurately located and identified in the survey notebook. All structures and pipes should be dimensioned. For the sake of clarity, it is often advantageous to show drainage separately from other detail, using separate sketches with cross references to regular detail notes.

Where a culvert crosses the baseline, locate the angle of intersection and determine the direction and distance to structures such as headwalls. Locate drainage and streams out to a distance of 30 m.

Drainage sketches should show not only structures such as endwalls, headwalls, inlets, manholes, but also lengths, types and sizes of pipes, location of ditches, etc.

6. Railroads

Where a railroad crosses or parallels the baseline, complete detail should be taken including type and condition of crossing, length and width, protective devices, location of rails, ditches, drainage, communication poles, etc.

It is often necessary to take detail and cross sections for some distance along the railroad baseline or random line parallel to the tracks. On important projects, the railroad baseline will be run out and connected to the survey baseline.

To eliminate the need for minus stationing, set the intersection of the roadway baseline (or other random line) at Sta. 10+00.

2.2.6 Highway Levels

As detailed in Section 1.4, Horizontal and Vertical Datums, until MHD officially adopts NAVD 88 as the vertical datum, the Department will designate either NGVD 29 or NAVD 88 as the vertical datum for use on a particular project or operation. The MHD Boston Survey Office can furnish copies of topo sheets upon which up-to-date control is plotted, as well as description cards for bench marks and other control. Bench marks are shown as squares; traverse stations, shown as triangles, often serve as bench marks also.

Two bench marks of known elevation should be recovered or placed at the beginning and also at the end of the leveling project. These should be First or Second-Order BMs. The crewchief should run check levels between adjacent benches. If other bench marks are found on the project or adjacent to it, they should be included in the level project. The surveyor is cautioned against using stray bench marks for level work on a project without first physically tying them into others to find if they are compatible. The crewchief should make a recovery report on every BM found, or searched for and not found.

Highway bench marks consist of a series of permanent and semi-permanent points which are identified by consecutive numbers or by numbers and letters. The points should be set where they will be accessible and not likely to be destroyed during construction. They should be carefully described in the notebook by type of mark, general description and station and offset. Heavy spikes (including railroad spikes) are often used as bench marks. They may be driven vertically into tree roots or horizontally into poles or trees so that they cannot be driven further in or off elevation.

Monel steel or brass plugs may be set firmly in concrete or stone. Square cuts in concrete or stone make excellent marks and are not easily confused with horizontal control. Care should be taken not to damage private property. Use of unsightly witnesses such as paint, stakes nailed to trees, etc. should be avoided.

Benches should be established at 300 m intervals or at elevation differences greater than 15 m. Structures require two adjacent bench marks; structures over water should have two benches on each shore. For surveys in urban areas, benches should be placed one or two blocks apart.

FGCC specifications for Third-Order levels are suitable for most highway work. (See Appendix A-12).

All bench marks should be actual turns and not side shots. At an end-of-run bench mark, the instrumentperson should "break" the set-up before a new backsight. Forward and back runs must be taken; single runs are insufficient.

Upon completion of the project leveling, the crewchief shall adjust the leveling to fixed benches first, and then to individual loops.

The single wire method of leveling may only be employed for the least precise work. Three-wire readings are preferable. Sample level notes are in Appendix A-6.

Equipment used in project leveling should consist of a good quality compensating-type level, rod levels and two Philadelphia or California (Frisco) rods adjusted for length. Backsight and foresight lengths should be kept about equal. The instrument person can balance sights by checking the lengths by stadia, or by pacing. Readings should be estimated to half the smallest division in most cases.

The circular bubble on compensating levels should always be kept in adjustment. A peg test should be performed once a week to ensure that the "C" factor does not exceed 7 mm per 100 m.

A compensator may stick, so the level should be lightly tapped before taking a reading. The operator should not re-level the instrument since the instrument height would change. The operator should be wary of the possibility of settlement, since the compensating level gives no indication of movement.

2.2.7 Cross Sections and Related Work

Cross sections show the vertical, or "elevation" view, of a project as compared with detail, the horizontal or "plan" view.

3-dimensional EFB-type data collection automatically produces suitable "preliminary survey" elevations provided:

* The traverse points occupied have been leveled,
* Care is taken by the operator in pointing on the prism, and
* Accurate heights of instruments (HIs), targets (HTs) or rods (HRs) are recorded and entered into the EFB.

Cross sections are normal to a baseline at regular intervals, usually 20 m or 10 m stations. Cross section data may also include odd sections, profiles, drainage and utility elevations, building sills and floor elevations, and other pertinent vertical information.

Related vertical work may include grid sections, ledge sections, different types of profiles, as well as punchings, soundings, boring layouts, etc.

Cross sections plotted from photogrammetry may be acceptable for design as preliminaries, depending on the accuracy of the mapping and save considerable field work expense. Some field work may be necessary to supplement the photogrammetry, such as for ground not visible from the air, when obscured by trees, brush, grass, water, etc., or when additional information is needed.

A) Preliminary Cross Sections

Preliminary cross sections are primarily for design, but are quite useful during construction as the basis for calculation of quantities and for reference purposes.

Where there are abrupt changes in terrain, cross sections should be taken both left and right even if the abrupt change is only on one side. Extra shots should be taken when the baseline profile changes markedly.

Features such as type of pavement, curbing, catch basins, utility structures, walls, etc., shall be clearly identified. Stations, offsets and descriptions should be consistent with the detail notes.

Typical cross section features to be identified are:

1. Crown and edges of pavement, breakdown lane, shoulder, curb, sidewalk, with readings to 0.02 m horizontally, 0.01 m vertically.

2. Walls, such as "balanced field stone", "stone masonry", properly described, showing height, thickness, etc.

3. Standard interval cross sections, noting character of ground, as "lawn", "cultivated", etc. Readings to 0.1m horizontally, 0.05 m vertically at 10 m spacings and at slope breaks, to beyond end of estimated construction limits.

4. Exposed ledge, size and location. Grid coverage may be required in large areas. During construction, ledge may be stripped and more thorough grid section coverage taken.

5. Plus and minus sections where a wall or some feature causes significant vertical change. (Slightly different offsets for the two should be used.)

6. Center of drives, dirt roads, paths, etc., profiles starting from baseline, with amount of skew, if any, noted; floors of garages or other buildings, top of loading platforms, etc.

7. Road drainage systems, showing drainage layout, manhole and

other structure elevations, inverts, sumps, and type of pipes.

8. Ground and sill elevations of buildings, or junction of siding and foundation; other pertinent elevations on buildings such as ground floor of warehouses, service centers, loading platforms, heights of openings, etc.

9. Top and bottom of steps, number of risers and treads.

10. Ground elevations of trees, which may be impacted.

11. Elevations at top of well casings, recording type, water elevation and depth, if possible.

12. Private sewage disposal components, such as cesspools, septic tanks, leaching fields.

13. Adjacent bridges and culverts, (see Bridge Surveys) showing flow lines, top of opening, roadway, water level, reporting on flood levels.

14. Culvert systems (see Section 2.2.8, Bridge Surveys). Obtain skew angle at baseline if system crosses the road, showing complete profile. A complete sketch of structures, with elevations, showing size and type of components should be taken. Brooks, streams or ditches should be located, with profile of bed, top of bank and cross sectional dimensions shown. The channel should be followed for a sufficient distance, say 50 m, and a spur line run, if necessary. Present water elevations should be obtained and any previous high water marks recorded.

15. Intersecting or adjacent roads, streets or railroad track. Cross sectioning should be done on an auxiliary baseline. At intersecting streets, gutterline and top of curbing should be taken for at least 30 m, if needed for pavement design.

16. All underground utilities that can be accessed. Elevations should be recorded and additional information obtained from utility companies, if available, on water gates, gas gates, manholes, gasoline station features, etc.

17. Overhead utilities such as power lines. Height should be taken by vertical angle only, not by throwing any kind of tape over the wires or by measuring with any type of rod, even fiberglass. No person or piece of equipment should be allowed within 4 m of overhead wiring.

B) Cross Section Methodology

Odd numbered MHD notebooks (HED 721) should be used for all vertical notes: cross sections, drainage inverts, soundings, boring notes, as well as for bench level runs.

At the start of cross sectioning, make relevant notes in index and on first page being used: town, road, job number, kind of work, activity, stations, crewchief and other survey personnel, date, page, etc. Note cross reference information such as layout name or number, baseline notebook, detail notebook, etc.

Take cross section notes from top of page down, running right hand sections increasing to right and left hand sections increasing to the left, as shown in Appendix A-6, Sample Survey Notes. Do not crowd notes and leave room for checking and calculations. Leave at least three spaces vertically between sections.

Dimensions at a point on the cross sections are shown as numerator and denominator, numerator being the offset, and denominator the rod reading.

When backsighting to a bench mark, record elevation and a brief description including station, offsets and source. A bench mark which is held should have its elevation shown in parentheses. Place HI value over a crow's foot so that it is clear which cross sections are taken from that particular HI. Clearly describe turning points so that they can be found and used again.

The instrument should be set firmly into the ground as it may be susceptible to settlement if located at a point for an extended period of time. At the end of the setup, the backsight value should be checked before making the foresight on a turning point (TP). Backsight and foresight lengths should be kept as near equal as practical.

Closure into a bench mark should be within about 0.01 m. If good closure is not affected, a new run through the TPs should be made and shown in the book. Wrong elevations can be corrected in the cross section notes by drawing a line through the old elevation and writing the correction above. All adjustments should be explained and referenced.

Features should be adequately described in cross section notes, using abbreviations where necessary. (See Appendix A-3, Abbreviations).

If a section or profile is not normal to the baseline, write "ask" for askew and show angle.

Encircle shots, which have no bearing on quantity, such as sills of houses, manhole rims, etc.

Some data such as drainage studies may be shown on separate pages for clarity, with appropriate cross reference made.

The crewchief should have other party members check the notes for such things as missing sections or elevations, calculations, etc.

C) Punchings

Punching rods are lengths of bars or pipes, which may be extended in length by coupling additional sections as the rod is pushed into the ground. Punchings are taken in soft materials like peat or muck. The surveyor pushes the rod into the peat, muck or similar material until the point of the rod hits hard ground and meets "refusal". Measurement is taken of the depth of penetration and recorded.

Cross sectioning is done first, with punchings taken on the cross section lines.

D) Borings

It is often necessary to lay out boring points for design. The surveyor locates the boring, records the boring designation and elevation of the ground at the point. If the point is on pavement, paint marks are used.

Bridge borings must be located to within 0.3 m of their specified position. If they cannot be located within said tolerance, the designer should be contacted. If they cannot be initially laid out by station and offset, they should be later redefined in reference to the baseline.

E) Soundings (see Section 2.2.8, Bridge Surveys)

Most soundings are taken in lakes and small rivers and streams, and occasionally in tidal or larger bodies of water.

Generally speaking, a baseline must be laid out on one or both shores so that the sounding can be located horizontally. On a small stream a range can be set up so that a boat can be kept on line. Range lines of soundings should be as near 90° to the thread of the stream as possible. A tape may be laid out, and distances measured directly, or a rope or wire may be extended across the stream and tagged at intervals. The baseline on the shore may be a random traverse line setup so that the soundings will be close to 90° from the center of the stream. If the stream follows a sharp curve, the soundings may be taken radially from a single point from PC to PT of the curve, with the angles of the radial lines recorded. Another method is to send the boat down a range line and have the instrumentperson locate the distance by stadia from the baseline.

For winter work when the ice is thick enough, small holes may be chopped through the ice as people do when ice fishing, so that soundings can be taken.

A water level gage is often needed near tidal-influenced water crossings. The gage should be tied in to two project bench marks. If the level of the water varies during the day, frequent reference to the gage should be made and recorded in the level notebook with dates and times of readings.

2.2.8 Bridge Surveys

Bridge surveys are basically three types: over roadway, over water, and over railway.

In the case of bridges over water, studies are made by the MHD Hydraulics Section, the Bridge Section and the Highway Design Section based on the data gathered in the bridge survey. A complete survey must be made of the stream, dam, or any other controlling features of the stream that would affect the passage of water under the bridge.

All the survey notes pertaining to the bridge and stream are to be kept on separate pages in the notebook and so indexed in the front of the book.

A) Bridge Detail

If there is any possibility that the existing bridge will be used in the construction of a new bridge, special care should be taken to accurately locate and measure the structure. If it is evident that the old bridge is in such condition or location that it has no possible use, the measurements may be taken with less care.

On the substructure, the angles of the abutments with the baseline, the location of tops and bottoms of batters, the widths of bridge seats and parapets, the location of the angles of wings and abutments, the length of wings and widths of copings should all be measured and the foundation located if possible. The type of masonry in the substructure and its condition should be noted.

The extent of detail to be taken on the superstructure is largely a matter of judgment. In all cases the general layout, such as curbs, trusses, girders, fences, sidewalks, copings, ends of bridge, etc., should be located. If the bridge is of an old truss, girder, or timber type, it will probably not be used. However, if it is of the beam and slab type and in good condition, it may be used and should be carefully measured.

While the highway plans are plotted to the scale of 1:500, the bridge plans are plotted to either 1:250 or 1:100, so that the accuracy on bridge detail locations should be far greater than on general highway work.

Levels should be taken on all parts of the substructure and superstructure, such as the bridge seats, top and ends of wings, bottom of beams, gutters, top of curbs at intermediate points and ends of curbs, tops of slabs, and on footings, if possible. All levels should be referred to NAVD 88. (See Section 1.4, Horizontal and Vertical Datums).

B) Bridge Grid

The bridge grid is taken in order that an accurate calculation can be made of excavation quantities for the proposed structure. In general, shots should be taken on a 3 m grid with additional shots as necessary for abrupt changes in contour. If the ground is fairly uniform, the shots need not be as frequent. They should extend about 20 m either side of the baseline and should cover enough ground longitudinally for any size or type of structure. The grid should be carried under the existing structure.

Wherever a railroad is crossed, the railroad baseline should be picked up and sections taken every 20 m perpendicular to the tracks for a distance of about 80 m either side of the location. These sections should cover the rails, ditches, slopes, and adjacent ground.

C) Stream

If the crewchief believes there is a possibility of a channel relocation, enough of the existing stream should be located to make a complete study of this change. In any event, the stream banks should be located for a distance up and downstream of at least 150 m. Any tributary entering the stream near the bridge site, either above or below, should be located for a distance of at least 150 m from its junction.

In addition to the bridge grid, cross sections perpendicular to the stream baseline and extending out beyond any known flood height should be taken every 20 m for at least 80 m, both up and downstream. Beyond 80 m, a profile should be taken to definitely establish the grade of the stream's bed.

For any down-stream tributary mentioned above, cross sections for 80 m should also be taken.

1. Dams

   Any dam immediately above and any below the bridge site which in any way affects flow through the bridge should be measured for the calculation of the flow. A section across the dam spillway, the abutments and the adjacent ground should be taken extending out beyond any known flood heights. The size, location and grade on any gates should be obtained. A typical section showing the shape of the spillway should be made. The approximate distance of the dam from the bridge should be noted and all levels referred to the same datum as at the bridge site.

2. Other Bridges on the Stream

   Any other bridge, up stream or down, that might affect the flow at the site should have its opening measured, and the grade of bed of stream and top of openings referred to the same datum as at the bridge site.

   If the bridge is over any part of a lake or pond, cross sections should be taken across any dam or pond outlet, as detailed in Section 2.2.8(C)1 above.

3. Water Levels

   Water levels on the date of survey and for a maximum flood should be obtained even though flood levels may be approximate. These levels should be taken immediately above and below the existing bridge and should be taken at each of the channel sections up and down stream, and beyond if necessary, to establish the top-of-water grade.

   The flood water height at the crest of any controlling dam or at any adjacent bridge should also be taken.

   If the bridge is over tide water, a tide gage should be set and continuous observations made over a period of a few days during normal tides. Observations should also be made as to whether the velocity of the water through the present opening seems excessive.

4. Character of the Stream

   Comment by the survey party should also be made as to the character of the stream, the bed of the stream, and the banks as follows:

Is the flow swift or moderate ?

Is there evidence of scour, either under the bridge or immediately below ?

Is there evidence of drift or debris in stream ?

Ice conditions (bad, moderate, or none) ?

Any evidence of harmful chemicals in stream?

Is bed of stream strewn with boulders, gravelly, sandy or muddy?

Are banks smooth, lined with trees, grass or boulders ?

Does surrounding land confine the stream or is it flat or swampy?

Does evidence indicate that the present size of the structure is inadequate?

Is the stream used for boating ?

2.3 CONSTRUCTION AND FINAL SURVEYS

2.3.1 Supervision and Administration

The Construction Division is responsible for the administration and supervision of construction projects undertaken by MHD. The responsibilities and duties of construction survey have evolved over the years and are detailed in MHD Standard Specifications, Section 5.07. The Resident Engineer is responsible for supervision of the construction work, and also for survey parties assigned to the project. While the Resident Engineer will assign the hours of employment and specific survey work, such survey parties also are responsible to the District Survey Engineer. The District Survey Engineer assigns and/or transfers parties between projects and exercises general supervision; he/she, or designee, will visit all projects periodically to ensure that proper surveying techniques are being followed and to give technical advice.

The Resident Engineer will be responsible for the time records of private parties and for endorsement of weekly reports. The Survey Engineer will process records of assignment, time sheets and payment vouchers.

Survey and Construction personnel must cooperate closely so that their respective operations will occur in the proper sequence in relation to the construction schedule. This schedule and all other project details including "basis of payment" items are the responsibility of the Resident Engineer. Close coordination is necessary to ensure that construction operations do not result in the destruction of survey markers.

2.3.2 Survey Work Performed on Construction Projects

Survey work on a construction project is performed by MHD or contract survey parties directed by the Resident Engineer, MHD Construction personnel, or the Contractor, each with specific responsibilities. Over the years, however, Survey's duties have tended to expand to include duties formerly performed by others.

The proper delineation of duties is:

A) Survey Section's Responsibilities:

* Establishment of baselines or centerlines of construction for main roadways, ramps, service roads, side streets and other major dry land items. Reproduction of baseline and centerlines, or lines offset to them when roadway cuts and fills have been completed. Levels may be taken on the points marking these lines.

* General bench mark control for the project.

* Original grade stakes at 20 m intervals.

* Preliminary and final surveys of pits (if borrow is paid by pit measure) and dredging areas, semifinal cross sections on ledge, peat, loam, etc.

* Control for structures, which shall consist of range lines on centerline of bearings or centerline of piers, face of abutments and wingwalls, horizontal and vertical control for beam seats along with bench marks close to structures for vertical control. Structures shall include but shall not be limited to bridges, culverts, dams, buildings and walls.

* Control for alignment of curbing or edging on ramps and at other complicated locations.

* Bound points and sideline stakes.

B) Resident Engineer's/Construction Section's Responsibilities:

* All necessary stakes for pipes and head walls; establish all catch basin and manhole locations as to line and grade.

* All necessary field checks on lines and grades established by the Contractor.

C) Contractor's Responsibilities:

* The Contractor shall employ qualified engineering personnel to insure adequate control and shall furnish and set stakes of the quality used by MHD for control staking. Rough stakes may be used to denote top and bottom of slopes, edge of pavement, gutter lines, etc.

* The Contractor shall furnish and set, at his own expense, all remaining stakes (such as batter boards, slope stakes, pins, offset stakes, etc.) required for the construction operations and he shall be solely responsible for the accuracy of the line and grade of all features of his work.

* The Contractor shall be held responsible for the preservation of all stakes and marks placed by or for Survey or Construction personnel. If any such stakes or marks are disturbed or destroyed by the Contractor, the cost of replacing them shall be deducted from the payment for the work as detailed in MHD Standard Specifications Section 9.05.

2.3.3 Preparation for Survey Work

The Survey Engineer should prepare for a construction project well in advance of the advertising date so that he/she will have sufficient time to collect needed data and to organize and begin survey activities.

As more construction plans are being prepared via computer aided design (CAD), the electronic files associated with the drawings may be in a suitable format to create a "stakeout" file for the electronic fieldbook (EFB). The Supervisor should obtain any available electronic data from the designer as part of his/her preparation.

He/she should obtain survey notebooks and other pertinent material such as layout plans, calculations and construction plans. He/she can assign parties to reestablish and mark up the baseline, locate and improve vertical control, place and mark offsets on grade stakes, take cross sections for construction, etc. Construction plans should be available so that he/she can locate the limits of construction, and make certain that points such as baseline ties, grade stakes, bench marks, etc., are beyond the construction area. Extra sets of plans, including cross sections, should be available after the bid opening.

Construction plans, especially those on major projects, often are quite complicated and should be thoroughly studied. Calculations should be commenced early and completely checked with discrepancies promptly resolved. The layout plans provide vital information on layout baselines, sidelines, MHB points, etc. Sideline data will have to be established, including exact location of bounds.

2.3.4 Survey Field Notes for Construction Work

When construction survey work is about to begin, the Survey Engineer will issue the preliminary survey notebooks pertaining to the job, as well as additional blank notebooks as needed, to the project survey engineer or crewchief.

Preliminary survey notes, tabulations and sketches used for design and the notes, sketches and calculations derived from construction operations are important survey records for the project. Also valuable are the horizontal and vertical control data and records of measurements that will be the basis for contract payments. These important records must be kept safe from damage or loss. When not in use, they should be locked in a fire resistant safe, if available.

If an EFB is being used to assist in the staking out, a manual notebook must be maintained as well for the reasons listed above.

Electronic data collector technology is not at a stage where the sketching can be recorded, stored safely, and retrieved reliably.

Notes in the books should be plain, accurate and thorough. A crewchief unfamiliar with the project should be able to pick up a notebook and proceed with the work. A construction engineer should be able to make his/her sketches and calculations for the project Black Book without difficulty. Those who are not specialists in engineering should be able to understand and interpret the notes.

On large projects, the Survey Engineer should keep an index of notebooks and all work therein on card files as are used in District Survey offices and all such records should be transferred to the District Office when the project ends. He/she should also keep a record of the various survey work performed.

Survey notes are legal evidence. Original information in the notebooks must not be erased. Revisions or corrections must be noted in such a way that the original is still legible, e.g. reference made to another page, or original lightly crossed out and new information added. All notes should be accompanied by identification of the survey party and date. Notes should be checked and initialed by the person doing the checking.

Baseline, layout and bound point notes and the like, may be added to preliminary survey notes in even numbered books, if there is room, or to new books. Similarly, level notes may be added to preliminary notes in odd numbered books, or in new books. The majority of the construction notes will be in level books. As two survey parties cannot work out of one book and the Resident Engineer cannot work with a book a survey party has in use and vice versa, there should be several books available on the project. Separate books are required for certain operations on large projects such as bridge surveys, borrow pits, certain types of excavations, and layout and bound points. Complete cross indexing of notebooks is important. Project notebooks will be used not only by Survey and Construction personnel who should be somewhat familiar with the contents of the various books, but also, for example, by Finals Section engineers, who will have no prior knowledge of the project.

2.3.5 Baselines

The Surveyor will be immediately concerned with the baselines. He/she should reproduce these, using the original ties (preliminary) in most cases, making all of the stationing plainly evident on the ground so that the baseline can be efficiently utilized.

The original long ties as well as short "hook" or "swing" ties that may already exist will generally be of temporary utility on the project since they are likely to obstruct construction operations.

These ties should be replaced by relatively indestructible long range ties ones that can be used for replacing the control at various stages of the project, as needed. Two sets of range ties crossing key points on the baseline are effective, as the baseline points can be quickly and accurately replaced with the aid of one or two transits and minimum taping. Reliance on a tape alone for ties can cause inaccurate results, especially if the tape is of a ribbon type and the ties are fairly long. While points can be replaced rapidly by use of an EDM, surveyors are reminded that ties are needed by others who may not have an EDM.

By referring to the construction plans, the surveyor can estimate a safe distance for ties, such as 5 or 10 m beyond slope limits for the closest point. If he/she does not yet have plans, as would often be the case before the advertising date, the limits of construction can be estimated and most long ties placed. The surveyor may be able to carefully tie in the baseline during the period from before the advertising date to the commencement of construction without being hampered by construction operations.

The surveyor should make certain there are good ties at PC, PI and PT locations as well as at control point on curve (POCs) and curve centers (CCs). There should be a sufficient number of points on tangent (POTs), which should usually be no farther apart than 150 m. Control points should be intervisible. Where possible, some surveyors prefer to run out ties at 90° to the baseline, or radially on curves. This method simplifies locating sidelines and bound points, setting offset lines, setting up curves, etc.

To protect and identify baseline points, an ample number of flagged witnesses, or "risers", with identification of the points plainly marked upon them, should be placed. Stationing should be carefully set and markings plainly written on stakes or printed on the pavement.

Where construction centerlines have to be run out on the ground, they should be calculated for closure to the baseline since they must fit both on paper and in the field. Additional calculations will likely be made, as the surveyor will design and figure ways to connect the various controls for efficient field layout.

2.3.6 Vertical Control

Bench mark (BM) control should be checked, and transferred or expanded as necessary at the same time baselines are being reestablished and tied in for construction. It is advisable to have the vertical control well established before construction begins and other activities tie up the survey parties.

The project surveyor will refer to the construction plans and check location and number of bench marks against construction details. He/she may find some have to be transferred as they will be endangered by construction operations and some may have to be added for convenience or insurance.

BMs should be witnessed and identified plainly and accurately. The stations, offsets and descriptions should be plainly indicated in the survey level notebooks.

Two BMs should be in place at critical locations such as at structures of importance, not only for construction convenience but so they can be checked one against the other. Use of temporary or poorly identified turning points as benches is discouraged. Strongly located turning points can provide additional level checks.

Bench marks which are located within the clearing and grubbing area on trees, roots or trunks, should be transferred before clearing operations. The loss of the weight of the tree may affect the elevation when the tree is cut. Other benches, such as those on boulders or ledge, could also be affected and should also be transferred.

Geodetic bench marks or any other survey marker should be preserved whenever possible. The MHD Boston Survey Office should be notified if any geodetic markers are at risk.

2.3.7 Grade Stakes

Grade stakes, also known as "construction stakes", or "side stakes", must be set prior to construction. They are stakes set in reference to the baseline station, usually placed in pairs, one left and one right, normal to the baseline and beyond the construction limits, at each 20 m station interval. An accurate horizontal measurement is taken with a steel tape between the station and the middle of the top of the stake, to the nearest 0.05 m, and levels are taken to the high corner of the stake to the nearest 0.01 m. These stakes provide horizontal and vertical control for construction operations. An example of highway grade staking is shown in Appendix A-6, Sample Survey Notes.

Although grade stakes do have great utility on a project, they cannot be depended upon for all construction control. The surveyor will find it necessary to reestablish the baseline at different stages of construction, and will need to use bench marks for vertical control.

The type of grade stake used as well as the manner in which they are laid out will depend upon the project, e.g., a superhighway vs two lane road, or an urban vs rural location. The grade stake is usually a 2" X 4" X 24" wooden stake, but in places where a stake cannot be driven or where another device would have greater utility, a hub, crow's foot, or tag nail may be used.

Where possible, grade stakes should be set so that they are on the cross section (normal to the baseline), intervisible, where they are not apt to be destroyed and close enough to the construction so that they can be readily used. The stakes should be driven solidly into the ground about half their length, with the wide face toward the baseline.

Stakes should be placed 5 to 10 m beyond the slope line, depending upon the particular project. On a small reconstruction job, for example, the stake may have to be close to the construction because of local features such as curbing, lawns, walls, etc. On a large project, the stake may often be 10 m beyond slope limits and a little beyond clearing, where it should be safe from harm.

For setting out grade stakes normal to the baseline, a right angle prism may be used for good results on short offsets, and for longer distances on level terrain. After the first stake is set, left or right, the other stake may be set by sighting line through the first stake and the baseline station. If they are not intervisible, it may be necessary to use the prism in both directions and set line stakes behind each grade stake.

The preferred grade stake location is normal to the baseline, left and right. When an obstruction is encountered, the offset stakes may be skewed near 90°, or two can be set on the same side of line.

Grade stakes that are set out during the initial stage of a large project may have to be placed so far out that they will have limited practical use for work close to the baseline. They will be used for determining locations in their vicinity, such as defining the sideline, edge of clearing area, slope limits, some drainage, etc.

Since they are not usually needed at once, elevations on grade stakes may be taken at a more convenient time such as after clearing operations have been completed. If more convenient, notes may be recorded in the staking book at the time elevations are taken.

After notes have been taken and checked, they should be given to the

Resident Engineer so the Stake Sheet can be completed.

2.3.8 Preliminary Cross Sections

The Survey Engineer or crewchief must ascertain at the initial stages of a construction project whether complete new preliminary cross sections will be needed or whether some new sections and some check sections will suffice.

Preliminary design sections, if taken originally by field methods, should generally be acceptable. Often there is a need to check, to extend or to take new levels.

If photogrammetric sections are to be used, they should be checked in the field to ensure that they are sufficiently accurate. Special attention should be given to areas in which it would be difficult to see the ground from the air because of obstructions such as high grass, brush, trees, water, etc., which might require that some new sections be taken.

Preliminary cross sections are used as a measurement basis for earth excavation, muck and peat, loam, unsuitable material and other construction payment items involving excavation or movement of materials.

Grid sections are useful for such items as ledge (Class A Rock Excavation), borrow and gravel, and work around structures.

Profiles are used for drainage, areas around structures, drives and so on, and for wider construction areas if depths are uniform or otherwise able to be calculated to needed accuracy.

Levels are also needed for profiles or cross sections for various excavation operations on construction projects. Since Survey personnel normally perform some of these operations while Construction personnel perform others, the Survey Engineer or crewchief must coordinate who will be doing which operation, and when it will be performed.

2.3.9 Cross Sections and Related Work

Several survey techniques such as normal cross sections, grid sections and profiles are utilized on construction projects as a basis for determining quantities, for checking preliminary design figures and for grade work.

As CAD technology improves, the Department may allow quantity measurements on construction to be taken using an electronic field book (EFB). For small areas, manual notes are most efficient. On large projects or borrow pits, the surveyor should discuss with the Resident Engineer and the District Survey Engineer the best method to obtain the quantities. Survey measurements for quantities are usually for calculation of volumes although areas and linear dimensions are also calculated.

The Survey Engineer (and/or crewchief) and Resident Engineer must cooperate so survey work will be timely and in accordance with the construction schedule and the methods of measurements to be employed. Consideration must be given not only to the MHD Standard Specifications and Construction Manual but also to the project plans, specifications, special provisions, agreements among the parties, etc. Regardless of one's background, it is often difficult to anticipate all of a project's survey needs.

Cross section work will utilize the techniques described in Section 2.2.7, Cross Sections and Related Work, with modifications as needed.

2.3.10 Semifinal and Final Surveys

In order for the Resident Engineer and/or the MHD Finals Section to compute quantities of material measured by cross section, the "before" (prelims) and the "after" (finals) sections are needed. Both the preliminary and the final shots should extend to at least the distance that will be designated as "old ground" in the finals (abbreviated "O.G." in the notebook).

Normally, final cross sections are taken "over the footsteps" of the preliminary. If the original sections were taken at 20 m stations, the finals should be also. Preliminary askew or odd sections should also be repeated.

There are several Construction "pay quantities" and "methods of measurements" that eliminate some Survey work entirely. The Resident Engineer should inform the Survey Engineer or crewchief when work normally performed by Survey personnel will not be necessary because alternate means of obtaining necessary data or measurements are being utilized. An example of a deviation from the general procedure is for "cut and fill" sections: while Survey would normally expect to take cross sections for finals at cut sections or mixed cut and fill sections, it may or may not be necessary to cover fill sections.

On ledge sections and other sections, which may involve calculation for payment to some predetermined pay line, fully executed final cross sections, may not be required. Ledge, for example, involves a pay line at a specified distance below the finished pavement and shoulders plus a specified distance beyond a slope line on the sides. Even though the prelims were taken on a grid, the finals in the roadway area can be covered by regular 20 m cross sections. However, grid sections are necessary on remaining ledge at slopes to old ground.

Many items on the project can be measured without cross sections. Profiles can often be utilized for driveways, areas around structures and culverts, and also with some materials such as dense graded crushed stone, gravel sub-base material, impervious soil borrow, sand loam, pavement material, etc. Construction personnel can measure and calculate such quantities with little or no Survey assistance. Area and linear measurements of quantities do not usually involve Survey, nor do measurements involving weight.

Surveyors assigned to construction operations should be familiar with the MHD Standard Specifications and Survey Manual so that conventional practices will be known. Discussion with the Resident Engineer will be needed so that Construction's needs for preliminary and final measurements are efficiently met.

2.3.11 Borrow Pits

Borrow is earth material such as topsoil, ordinary borrow, gravel, sand, etc., taken from outside of the project layout for use on the project.

The preferred method of measuring borrow is to measure it in place on the project. When that method is not practicable, the Resident Engineer may request Survey to measure the borrow pit. The general procedure in measuring a pit is to take elevations in a grid pattern after the pit is stripped of non-payment material and to take elevations over the same grid pattern after material has been removed.

Depending on conditions and types of materials, other methods usually not requiring Survey are available and described in the MHD Standard Specifications.

A) Preparation

Property lines should be well marked to prevent encroachment. This is the Contractor's responsibility. Pit limits should be defined and marked in a prominent manner.

The Resident Engineer and Survey Engineer should agree on procedures for clearing and stripping. It may be desirable to place some main control lines after clearing and before stripping and to have the pit stripped before the grid is laid out and sections taken. Stripping will be inside the pit limits, and sections taken over them.

B) Notebooks and Sketches

A pit must have its own odd numbered notebook, to be used for all pit work, sketches, control work, cross sections, etc. A complete sketch and description of the pit should be entered into the notebook, showing its location in reference to highways, baselines, its ties, and other data. All horizontal and vertical work should be recorded.

C) Pit Baselines

The main baseline should be located longitudinally through the pit, parallel to side banks and normal to end banks. It should extend well beyond pit limits. It should be tied into a local road, if practicable.

If the main baseline has to be assumed, as it would be at an isolated location, magnetic bearings may be used for the assumed direction.

If a pit abuts or is close to the project, it should be tied into the project control (baseline) with a traverse that can be closed.

Baseline control for the pit should be well tied in so that it can be reproduced in the future.

D) Bench Marks

Two or more bench marks should be established at safe locations at the pit. If the pit abuts or is close to the project, the bench marks must be on the same datum as the project vertical control. At least two bench marks on the project should be used. If the project is not close enough to the pit other bench marks may be used.

Where there are no known bench marks available, an elevation may be assumed. The assumed elevation should be a high enough number so that no pit elevations will be less than zero.

Leveling should be run highway style, closed and adjusted.

2.3.12 Bound Points

The Survey Engineer should begin computations during the initial stage of the project and set bound points as time and construction conditions allow.

The baseline, control traverses and bound point data should be on an accurate coordinate system. On small projects the crewchief can usually make needed computations with a hand calculator, but most coordinates for larger projects should be available.

A complete record of the bound work including the location of all bounds, dates set and by whom, angles and distances used to set them, all ties, etc., should be entered in an even numbered notebook.

As bounds are permanent witnesses to the legal layout as well as an aid in reproducing the baseline in the future, they should be accurately and carefully located on the ground. Bounds are defined in position on the layout plan by station, bearing and distance, or by coordinate value. Although they are mathematically accurate, they may not be ideally located from Survey's point of view. Although it may involve many calculations, the surveyor may desire to set the bound points "indirectly", from a nearby control point on the baseline or from a control traverse point close to the bound.

Bound points, especially those set indirectly, should be checked independently of the method used to set them. Checks can be made to other points, to the baseline, or to another bound.

A bound point set by Survey is usually a hub with a small nail or e-pin, with the hub set solidly into the ground. Close ties 2 m or so from the point may consist of hook ties to stakes or trees. Use of straddle stakes, from which string lines may be run to cross over the point, are usually preferred by the worker actually setting the bound. Strong, long ties should also be utilized to enable crew members to check or relocate the point.

Tie distances should be written on stakes and other ties. A witness stake, with bound identification written on it, should also be set.

"Pinning" a bound usually is done when the project is nearly completed and after the bound has had a chance to settle. An escutcheon pin is set in lead in the drilled hole on the bound and is an indication that the surveyors have made a final check on the point and that the location is accurate.

The bound should never be pinned from short ties. The surveyors should run the bound point in again from the baseline or from good control and check the work against the long ties.

2.3.13 Culvert and Bridge Surveys

Section 2.3.2, Survey Work Performed on Construction Projects, details work to be performed by Survey and Construction personnel and by the Contractor. It is sometimes necessary for Survey personnel, at the direction of the Resident Engineer, to perform work normally considered Construction's duties. Survey should not, however, perform work that is the Contractor's responsibility.

Survey should furnish necessary staking for pipes and head walls and establish line and grade for catch basins and manholes. Survey may also be directed by the Resident Engineer to check control work done by the Contractor.

Section 5.07 (E) of the MHD Standard Specifications states:

> Control for structures, which shall consist of range lines on centerline of bearings or centerline of piers, face of abutments and wingwalls, horizontal and vertical control for beam seats, along with bench marks close to structures for vertical control.

In many cases the surveyor will have to extend lines for a considerable distance to avoid destruction of control from construction activity.

2.3.14 Bridge Staking

Starting with the baseline, centerline of construction, or whatever line is used on the bridge plans to locate the structure:

a) Stake out, range and tie in the centerline of each abutment and pier.

b) Stake out, range and tie the concrete line of the wingwalls. Place a stake at the intersection of the concrete line of wingwall and face of abutment.

c) Stake the position of each outside beam, measure space and compare with plans.

d) If structure is not at right angles, compute square span from centerline of bearing to centerline of bearing. Check-measure these as staked out.

e) Measure distance from concrete line of wingwall to concrete line of wingwall along line of face of abutment and compare with plans.

f) Measure overall length of piers and compare with plans. Make a sketch showing how the structure was laid out, ranged and tied. Start with baseline measurements that were used to arrive at plan stations. Enter the information in an even numbered notebook, with check measurements shown on the sketch.

2.4 SURVEYS FOR PHOTOGRAMMETRY

The Department generally does two types of surveys for photogrammetry projects, a survey to establish horizontal and vertical control and a field-check survey to determine the accuracy of a mapping submittal. The control should be established first to at least Third-Order standards, then the photo check points should be obtained from the adjusted control.

Cloth targets or targets painted on the ground are the desired means of controlling aerial mapping. They provide a sharp, well-defined point and do not require photo interpretation by the surveyor in the field. Disadvantages to targets placed prior to flying are they may be removed or destroyed prior to flying, may fall in shadows when flown, take additional time and more points than necessary must be placed to ensure enough for each model.

Common practice is to have the project flown when conditions are optimum for the photography and then have the Photogrammetrist select suitable points from the photography. In this case the surveyor is given a set of contact prints with selected points, pin-pricked and circled on the back, with a description. Some points may require either a horizontal or a vertical position and some will require both.

More discrepancies in ground control arise from incorrect identification of points on the photograph than from erroneous survey data. If there is any doubt in the surveyor's mind as to whether the point selected on the photograph can be identified on the ground, has been disturbed, or is otherwise undesirable for use as a control point, another nearby point which can be clearly identified on the ground and on the photograph should be selected. When an alternate point is selected, the surveyor must note the new description on the photograph and in the fieldbook, and provide enough details to eliminate any confusion.

2.4.1 Field Procedures for Photogrammetry Control

The ground control survey must be accurate if the photogrammetry is to be accurate. While higher order surveys are preferable, Third-Order traverses and levels are usually adequate. Whenever possible, horizontal control points to be used for the mapping should be incorporated into the traverse. Other required horizontal positions may be obtained by side shots provided there is suitable redundancy to ensure accuracy. A minimum observation would be two complete sets of two readings, one direct and one reverse on the backsight and the same on the side shot in each set, with a mean of five EDM distances in the direct position and five in the reverse.

Vertical control point elevations should be obtained using level and rod only. Prior approval of the MHD Boston Survey Office and the Photogrammetry Engineer is required for use of a total station to establish control elevations electronically.

2.4.2 Field Procedures for Checking Photogrammetry

The mapping that is generated based on the control points established in the above procedure will need to be checked for accuracy. The purpose of this field verification is to determine if the stereomodels have been set up properly and the mapping compiled accurately. Objects or selected points that are plotted on the mapping must be verified horizontally and vertically.

The surveyor will be provided with a set of preliminary prints of the mapping. The number and spacing of check points will be designated although the surveyor will have some leeway in choosing locations to expedite the work. Checking vertical positions by level and rod and checking horizontal positions by tape usually accomplish this work. Because most MHD photogrammetric mapping is oriented along highway corridors, it is flown and plotted in strips where lateral dimension checking can be done by taping relatively fast.

The locations checked and the results should be noted in red on the prints and in the fieldbook and returned to the MHD Boston Survey Office so that a table of field results can be compiled.

SECTION THREE

SURVEY TECHNIQUES

3.1 CONVENTIONAL FIELD SURVEY EQUIPMENT

3.1.1 Capsule Description of Conventional (non-Global Positioning System) Surveying Instruments

a) Theodolites/Transits

These instruments are primarily to measure horizontal and vertical (zenith) angles and to establish line and grade. While the major difference between the instruments is that the reading system is external on a transit and internal on a theodolite, both instruments perform similar functions and will be collectively referred to as theodolites. Horizontal angles are derived from horizontal circle readings and vertical angles from vertical circle readings. Stadia hairs, used in conjunction with a leveling rod, may be used to obtain horizontal distances and elevation changes. Electronic theodolites have digital readout, which relieves the surveyor from reading scales or micrometers.

b) Electronic Distance Measurement Instruments (EDMs)

An EDM derives a slope distance from the instrument to a reflector by measuring the outgoing and incoming wavelength of various electronic sources. An EDM is generally used in conjunction with a theodolite to derive horizontal distance and elevation change from the slope distance and zenith angle. Many EDMs mount directly onto theodolites. While some newer EDMs can measure a distance to an object over short distances (less than 50 m.) without the use of a prism, use of a prism is required for all MHD work.

c) Levels

The level is used to determine differences in elevation; its line of sight is fixed at horizontal when the instrument is leveled. When a leveling rod is observed, the elevation change from the instrument's line of sight down to the bottom of the rod can be obtained. Levels are often equipped with stadia hairs for distance measurement and often have a very rough external scale for horizontal angle measurement.

There are three basic types of levels: spirit, compensator and electronic. Electronic levels have a compensator and utilize digital readout to read a barcoded invar level rod to determine elevation differences and distances from instrument to rod.

d) Tapes

A tape can be of cloth, fiberglass or steel construction and is used to measure horizontal distance between two points.

e) Total Stations

This instrument is an integrated theodolite/EDM with digital readout of horizontal circle, zenith circle, and slope distance. Most total stations have a variety of on-board software for computation of horizontal angle, horizontal distance, elevation change, coordinates, and coordinate inversing. The total station has serial communications so an electronic field book (EFB) can automatically store desired information and perform some instrument commands such as zeroing out the horizontal circle. Some instruments have EFB on-board capabilities with data storage primarily on PC-MIA compatible cards, very durable hard disks which are the equivalent of structurally-sound floppy disks and are a rugged mechanism for data storage under field conditions.

3.1.2 Detailed Description and Applications for Conventional (Non-Global Positioning System) Surveying Instruments

A) Theodolites

This discussion on theodolites is also applicable to transits, which are similar instruments, and to total stations, which contain a theodolite as an integral part.

Most theodolites require a tripod different than that used for transits. The type of tripod used for levels is also suitable for most theodolites.

A tripod is a three-legged device; the legs are usually constructed of wood or aluminum. The end of the legs contains a pointed device connected to a piece of metal which one can step on to secure the legs by pushing them into the earth. In cases where the legs cannot be pushed into the earth (such as on a concrete surface), care must be taken to continuously monitor the stability of the instrument. Sandbags or similar items can be used to secure the bottoms of the tripod legs or devices such as tripod triangles or chains can be used to ensure stability. Wooden footpads can be used to prevent settlement on bituminous or similar surfaces.

The tripod has a flat mounting surface at its top to which the bottom plate of the theodolite is secured via a knurled threaded knob on the tripod. The knob threads directly into the bottom of the theodolite's tribrach base for a secure fit between the two devices. Tribrachs, which are detachable from the theodolite, are very useful in "leap frogging" in traverse operations, a procedure that eliminates many problems with measurement of height of instrument and target.

Tripods either have fixed length legs or adjustable legs, which can be withdrawn or extended. While the former ensures stability, the latter is more commonly used because it is adaptable to different instrument heights, undulating ground surface and efficiency in setting up over a point. Twist knobs are one of the standard mechanisms for release and tightening the devices, which allow legs to be extended or contracted. Tripods are approximately 1 meter long when contracted and 2 meters long when fully extended.

A theodolite often has to be set up so its vertical axis is directly over a survey point when the instrument is level. On older instruments this was accomplished using a plumb bob, which was attached to a hook set directly below the theodolite. The optical plummet has generally replaced this method, which is now standard for most instruments. The optical plummet is a mechanism, which allows the surveyor a visual line-of-sight vertically down from the center of a leveled instrument. A right angle prism is usually part of the optical plummet system so that the surveyor can look horizontally through an eyepiece and actually see vertically down from the center of the instrument to the survey point. A precise pointing mechanism, usually a bull's eye circular cross hair, is part of the viewing system of the optical plummet. The cross hair can be focused separately from the ground. The same leveling and setup principals apply to prisms and targets mounted on tribrachs and tripods. For targeting systems, which do not have a vial bubble, one must rely solely on a bull's eye bubble; its proper adjustment is critical.

A theodolite's two functions are to measure horizontal and zenith (vertical) angles. The horizontal angle is derived from the difference between two horizontal circle readings of two independent lines and the zenith angle is derived from zero on the vertical circle being at the zenith of a leveled instrument. Some theodolites measure vertical (above or below a horizontal line) angles instead of zenith (down from straight up) angles. While the angular readout can be in grads and/or mils, most instruments have angle measuring systems, which are degrees-minutes-seconds, based. Decimal degrees are not standard in surveying practice except as an intermediate value for computational purposes.

A viewing system for pointing the instrument is usually a telescope with a viewing focus, which rotates both vertically and horizontally about the intersection of the instrument's horizontal and vertical axes. Horizontal and vertical cross hairs, which should have a separate focus device, are used for precise pointing. The vertical cross hair usually contains stadia hairs for estimating horizontal distance and elevation change when used with a level rod and zenith angle.

There are several methods to determine the angular accuracy obtainable from a theodolite or total station, the least reliable of which is the Least Count method. The least count of a theodolite or total station is that value that can be read or recorded without interpolation. This was a reasonable determination for older vernier-type transits but it is inadequate for modern theodolites. Certain scale reading theodolites have a least count of 1' but can be easily and accurately interpolated to 0.1' and often have a standard deviation on one set of angles of under 3". Other theodolites may have a least count of 1" but are found to have a Standard Deviation (SD) of about 5".

An indicator of an instrument's accuracy that is usually reliable is the accuracy rating provided by the manufacturer. The owner's manual for most instruments lists an angular Standard Deviation, which the instrument is capable of achieving. This value is based on the International DIN specifications, which have standardized tests and procedures for rating theodolites and total stations.

Probably the best method is the field test that determines the instrument's Smallest Measuring Ability (SMA). The instrument's SMA is obtained by taking repeated measurements over lines of 200-400 meters in length. The test should include four well defined sights, with at least four direct and four reverse readings and repeated at least three times. The standard errors for the three tests should be relatively consistent, or testing must continue until consistency is achieved. The standard deviation becomes the instrument's SMA. The least count for an instrument could be smaller or greater than its SMA, depending on how the least count relates to the internal optics and motions of the instrument. SMA is similar to the DIN specifications for theodolites and total stations but with less rigidity in the testing process.

The theodolite contains tangent (motion) screws for both horizontal and vertical pointing. When loosened horizontally, the instrument revolves around its vertical axis freely. The telescope freely rotates vertically when the vertical motion is loosened. When secure, each motion has precise pointing mechanisms called slow motion tangent screws. These are usually "built in" components of the general tangent screws.

Repeating theodolites are the most common type. American-style transits are repeating instruments as are some total stations. Repeating, or doubling angles is an important procedure when an instrument has a high SMA. To get a precise reading, angles turned should be added and calculated so that the final value is obtained by dividing the sum by the number of times the angle was observed. This is the best method when an instrument's SMA is 10" or above.

Directional theodolites differ from repeating instruments in that there is no lower motion so that angles are being measured continually. Most precise theodolites and total stations are directional instruments although some total stations have modes, which enable angles to be summed. Most instruments and data collectors are set up for directional angular input. One angle or set is defined as the average of one direct and one reverse angular observation. Project needs dictate how many sets of angles must be observed.

With modern total stations and electronic fieldbooks, the need for a lower motion is not critical and has been eliminated on many instruments. Likewise, many precise older theodolites were called positional or directional instruments where horizontal angles were derived from the circle readings and were not supposed to be used for layout or angle doubling. These instruments were designed for control traversing where zeroing out an instrument or retaining an angular value were not critical concerns.

Leveling a theodolite is accomplished by using three leveling screws, which are usually between the bottom plate of the tribrach, which attaches to the tripod and the rest of the theodolite. Most instruments contain a bull's eye bubble for rough leveling and 1 or 2 alidade bubbles for more precise leveling. General practice is to level the bull's eye bubble and then the alidade bubble(s). Turning the leveling tangent screws means the vertical axis of the instrument will be moving off the point on which it is centered.

Many newer theodolites and total stations have axis compensators, which will attempt to keep an instrument leveled once manual procedures have been performed.

Nearly all theodolites allow the instrument's scope to be rotated in a complete vertical circle. Considering that an instrument's vertical circle is zenith-based with zero on the reading scale straight up, in one direction downwards (direct or circle left) the scale reads from zero to 180° (straight down) while in the other direction (reverse, indirect, or circle right) the scale reads from 360° (equivalent to zero) down to 180° degrees straight down.

Measuring horizontal angles by averaging an equal number of times in direct and reverse eliminates both vertical and horizontal collimation errors. The same is true for zenith angles except a simple average cannot be used because the correction will be of the same sign to both direct and reverse readings. In a perfectly collimated instrument, the sum of a direct and reverse vertical reading will be 360°00'00". As an example, precise direct and reverse pointings result in zenith angle readings of 89°59'20" and 270°00'20" respectively. The sum is 359°59'40" and indicates ten seconds should be added to each reading.

B) Electronic Distance Measuring Instruments (EDMs)

EDMs, developed from experimentation by physicists trying to measure the speed of light, have revolutionized surveying since for most purposes the tedious job of measuring distance in tape lengths is no longer required. In addition, combining an EDM with a theodolite's zenith angle produces both horizontal distance and elevation change from the instrument's center to the reflector. Measuring height of instrument and height of reflector above occupied points enables the aforementioned elevation change to be converted to an elevation change from instrument-occupied survey ground point to reflector-occupied ground point, often referred to as a "mark to mark" elevation change.

A sequence of mathematical operations may be required for reduction of a slope distance/zenith angle in combination with height of instrument/height of reflector. The slope distance may require correction for temperature, pressure and combined instrument/prism offset. The four values are used in conjunction with earth curvature and atmospheric refraction correction to obtain a horizontal distance and a mark-to-mark elevation difference. The horizontal distance may require reduction to the ellipsoid and a scale factor applied to obtain a grid (state plane) distance.

Care must be taken if the EDMs vertical axis is offset from the theodolite's similar axis. This requires an instrument-offset correction, which needs to be applied to all measured slope distances.

It is also necessary to consider that the distance is measured from the EDM while the zenith angle is measured from the theodolite, which is usually below it. Since the distance from the EDM to the theodolite is very small, it usually only requires attention on short lines (less than 30 meters) which have unusual zenith angles (deviations from horizontal of more than 30 degrees). On most total stations the EDM and theodolite center are vertically coincident and thus this offset is eliminated. The height of instrument should always be measured to the optical center of the theodolite, as that is where the zenith angle is measured.

EDM-produced slope distances are affected by temperature and pressure. "Corrections" for these variables are scale factors, which are multipliers to the measured value and are so near a value of one that lines less than 200 m in length are usually affected less than 0.001 m. The correction can be considered insignificant when measured lines are shorter than 200 m. Before an EDM is used, one should test it with some temperature and pressure extremes to see when the corrections become significant. In control traversing, it is imperative that correct temperature and pressure values are entered and that atmospheric corrections are made.

The corrections are computed through equation or vendor-supplied nomographs which require user interpolation. Most total stations allow field-entered correction values so that the distance measured will already be corrected. Some total stations now have on-board thermometers and pressure gauges, which communicate directly with the EDM so the user does not have to enter any correction values.

EDMs require a prism at the point to which the measurement is taken. Standard prisms are approximately 0.07 m in diameter while "peanut" prisms, used for some short line measurements, are approximately 0.02 m in diameter. In control traversing, long lines may require the use of more than one prism.

Larger prisms are secured into a holder, which mounts on a prism pole, or into a tribrach, which can be mounted on a tripod. The holder usually can secure the prism with a zero offset or a 0.03 m offset. It is of utmost importance that the combined affect of EDM instrument offset plus the prism offset be properly determined. Most newer model EDMs have internal dip switches or dials which enable input of the combined offset correction.

Once the combined offset has been entered, it is important to check its integrity. A distance of approximately 30 m should be taped on flat ground as a baseline comparison to the EDM-measured value. If these two values are in disagreement beyond random error limits (more than 0.005 m) it is likely that the wrong correction was dialed into the EDM and needs to be corrected. Offsets can change by a small amount just because of everyday jostling of equipment.

Prism poles experience a significant amount of hard field use and should be checked routinely by this baseline process. A significant departure from the taped value would indicate the bull's eye bubble on the prism pole is in need of adjustment.

It is useful to have a well-monumented baseline readily available for checking EDMs. It is also important to take an EDM to the nearest calibration baseline on a routine basis (2-4 times per year) to check its measuring ability over a variety of line lengths. Tribrachs and optical plummets should be calibrated before the baseline is visited. (See Appendix A-8, Massachusetts Calibration Baselines).

If prisms and tribrachs are interchanged between crews and/or total stations, it is extremely important to perform a quick distance comparison to a taped distance. The most common error in EDM use is incorrect prism offset.

C) Levels

The level is the standard instrument for determining elevation differences, especially where the accuracy derived from trigonometric means may not be sufficient.

Most levels are defined as either spirit or compensator (also called "automatic"). The line of sight for a spirit level is determined by leveling a tubular spirit level attached to the instrument. The sensitivity of the liquid bubble determines to a large degree the precision of the instrument. There are many different models depending on the precision and accuracy desired.

The controlling mechanism on Dumpy and Wye levels, as on a transit telescope, is a spirit level. The instrument used for most of the first-order level runs performed by United States Coast and Geodetic Survey during the early to mid 1900s was a Fischer Level, a Dumpy Level modified for that purpose.

The Spirit Level has generally been replaced by the Automatic (or Compensator) Level because of its simpler operation and easier-obtained accuracy.

On an Automatic Level the line of sight is automatically kept horizontal by the compensator, a set of prisms or mirrors that swing freely in response to gravity. The compensator may swing from wires or be attached to a pendulum or flexible spring. The cost and accuracy of Automatic Levels cover a wide range from basic construction levels to sophisticated first-order levels with reversible compensators and built-in parallel plate micrometers.

Most precise leveling performed since the 1950s was performed with automatic levels equipped with a built-in or add-on parallel plate micrometer. The micrometer attaches to the objective lens of the instrument and permits the level rod to be read more precisely. This increases the accuracy attainable by any level and enables most levels to be used for precise leveling applications.

A theodolite or total station can be used as a level provided the collimation of the instrument is correct, that is the line of sight is level when the zenith angle is 90°00'00" and 270°00'00".

Levels are generally not set precisely over survey points and thus have no optical plummet. Most levels have the capability for use of a plumb bob for rough point centering. Levels mount on tripods in the same manner as theodolites. The telescope on a level can only rotate horizontally because preservation of a horizontal line of sight is critical. The telescope has both an image and a cross hair focus. Most levels have a horizontal slow-motion tangent screw similar to that on theodolites. The vertical cross hair contains a distinct center position with upper and lower stadia hairs. The stadia constant is usually 100 but should always be checked. The stadia hairs are utilized to check the middle cross hair reading and to estimate the distance from instrument to the level rod.

Rods used for conventional leveling are usually of wood construction with a metal strip attached to the face with either centimeter or half-centimeter graduations. The rods can be comprised of either two or three sections with a total length of either 3, 4 or 5 meters. Telescoping fiberglass rods as long as 7 meters are available but these are primarily used for the lowest precision work.

Standard level rods have direct reading ability generally between 0.001 m - 0.01 m depending on the type of rod being used for a particular application. Interpolation between least count can be estimated by observation or can be improved with the use of a vernier system, which attaches to the rod. Level rods are approximately 2 m in length contracted and 4 m in length when extended. A level rod should be vertical when being read, a procedure made easier by attaching a bull's eye bubble called a rod level. When a rod level bubble is not available the rod can be rocked back and forth in the direction of the level with the instrument operator reading the smallest value as the vertical reading.

The level rod is an important part of the leveling process. The most precise leveling instrument cannot provide high-order control unless matched with a proper rod. Most precise applications require a one piece invar leveling staff, usually 3 m in length, of which there are two types, single scale and double scale. The scale on single scale rods consists of a delicate invar strip with 0.005 or 0.01 m graduations. Double scale rods are the most precise. They have scales on both sides of the rod, one side set at 0.0 (low scale) and the other side set at a known offset (high scale). While high accuracies are attainable on a single run using specific leveling procedures, FGCC First-Order, Class 2 and Second-Order specifications usually require double runs. Single-run lines may be acceptable for some First-Order, Class 2 and Second-Order applications if the invar rods have different offset values between high and low scales. Having different scale differences allows detection of the most common error in precise leveling, switching backsight and foresight readings. Other important components of a precise leveling process include portable turning points (plates or drivable pins) and braces to keep the rod vertical.

For those levels, which read rods automatically, the invar staff is precisely bar coded. It can only be read under conditions conducive to a normal user's visual reading of a level rod. This type of level contains on-board software for data collection and also has the capability to connect with an external electronic fieldbook system.

D) Tapes

Prior to the development of EDMs, the tape was the standard means for precise distance measurement. While many standard field procedures now use the EDM for distance measurement due to its efficiency, certain field operations still warrant the use of a tape. An example is measurement of bridge components less than 30 m in length. Steel tapes are used for precise measurements of short lines, where they provide a more accurate determination than an EDM or total station.

Steel and coated steel tapes are standards for precise measurement because of their durable nature. Cloth and fiberglass tapes are much easier to maintain and are used primarily where high accuracy is not a concern. A typical steel tape is 30 m in length with a least count of 0.001-0.005 m. Cloth tapes vary in length but rarely exceed 30 m due to comparatively high elasticity.

Any tape needs to be routinely checked against some measured baseline because normal use will eventually affect its length. Tapes are also subject to the systematic errors of temperature, pull and sag. These errors require correction, if deemed significant in size, for a given measured length. On short lines (less than 15 m) the systematic errors are generally smaller than the survey accuracy requirements.

E) Total Stations

A total station is designed as an integrated theodolite/EDM system. This means the EDM is not mounted on the theodolite and, on most total stations, the EDM is co-incident with the theodolite's line of sight.

The above discussion on theodolites and EDMs applies directly to total stations in terms of components and measuring system clarification. The slope distance, zenith circle, and horizontal circle are all provided by digital readout.

All total stations come with on-board software, which is accessed through buttons and keyboards. Typical on-board software provides for zeroing of the horizontal circle, reducing slope distance and zenith angle to its horizontal and vertical components, calculation of a remote distance, identifying angle and distance repetition errors, and calculating coordinates as measurements are made. Care must be taken since these calculated quantities are often derived from single measurements, which may not be corrected for systematic errors, and also that coordinate calculations have a consistent directional orientation defined.

All modern total stations have the capability to interact with a data collector or electronic fieldbook (EFB). Manufacturers often provide their own EFB systems or third party EFB systems can be utilized with a variety of total stations. Some total stations have integral on-board EFB systems, which eliminate the need for a serial communication cable and external EFB hardware and software. PC-MIA cards are becoming the standard for EFB storage of the collected field information or coordinates provided from the office computer for layout purposes.

3.2 FIELD OPERATIONS WITH CONVENTIONAL SURVEY EQUIPMENT

MHD control surveys follow Federal Geodetic Control Committee (FGCC) guidelines (See Appendix A-11) for procedures, instrumentation and monumentation.

Different job requirements dictate different field equipment and survey procedures. The description of a point and its 3-D coordinates are the desired data for some types of survey. In route surveying, it is necessary to relate a point to a defined centerline and to have much of the collected data linked to a defined project centerline by station and offset.

3.2.1 - Capsule Description of Some Types of Surveys

a) Traverse

Any form of traversing is a control-type survey without the control survey's high-accuracy requirements. Traversing is loosely defined as a procedure for obtaining 2-D or 3-D coordinates for survey stations where vertical data is not necessarily desired. Traversing, along with GPS, often provides the coordinate base necessary for surveys, which produce topographic information.

b) Differential Leveling

The use of a level for control-type surveys determines the elevations of points, which will later be used for topographic purposes. While traversing and GPS can produce elevations, they are often not of sufficient accuracy. Differential leveling is the proven means of obtaining highly accurate elevations.

c) Topographic Data Collection

The elevations of topographic points and lines are determined and collected and may or may not be used in a ground model. Attributing of this information is especially important in linking the data to the proper symbol from the electronic plan's library of symbols. Additional attributing can describe the significance of the point (tree type, diameter, crown width, etc.) and must appear as text information in topographic representation.

d) Digital Terrain Model Data Collection

Spot elevation and break line data is collected for the generation of a Triangulated Irregular Network (TIN), which produces the Digital Terrain Model (DTM) of the ground surface.

The DTM is extremely useful as any type of elevation information (contours, profiles, cross sections, 3-D views, etc.) can be generated from it.

e) Cross Section Data Collection

If a DTM or contours are not desired products but a route design will require some volume calculations, collection of cross section information at defined intervals along a centerline(s) can be desired survey information. Various procedures can be used to collect this information.

f) Stakeout

As opposed to survey data collection, stakeout involves the field layout of specified points, which were calculated in the office. These could include lost or new boundary locations, centerline or right-of-way points, building layout, etc. If elevation is a desired product, one must be able to locate the desired elevation (up or down) relative to the surveyed point (e.g., cut or fill elevations on grade stakes).

3.2.2 Detailed Description and Applications for Some Types of Surveys

A) Traverse

Traversing is the usual form of conventional survey control extension. Traverses are simply interconnected lines of distance and angle measurements.

Most MHD traverses start and end on pairs of geodetic control points of known state plane coordinate values. The initial and terminal bearing or azimuth are computed from a grid inverse on the control points. Some traverses may start or end on a single point which has a known grid azimuth to a reference object (spire, water tank, tower, etc.). Any other control points, which lay in or near the path of the traverse, should be incorporated into it.

The two primary forms of traverse are the loop (closed) and the link (open ended). The loop begins and ends at the same point of known coordinates. The initial bearing for the traverse can be derived from coordinate inverse, through astronomic observation, or in rare cases is assumed (a practice not recommended). The link traverse starts and ends at two different points of known coordinates. Determination of the starting bearing can be similar to that of the loop, or it can be derived from the bearing between the known coordinates of the endpoints. Computationally, this last procedure requires starting on an assumed bearing. When the end station is reached, the traverse can be rotated to the bearing defined by the link traverse endpoints although MHD does not recommended the practice. Since simple link and loop traverses can close well despite systematic errors, they should be checked carefully.

In route surveys, the control traverse often becomes a series of interconnected link and loop traverses. The surveyor should make additional angle and distance measurements between intervisible stations whenever possible to increase the number of checks on the accuracy of the survey and add more geometry to the survey "network." The traverse should tie into all control points in the area so that the integrity of the existing coordinates is verified.

When traversing, prisms should always be mounted on tribrachs rather than prism poles to ensure quality measurements. Whenever possible, a prism should be mounted on a tribrach on all backsights. This provides a check on the change in distance and elevation shot when the backsight station was occupied and also when the presently occupied station was foresighted. It also eliminates some environmental systematic errors when the elevation difference measured in opposite directions is averaged.

If elevation is a desired product of the traverse, it is critical to measure precise heights of instruments and heights of targets at all setups and sighted stations. It is also important to sight prisms precisely, both vertically and horizontally, and to ensure that the instrument's vertical collimation is correct and level at 90◦00'00" and 270◦00'00".

Theoretically sound practice is to measure both angles and distances an equal number of times in the direct and reverse (D&R) positions and then to average. This eliminates the instrument's horizontal collimation errors. The average of a D&R reading is considered one repetition or set.

Requirements for a particular job will dictate needed accuracies for the angles and distances, an acceptable least count for an instrument, the number of repetitions required, closure limits/precisions, monumentation, etc. A common angular accuracy requirement is a standard deviation (SD) of 3" for one repetition; a common distance accuracy requirement is 5 mm +/-5 ppm for one measurement.

Typical specifications for an MHD project requiring Third- Order, Class I horizontal and vertical control might include the following:

1. Control:

   a) Horizontal - A minimum of four MHD or NGS Second-Order, Class II (or above) control points, with two pairs intervisible.

   b) Vertical - A minimum of three MHD or NGS Second-Order, Class II (or above) control points.

2. Monumentation:

   a) Horizontal: All traverse points must be stable, flush, semi-permanent marks with at least 40% being 1-meter iron rods 0.015 m in diameter, driven flush to the ground.

   b) Vertical: A bench mark every 500 m consisting of a semi-permanent mark not subject to movement.

3. Instrumentation:

   a) Horizontal and vertical angle instrument having a least count of 1" and able to produce a SD of 3" in one set of D&R readings.

   b) Distance: EDM with a SD of at least 0.5 mm +/-5 ppm per measurement.

   c) Elevation: Level capable of leveling a 1 km line to a SD of 1.5 mm for a double run.

4. Field Procedures:

   a) Horizontal Angles: four sets D&R, each set within +/-5″ of mean.

   b) Zenith Angles: 2 reciprocal sets to agree within 10".

   c) Distance: Reciprocal, at least four in each direction to agree within 10 mm. If distance is obtained directly from instrument, Zenith Angle must be correct to within 10".

   d) Levels: 3-wire single run between control, single hair to level traverse points.

5. Closures:

   Azimuth - 10"√(No. of lines in traverse)
   Position - 1/10,000 after azimuth adjustment
   Vertical - 0.012 m√(Dist in km of line between control BM)

6. Final Products:

   Copies of all control used with published values, description and sketches of all points established, copies of traverse and level adjustments and a listing of state plane coordinates on NAD 83 and elevations based on NAVD 88 for all traverse points and bench marks established.

B) Differential Leveling

A level and level rod are commonly used to determine elevation differences between survey points. By tying elevation differences to one or more points of known elevation (bench marks), the elevation of points can be determined by differential leveling.

The quality of the level in terms of maintaining a horizontal line of sight, the magnification power of the telescope, and keeping the rod vertical all play a role in the accuracy of elevation determinations. The requirements for a job should include parameters such as these in addition to desired closures as a function of the number of setups or distance leveled in kilometers.

Differential leveling is particularly sensitive to systematic errors of non-horizontal line of sight and environmental conditions of earth curvature and atmospheric refraction. Balancing backsight (BS) and foresight (FS) distances during the leveling process eliminates most of these errors. All control leveling above FGCC Third-Order requires an invar one-piece rod. Virtually all precise leveling should be done with a compensator level equipped with a parallel plate micrometer and invar leveling staffs.

For control purposes, standard differential leveling field procedures normally require that all three wires (upper, middle, and lower cross hairs) be read and meaned. This allows a quick check that the rod intercept difference of the upper and middle readings compared to the interval of the middle and lower cross hair readings indicates reasonable closure. It also allows computation of the distance from the level to the rod by multiplying the difference between the upper and lower cross hair readings by the stadia constant. Once a backsight is chosen, the foresight should be paced so that its distance will approximate the backsight distance as closely as possible. The backsight and foresight readings should be taken as close together as possible time-wise to eliminate changes caused by atmospheric conditions and settlement. Productivity and precision are enhanced by use of two level rods. Keeping the level vertical when moving between setups also enhances data quality. The backsight and foresight distances should be summed throughout the leveling process. Any setup which has an imbalance between BS and FS greater than 10 m shall be redone immediately to bring the distances closer together.

As a level line nears completion, the sum of the backsight distances should be compared to the sum of the foresight distances. Instead of matching a backsight distance to a foresight distance, the surveyor should balance the sum of the backsights as nearly as practicable with the sum of the foresights.

Correction for non-horizontal line of sight requires the use of a peg test (described in Section 3.4.2 (D), Differential Leveling) to determine the amount of departure (elevation or depression) from horizontal per 100 m of sight distance. A peg test should be performed before and after a differential leveling project to check if the inclination/declination stayed consistent, or if an inordinate amount of inclination/ declination exists. For Second-Order leveling or better, a peg test should be performed daily.

The following field procedures should be used to eliminate most errors during data collection:

1. Balancing sight distances eliminates most collimation and refraction errors.

2. Using two rods eliminates many systematic errors, if:

   a) BS and FS are read as close together time-wise as possible;
   b) Same rod is read first at each setup;
   c) Same rod is used to start and end each level run;
   d) A minimum ground clearance of at least 0.5 m is maintained.

Important checks are reading all three wires and meaning the three readings, and reading the rod inverted as the total length of the level rod is known. Precise leveling requires double running all lines forward and back (or use of the micrometer/invar staff method previously described) with a close comparison of closure between the two elevation differences determined for the same level run.

C) Topographic Data Collection

Topographic data collection differs from traversing for two major reasons:

1. The data has no mathematical quality checks since it is usually a series of sideshots.

2. The attribute (non-measurement) information collected is of critical importance in describing what feature is at a particular location.

Topographic data is usually collected with the total station by occupying a station of known horizontal (and possibly vertical) location. A station is backsighted in a known direction usually derived from the horizontal coordinates of the backsight. If the occupied station does not have a known elevation, a backsight to a point of known elevation must be made if elevations are desired (a backsight with 3-D coordinates resolves both the horizontal and vertical requirements). To derive an elevation from a backsight, a prism must be placed there so a slope distance and zenith angle can be measured. For horizontal direction, a prism is not required on a backsight but it is highly recommended as the measured horizontal distance can be compared to its value derived from coordinate inverse.

Since topographic data collection is often made from traverse stations (points of known coordinates), at least two other traverse stations can often also be sighted. This allows "double" backsights to be measured to each of the visible traverse stations, which provide added checks on the horizontal distance and elevation changes as measured. The horizontal angle between the two backsights, as compared to what is derived from coordinate inverse, can be checked by this method.

In some cases, a point of known location cannot be occupied to perform topographic data collection. Resection, a process of measuring to at least two known stations with angles and distances, or to three or more stations with angles only, is not recommended, as it is often susceptible to weak geometric solutions. Using this method, the elevation is derived by measuring a slope distance and zenith angle to any point of known elevation. As in the case of a conventional topographic setup, measuring to extra stations affords mathematical checks on the produced coordinates.

When the elevations of the topographic data are required, it is of utmost importance to measure height of instrument and height of target as precisely as possible.

Occupying a topographic feature with a prism pole is often impossible or impractical. A typical problem, locating a tree, is called an eccentric station since it is impossible to measure to its center. The eccentric station is located by using the prism to measure forward, back, left, or right of the object.

Two methods exist for eccentric measurement:

1. The offset distance (such as the tree radius) is measured and the slope distance, zenith circle reading and horizontal circle reading to the offset prism are recorded. The distance and direction (if a left or right offset) is corrected to the center of the object.

2. The slope distance, zenith circle reading, and horizontal circle reading are recorded to a left or right offset prism. The instrument is then pointed at the object for a second horizontal circle reading from which a horizontal angle can be derived from the prism to the object. Assuming a right angle at the prism enables the offset distance to be resolved and coordinates for the object established. This method requires two shots but does not require measurement of the offset distance.

Taping in orthogonal (forward, back, left, right) directions is another form of data collection for planimetric position only. It is effective for eliminating instrument setups from which only a few points can be measured and is normally recorded in conventional fieldbooks. An example is measuring to two corners of a building, which are visible from an instrument setup. The "back" corners of the building can be located relative to the other two corners by taping in orthogonal directions relative to them or any other surveyed points. Taping is simply a form of traversing where the tape replaces the EDM and all horizontal angles are 0°, 90°, 180° or 270°, or measured with a right angle prism or by human interpolation. Taping is not a precise form of locating, but it can dramatically speed up topographic data collection.

Another form of planimetric location by taping is location by station-offset. This procedure assumes known centerline stationing and all topographic features are located at a left or right offset distance perpendicular to a centerline station. The distance is simply taped perpendicular to the centerline.

If a level is set up on a centerline station and a station with known elevation is backsighted, using upper and lower stadia hairs for distance measurement and the middle cross hair for elevation determination allows 3-D station offset positions to be determined. The instrument person is responsible for interpolating a right angle off the centerline for the level's line of sight in the offset measuring process.

Attributing data is critical in topographic surveys. One of the simplest attributes is whether the point measured to is a ground shot (for inclusion into a DTM) or a feature point, which is not to be included in a ground model, such as a measurement to a corner of a building overhang. Most of the points in a topographic survey are ground shots, while the number of feature points is quite few in comparison.

Another form of attributing involves assigning a station, offset, or cross section designation to a measured point. Examples are: a point is located at Station 82+18.00, a point is offset 12.6 m left of Station 82+18.00, or a point is part of the cross section data at Station 82+18.00.

Still another form of attribute information is a description of the measured object, often called a feature code. Feature codes are needed for both points (tree, manhole, property corner, etc.) and lines (edge of pavement, power line, building, etc.). MHD has a table of standard feature codes consisting of symbols, line types and colors for use on all Department projects. All field crews working on a project should be using the same feature coding system.

It is possible for a measured point to be both a point and part of a line, such as a power pole through which a power line runs. It is also possible for a point to be part of more than one line, such as when a point is the intersection of two centerlines.

Lines can be straight or curved (splines), and one unique line can have both straight and curved components. The measured points of a line receive the curve or straight designation. An incoming tangent, a horizontal curve and an outgoing tangent is defined by two straight points on the first tangent, a curve point anywhere on the horizontal curve and two straight points on the outgoing tangent. The line is then defined geometrically without having to locate the PC, PI, or PT of the curve. Lines which close upon themselves (buildings) should only require this designation, and not require a re-measurement to the initial building corner that was measured.

The final component of attributing is the descriptive information about points and lines not included in a feature code, e.g., tree type and diameter, manhole type and diameter or property corner description. Street name, driveway material or a utility line description are examples of descriptive information about line feature codes. The descriptive information is attached to the feature code and the point location. It can usually be reviewed on the topographic map.

Whether using a data collector or a conventional fieldbook, drawing a field sketch of the topographic survey in a conventional fieldbook is of extreme importance for understanding the information later in an office environment. This is especially important for drainage data.

D) Digital Terrain Model (DTM) Data Collection

DTM-specific data is often collected as part of the topographic survey data collection process. All topographic features shot as ground points are included as spot elevations for building the DTM. In addition, other spot elevations need to be recorded to fill in the areas where topographic data did not exist. The density of spot elevation information required is a function of the job requirements and the terrain undulations.

A break line is a specific type of line feature code, which is required for most DTMs. Spot elevation information without any break lines in a DTM, will create a smooth surface with no abrupt changes in grade. Ground surfaces are rarely smooth in all areas as man-made improvements create many sharp forms of breaks (abrupt changes in grade) and some breaks are natural.

A "V" shaped ditch running alongside a road is an example of three break lines which will run fairly parallel with the road. The most obvious break line would be the bottom of the ditch. The two edges of the top of the ditch are also often break lines, as the change of grade perpendicular to the direction of the road is quite abrupt at these points. Top of curb and bottom of curb are also good examples of break lines. A line is surveyed topographically along the top (and bottom) of the curb. The points on the line can be designated as straight or curved as detailed in Section 3.2.2 (C), Topographic Data Collection.

Break lines are required to properly define a rock outcrop or other sharp changes in grade that occur naturally on many hills. The ground surface model will not be smoothed at the break lines and instead will show an abrupt change in grade. Profiles, contours, and 3-D views from the DTM should show these breaks.

E) Cross Section Data Collection

Before computerization made the concept of a DTM possible for ground modeling and volumetric computations, the development of cross sections was the standard procedure. A cross section is a profile of the ground surface at a given station perpendicular to the centerline. Any man-made features existing along the cross section line are generally shown on the cross section and labeled. Typical items as centerline, edge of lane, edge of pavement, curb, sidewalk, right-of-way line, etc. should be labeled on a cross section and thus must be surveyed and identified during the data collection process.

The distance between cross sections and the length of each cross section perpendicular to the centerline are dependent on the job requirements. Intermediate cross sections, not at a standard interval, are used to highlight dramatic changes in grade or elevation since cross section practice assumes a smooth transition between adjacent cross sections. Conventional cross sections also include subsurface structures, such as a manhole rim or catch basin, which need an elevation reference.

The first step for surveying cross sections is to identify the stations along the centerline. The cross section is then surveyed using a theodolite/EDM, a total station or a level.

Using a theodolite/EDM for cross sectioning is similar to topographic data collection. A station with known coordinates is occupied and a known backsight is used to derive direction and possibly elevation if the occupied station's elevation is unknown. Measurements are then made to selected cross section points, which are attributed as such and identified as to stationing. Software is used to compute the offset from the centerline to the measured point, as an office procedure. Software usually "moves" the measured position to a perpendicular to the centerline to be more graphically pleasing since that was the intended prism pole location.

Using a level, the centerline station could be occupied with a backsight to a point of known elevation. Foresights are taken to cross section points at that station with the distance left or right of centerline derived from reading upper and lower cross hairs. The level rod sometimes needs to be offset from the perpendicular to the centerline so that it may be read, but it is assumed that the determined elevation is the value on the perpendicular at that distance.

If it is not desired to occupy the centerline station, the distance of desired points left or right of centerline can be measured by taping. The level can then be setup anywhere and measurements made to a rod at those points. The distance need not be derived from the level and rod as a tape was used to determine the point's offset relative to the centerline.

F) Stakeout

Stakeout is setting points in the field that were usually derived from office computations. Horizontal stakeout is a direct process where the monument, pin, or stake is set at the desired horizontal position. If elevation is required in the stakeout process, it implies a later earthwork and/or construction operation to establish a final defined elevation at that point. The survey point is placed at an elevation and referenced as to how far the final elevation is above or below the survey elevation.

Layout is usually from angle/distance using a total station or by station-offset from an existing centerline or baseline. The elevation component is derived from slope distance, zenith angle and height of target in the former case, and by level and level rod for the latter.

Layout with a total station requires coordinate inversing. If an occupied station, backsight station and desired layout station all have known coordinates it is possible to derive a horizontal angle and horizontal distance for layout. If the coordinates are state plane, the coordinate inverse distance will be grid and need to be divided by scale and elevation factor to obtain a ground value.

If the point cannot be laid out from the desired occupied station, another nearby station with known coordinates needs to be selected with an appropriate backsight and the coordinate inversing process repeated to obtain layout dimensions. A short spur traverse to set an intermediate point may be required to establish a position from which the layout can be performed.

Once the layout point is occupied, angles and distances to other visible stations can be checked. This is especially useful for important layout points like property corners.

Horizontal layout by station-offset is the use of a tape to locate the desired stationing along the baseline and then taping the offset distance at a right angle to the baseline. Scale factor and elevation factor need to be considered if the station-offset dimensions are in grid.

Vertical layout is establishing the survey point at an elevation by slope distance/zenith angle/height of target, or by level and level rod, and then noting the vertical dimension, above or below the survey point, to the desired elevation. If the survey point is marked by a stake, the elevation difference and its direction is recorded directly on the stake.

Stakeout is identical in nature to "as built"-type quality checks where the survey is verifying the location of a structure relative to the design plans. Instead of staking where a position is, a surveyor is verifying that something is in the correct horizontal and vertical position. The as-built data checking is again based on coordinate inversing or station offset information.

3.3 FIELDBOOKS AND DIGITAL DATA COLLECTION

Historically, the surveyor's fieldbook has stored all of the numerical, graphical and descriptive data required to produce the desired final product. The surveyor's notebook also usually contains considerable information about the job, which may not affect the final product but is invaluable for survey indexing and validation purposes if questions arise regarding the survey.

The ability to collect survey data in digital form using a total station and data collector (electronic fieldbook, or EFB) has the potential for replacing much of the measurement information entered by hand in a conventional fieldbook. An EFB also usually has the capability of storing keyboard-entered attribute and other descriptive information, as well as non-total station survey measurement data such as taping, leveling (rod readings), station-offset, and eccentric distances.

If an EFB is being used but does not allow collection of desired attribute or descriptive information, this information must be recorded in a conventional fieldbook along with the EFB data and all field sketches. Use of a MHD Survey Notebook is required for all surveys conducted by or for the Department.

3.3.1 Capsule Description of Fieldbook Information Sections

Fieldbook data can be divided into the following sections:

a) Header - Descriptive information which contains the purpose of the job, its location, etc.

b) Calibration - Information on type of instrument(s), a test of its measuring ability, weather, field crew personnel, etc.

c) Setup - Required information about each instrument setup.

d) Observation - Required information for each measurement.

e) Linework - Usually applies to detail for topographic surveys where one is defining what a line represents (feature code) and which survey shots should be connected to one another.

f) Remarks - Generally descriptive comments which are not necessarily about specific survey points.

3.3.2 Detailed Description of Fieldbook Data Sections

A more detailed description of sections that can be used to provide needed fieldbook data is as follows:

A) Header

A header is information about a project. An example might be:

"This is Section B of a road widening project for Rte. 20 between Smallville and Pleasant Lake. This data is located approximately 8 km. east of Smallville and includes topographic data from Sta. 82+21 to 88+00. The existing two lane road is in a rural area, with unpaved shoulders and hilly terrain."

If an EFB is being used, the header should contain the name of the file as it was collected in the field.

B) Calibration

While some of this information could be included in a header, calibration information provides data on survey conditions, such as equipment used, field crew, and weather. Calibration relates to daily activities, while a header pertains to a segment of a project. Calibration information should include:

1. Comments - information about unusual field or equipment conditions
2. Temperature - degrees Celsius
3. Pressure - mm or mb of mercury
4. Weather - a description of conditions
5. Instrumentperson - name or initials
6. Notekeeper - name or initials
7. Rodperson(s) - name or initials
8. Instrument(s) - brand and model
9. Serial number(s) of instruments
10. Stadia factor - if relevant
11. Instrument calibration check(s)

Instrument checks for levels and theodolites/total stations are described in Section 3.4, Conventional Field Data Processing. A level should have a peg test performed on it to determine any non-horizontal nature of the line of sight. A theodolite or total station should be pointed at a distinct point an equal number of times in the direct and reverse positions (recording horizontal and zenith circle readings) to determine the instrument's vertical and horizontal collimation errors and to obtain an estimate of any operator pointing error. An unusually large calibration error or change in value from the previous calibration error may be an indication of a problem with the mechanics of the instrument.

C) Instrument Setup

For differential leveling, data on instrument setups is recorded for necessary office processing. Measuring to a point of known elevation is required to determine the HI, the height (elevation) of the instrument's line of sight.

Components of a setup record for theodolite/EDMs or total stations should include:

1. Comments - Information about unusual field or equipment conditions.

2. Station name - Numeric or alphanumeric identification for a survey point, to be used every time the station is measured to or occupied; use facilitates association of coordinates with the station name.

3. Line attribute - Defines whether it is a curve or straight point on a line; not usually applicable to setup stations.

4. Feature code - May include further attributing such as a zone number or description.

5. Ground, feature, or cross section - Ground and cross section points are placed in a DTM while feature points are not. Cross section points are attached to a station.

6. Height of Instrument (HI) - A value not necessary in 2-D surveys, but needed in any 3-D survey performed with a theodolite/EDM or a total station. The meaning of the term has evolved from "elevation of the level" to "distance the instrument is above the mark".

Setup records should be consistent and easily interpretable.

In some EFBs the backsight is associated with the setup record. In other systems it is treated as an observation, which always contains a horizontal circle reading, and may or may not contain a zenith circle reading and slope distance.

D) Observation/Measurement

Data will vary based on whether the data is collected by theodolite/EDM, total station, level (station-offset also possible), or orthogonal taping. Data consistent with all collection methods that should be recorded include:

* Comments - Information about unusual field or equipment conditions.

* Line attribute - Defines whether it is a curve or straight line; often not applicable to data collected with a level.

* Feature code - May include further attributing such as a zone number or description.

* Ground, feature, or cross section - Ground and cross section points are placed in a DTM while feature points are not. Cross section points are attached to a station.

* Sighted station name - Numeric or alphanumeric identifier for a survey point, to be used every time the station is measured to or occupied; use facilitates association of coordinates with the station name. A backsight station may be so designated, or it may be implied the first sighted station is the backsight. In some topographic surveys a point name for sideshots may not be required since only location and attribute data is desired.

1. Theodolite/EDM or total station data should also include:

* Horizontal circle reading - While this could be implied as zero on a backsight in some cases, it should be recorded to verify its value.

* Zenith circle reading - Not required if a prism is not at the sighted station or if a 2-D survey is being performed and the horizontal distance recorded is derived from the zenith angle and slope distance.

* Distance - Usually the slope distance, but may be a horizontal distance derived from the slope and zenith angle.

* Height of target - Not required if no distance is being measured to the point or if it is a 2-D survey; a necessity for any 3-D survey.

* Eccentric direction and value - Not required if the prism is over the survey point. If there is eccentricity, the direction (forward, back, left, right) of the prism relative to the object is required along with the horizontal distance from the prism to the survey point.

* Direct/Reverse "face" - If the zenith circle reading is not required but repetitions are being made, it is important to designate what face the instrument is in for proper reduction of the raw survey data. If the zenith circle is recorded, the instrument face is known.

* Position number - This is primarily reserved for control traversing where 1 direct and 1 reverse pointing represents a position. It can be used in other forms of surveying when one wants to reorient on a backsight station and then continue collecting other data at that occupied point. Toggling to the next position number requires measuring again to a backsight.

2. Data collected with a level should also include:

* Stationing of the measured point if horizontal location is desired.

* Offset of the measured point relative to the input stationing if horizontal location is desired.

* Rod readings - Should always include the middle cross hair reading and can include the upper and/or lower cross hair readings depending on the application.

* Route - A defined line, which contains points with stationing that make up the centerline to which the input stationing information is referenced. This allows coordinates to be computed on station-offset points if enough coordinates have been associated with some of the stations along the route.

3. Data collected by orthogonal taping should include:

* Occupied point name - Needed for coordinate computations.

* Backsighted point name - Also required for coordinate computations.

* Direction - Forward, back, left, or right relative to the occupied and backsighted stations.

* Distance - The measured horizontal distance from the occupied station to the station being measured to.

E) Linework

For topographic data collection, one of the most important pieces of information collected is how points are connected to form lines which define features such as centerline, edge of pavement, building, shrub outline, etc. A variety of procedures have been developed for use with both conventional fieldbooks and EFBs to define what points should be connected by lines. Many of these are based on collecting the points in order and having a change in feature code define a new line. There will be lines whose points cannot be collected in order and a system must be capable of the most generic case where a line is entirely defined by the names of the survey points which were collected. The linework is immediately tied to the points being identified as either straight or curved.

It is important to note that a point may have a feature code, while a line through it could also have a feature code (e.g., power pole and power line). Some points may not have point feature codes but will be part of a line that is feature coded (edge of pavement). Other points may not be part of lines (lamp post, manhole, etc.). A point may also not have any point feature code or be part of any line if it is a spot elevation and has properly been designated as a ground point.

Manually connecting coordinated points to form lines by computer in the office is a very tedious and error-prone process. Field survey collection procedures should facilitate the necessary office work to follow. The field note system must include a well defined procedure for line definition.

F) Remarks

Remarks are items of field information, which may help office processing of the survey data. It is common procedure to have a different person processing data in the office from the one who performed the field data collection. Remarks are useful in determining field procedures and finding mistakes. For example, a remark that the field person was not sure whether an occupied point was "A25" or "A26" might be resolved through an office check on the distance to a backsight to help confirm proper identification of the station.

3.3.3 Electronic Fieldbook (EFB) Data

The use of an EFB to collect and store raw data should follow the procedures previously described in this Section. Original raw data should always be preserved and any changes or corrections made to field data, such as station name, height of instrument or target, should be documented. Raw field data should also be preserved in hard copy output form similar to how a conventional fieldbook is preserved.

Procedures for proper office processing of data are described in Section 3.4, Conventional Field Data Processing.

3.3.4 Survey Notebook Data

Field survey data and sketches which cannot be efficiently recorded in the EFB should be recorded in a conventional fieldbook and stored with copies of the electronic data.

3.3.5 Notekeeping Examples

Appendix A-6, Sample Survey Notes, contains examples of various MHD notekeeping formats.

3.4 CONVENTIONAL FIELD DATA PROCESSING

This Section details some field procedures, which enhance office processing. It is important to use field techniques, which reduce the number of office computations. A field procedure, which identifies and corrects theodolite vertical collimation error, for example, will eliminate the need for later office correction and adjustment.

3.4.1 Procedures to Help Ensure Proper Processing of Field Data

Procedures to help ensure proper processing of field data include:

a) Use of field techniques and office procedures to eliminate or minimize all significant systematic errors.

b) Office processing procedures should take advantage of every redundant field measurement that can reasonably be made and should be geared to the reproducibility of final results (usually in coordinate form) from the raw survey data.

c) Proper analysis procedures should include the geodetic aspects of conventional survey practices such as the relationship between geographic and grid angles and distances.

3.4.2 Removal of Systematic Errors

Most systematic errors are caused by incorrect or inconsistent field procedures or measurements. "Significant" systematic error is one which creates an error of more than: 0.01 m in any slope or horizontal distance derived from total station, EDM/theodolite or taping; 0.02 m in any trigonometry-derived elevation change; or 0.002 m in elevation change derived from differential leveling.

Horizontal angle systematic error, which causes an error in position, is a function of the length of the line and is derived from the distance multiplied by the sine of the systematic angular error. Trigonometric elevation difference error is derived from the distance multiplied by the sine of the error in zenith angle.

A) Taping

Systematic error in taping can be derived from the conventional equations for temperature, pull and sag. A better procedure is to make a measurement against a known baseline (usually 30 m in length). The surveyor can observe what the tape length is for a given temperature, sag and consistent operator pull. Those conditions can then be related to the survey field conditions and a correction applied, if necessary.

B) Electronic Distance Measurement (EDM)

Accurate EDM measurement requires a prism on the other end of a line, and thus any systematic error determined is actually a combined EDM/prism value. The most common systematic error in EDM measurement is incorrect prism offset. The prism offset should be known and recorded in the fieldbook or data collector.

Three procedures, based on measurement of horizontal lines, can be used to determine systematic error:

1. A tape is used to lay out a 30 m (or some other distance) baseline. The EDM measurement is compared to this baseline. The assumption is made that the taped baseline is more accurate than the EDM measurement.

2. Three collinear points (A, B, C, with B in the middle) of unknown distance are established. If EDM distances AB + BC does not equal AC then an error in the EDM/prism exists and can be calculated because the summed value contains twice that error while the direct measurement contains that value.

3. An EDM calibration baseline is utilized to determine any constant error from (1) or (2) above and also any existing scale error, i.e., different errors existing at different calibration distances. National Geodetic Survey's publication, NOAA Technical Memorandum NOS NGS-10, discusses the derivation of scale and constant errors from measurement of an EDM calibration range. Note that the procedures of (2) can be used on a baseline as the points are intended to be collinear.

In all three cases, all measurements need to be corrected for systematic errors of temperature and pressure before a correct analysis can be made. In the absence of a pressure determination, an estimate can be derived from the elevation of the survey area.

C) Zenith angle (vertical) collimation error:

The zero mark of a zenith circle of a theodolite or a total station is often not at the zenith when an instrument is leveled. The deviation of this zero mark from the zenith is obtained by precise pointings of a distinctly observable point in the direct and reverse positions. Repeated measurements ensure the validity of this calibration process. (See Section 3.2 - Field Operations with Conventional Survey Equipment). The direct zenith circle reading plus the reverse zenith circle reading should total 360°. The deviation from this value is twice the vertical collimation error. If direct and reverse values are measured as 80°00'20" and 280°00'10" respectively, the sum of 360°00'30" indicates that 15 seconds should be subtracted from all zenith circle readings. This correction is especially important on sideshots, as this error will not be eliminated since sideshots are generally only measured with direct readings.

The lengths of lines being measured and the magnitude of the vertical collimation error should be considered before corrections are made. Since most zenith angles are near horizontal, the chance of a vertical collimation error changing a horizontal distance by more than 0.01 m is highly unlikely. The elevation differences are generally affected more, but the accuracy of long lines in trigonometric leveling is rarely better than +/- 0.02 m. A large collimation error will also affect the computed horizontal distance as well as the elevation difference. If the instrument automatically uses the displayed vertical angle to compute the horizontal and vertical distance, it is most important to have the correct vertical angle displayed since D&R measurements will not correct for it.

Horizontal collimation error can also occur but is rarely of a magnitude worth correcting. To check horizontal collimation at the same time as vertical collimation, horizontal circle readings should also be recorded. The direct reading should differ from the reverse reading by 180°, with the difference being twice the horizontal collimation error. An error of greater than twice the Smallest Measuring Ability (SMA) of the instrument is significant.

Many total stations have internal compensators and software to apply corrections to the horizontal and zenith circles. Even after such corrections, a check should still be made for collimation error. Collimation values fluctuate and should be checked daily and recorded in a fieldbook or data collector.

D) Differential Leveling

The peg test is the standard procedure for determining the inclination/declination error in the instrument's line of sight. The simplest peg test involves determining the elevation difference between two points by a setup at the midpoint and a setup near one of the points. The midpoint-derived elevation is corrected for non-horizontal line of sight since the backsight and foresight distances are equal. The difference between it and the elevation determined from the endpoint is the inclination/declination error over the distance between the endpoints, usually referred to as inclination/declination per 100 m. The same value can be obtained by sighting two rods from different distances and comparing the derived elevation differences to obtain inclination or declination per 100 m. This value is known as the collimation ("C") factor. Any level with a "C" factor greater than 0.007 m/100 m should be adjusted. A "C" factor of less than 0.005 m/100 m is required For precise leveling.

3.4.3 Systematic Error From Refraction and Curvature

Differential leveling and zenith angles are affected by the systematic errors of atmospheric refraction and earth curvature. While the two errors are independent, both are generally considered together as one equation can effectively model the combined effect of both errors. The two errors affect elevation differences and zenith angles in opposite directions, but the atmospheric refraction error is only approximately 13% of the error for earth curvature. Earth curvature creates larger rod readings in differential leveling (a negative correction to the rod reading) and smaller zenith angles (a positive correction for direct position values). The equation for the correction to elevation differences is:

$$E_{C\&R} = 0.0675 \times D^2$$

where $E_{C\&R}$ is the error due to earth curvature and atmospheric distance, and D is the distance, both in kilometers.

For extreme zenith angles (25° above or below horizontal) and measured distances greater than 1 km, a more complicated equation solves for the zenith angle error and also provides a small correction to the horizontal distance. Only in precise control surveys and an extreme zenith angle case should the more complicated equation ever be considered. Many total stations can apply an earth curvature and atmospheric refraction correction. The instrument manual usually contains the equation used.

Use of the equation for a variety of distances will show the correction to be a negligible value. It grows geometrically due to the squared term, but one should recognize the weakness of vertical accuracy in trigonometric leveling of long lines.

For differential leveling, the balancing of corresponding backsight and foresight distances eliminates the error due to curvature and lessens the effect of refraction. Refraction is a function also of the distance above the ground which is being sighted and is more pronounced for rod readings below 0.5 m. For differential leveling for control purposes, rod readings of less than 0.5 m should be avoided. Because of the squared term, balancing backsight and foresight distances for a level run does not necessarily eliminate the error due to earth curvature and atmospheric refraction. Due to the short foresight and backsight distances usually utilized in differential leveling and the standard practice of trying to balance backsight and foresight distances, earth curvature and atmospheric refraction correction can be considered negligible for all but very precise work.

A daily peg test should be included as part of control differential leveling procedures. The field notes for control differential leveling should always include three wire readings and a running total of backsight and foresight distances.

Control leveling performed by MHD is usually to Second-Order, Class I specifications, double run, using automatic levels with parallel plate micrometer and double-scale invar leveling staffs.

Most systematic errors in differential leveling can be canceled or at least minimized by following sound field practice. Some recommended procedures, which should be used, are:

a) Balancing backsights and foresights. Its value cannot be over stated since errors of off-level line of sight, curvature of the earth, and to a large extent, refraction are eliminated.

b) Using two rods, with BS and FS read as close together as possible. This reduces refraction and settlement errors.

c) Never reading a rod below 0.5 m because heat waves are more prevalent and refraction inconsistent near the earth surface.

d) Reading the same rod first at each set up, with the BS first on even numbered setups and the FS first on odd numbered setups.

e) Starting and ending a line on the same rod requires an even number of setups and reduces error caused by rod error.

3.4.4 Reduction of Repeated Raw Survey Measurements

Repeating survey measurements is standard practice for all traverse (non-sideshot) operations. It is possible to achieve suitable survey accuracies and closures without repeating traverse measurements. However, the advantages derived from repetition, the elimination of systematic errors through equal numbers of direct and reverse observations, and the possible increase in precision through averaging repeated measurements, far outweigh the short increase in observation time.

In addition to repetition, another recommended practice when using a total station or EDM is having a prism or reflector height measurement on all backsights. This enables checks on the horizontal distance and elevation change measured in opposite directions and on errors which repetition will not disclose such as in height of instrument (HI), height of target (HT) or reflector (HR), instrument setup over the point and reflector setup over the point.

If performing a series of sideshots from a control point backsighting another control point, a prism on the backsighted station can provide an excellent mathematical check by comparing the horizontal distance and elevation change to values derived from the control coordinates.

Backsight horizontal distance checks should generally be within 0.01 m + 20 ppm. A 1000 m distance should check within 0.01 + (1000 X 20/1000000) = 0.030 m. Elevation changes derived from trigonometric leveling should check within 0.02 m + 100 ppm. A comparison to existing coordinates, which may not be perfect, could contribute to the magnitude of the comparison.

The reduction of repetitions is generally a simple averaging process. As a by product of that process, standard deviations, standard deviations in the mean and the maximum spread of any single measurement from the average can be computed.

The maximum spread should be the major factor in assessing the quality of a series of repetitions. Horizontal distances should generally not have a deviation of more than 0.01 m + 10 ppm; trigonometric elevation difference maximum spread should not exceed 0.02 m + 50 ppm; differential rod readings which are repeated should not exceed twice the least count of the rod; in three-wire leveling the average of the upper and lower wires should not differ from the middle wire reading by more than two times the least count of the rod. The horizontal angle maximum spread should not be more than twice the Smallest Measuring Ability (SMA) on lines longer than 250 m. On shorter lines, the sine of maximum spread times the length of the shortest line (backsight or foresight) should not exceed 0.01 m.

A particular project may require specific standards for standard deviation in a single observation or standard error in the mean. The maximum spread from the mean remains the best indicator of the possibility of error in a repetition.

An obvious outlier should be discarded and not used in determining the mean. The most common standard for an acceptable angle is +/- 5" from the mean. When there is an unusually large spread between angles or if the angles appear to be creeping up or down in a consistent manner, the instrument and reflector setup data should be checked.

3.4.5 Reduction to a Datum

While some surveys may be based on assumed plane coordinates, most highway projects are of sufficient dimension that geodetic aspects must be considered. Computation of coordinates with respect to a datum may involve production of:

A) Geocentric Coordinates: an earth centered system within which GPS primarily works. If a combined GPS/traverse control network has been observed, this may be the most realistic system for coordinate production as the conventional measurements are also definable in this system. No application of scale or elevation factors to horizontal distances are required in such a system. Software can readily convert values to state plane coordinates.

B) Latitude, Longitude and Elevation: the conventional geodetic coordinate system is well documented and applicable to their computation based on survey measurements. Unfortunately, the equations are not simple and are unavailable in most commercial survey reduction software. Horizontal distances must be reduced to the datum surface (ellipsoid) before coordinates are computed.

C) State Plane (SPC) or Universal Transverse Mercator (UTM) Coordinates: These are probably the most widely used geodetic reference systems for reduction of conventional survey traverse data, with state plane predominating for survey applications. Plane survey equations can be used if scale and elevation factors are applied to horizontal distances and convergence angles and LaPlace correction factors (a range from about 3.5" in the Boston area to about 10" in western Massachusetts) are applied to any astronomic azimuth determinations. The Massachusetts State Plane Coordinate System is a Lambert Conic Conformal Projection made up of two zones - Mainland and Island. Scale factors (the projection of a line from its position on the ellipsoid to the state plane projection) remain constant in an east-west direction and change most quickly in a north-south direction. The convergence angle changes according to relative position and changes predominately in an east-west direction.

It is critical to label distances as slope, horizontal, ellipsoidal/sea level, or grid. Likewise directions need to be defined as geodetic, astronomic, or grid when state plane or UTM coordinates and computations are being utilized.

3.4.6 Relationships Between Different Types of Distances, Angles and Azimuths

A) Distances

1. Mark to Mark - Spatial distance between stations. Can be obtained from slope distance by use of zenith angle or elevation difference, or by inversing geocentric coordinates (cartesian) obtained from GPS.

2. Slope - Distance measured from instrument to reflector.

3. Horizontal - Application of zenith angles or height difference to slope distance.

4. Ellipsoid (Sea Level) - Horizontal Distance X Elevation Scale Factor

Elevation Scale Factor:

$$\text{NAD 1927} = \frac{R}{R + \text{Elevation}}$$

$$\text{NAD 1983} = \frac{R}{R + (\text{Elev.} + \text{G.H.})}$$

R = Radius of Earth, approximately 6,372,200 m.

Elev. = Average Elevation of Line, in meters.

G.H. = Geoidal separation between Ellipsoid and Geoid (approximate Sea Level in Mass.), a negative value varying between 27 and 30 meters. This means that Ellipsoid heights are always approximately 30 m lower than orthometric or Sea Level height. With a G.H. in Mass. of between 27 and 30 m, using a value of 30 m for all calculations will produce an ellipsoid distance within 1 or 2 ppm.

The formula becomes:

$$\text{Ellipsoid Distance} = \frac{R}{R + (\text{Elev.} - 30)}$$

5. Grid = Ellipsoid Distance X Grid Scale Factor

The Grid Scale Factor is determined by the line's position in the state plane coordinate zone. In the Mass. zones, it is a function of latitude: 1.0 at the zone's two standard parallels and decreasing to 0.9999645 at the mid point of the zone. The factor is usually obtained from conversion software or from the table in Appendix A-5.

6. Uses of Distances:

a) Horizontal distances are used to define physical objects on the ground. Any topographic or detail survey or measurement which defines the true dimensions and location of an object should consist of horizontal distances.

b) A mark-to-mark distance is a true distance used with its earth-centered Cartesian coordinate differences in GPS surveys.

c) An ellipsoidal distance is used when working with geodetic or geographic coordinates, i.e., latitude and longitude on the ellipsoid. The NAD 83 model is GRS 80.

d) A grid distance is used when working in the state plane coordinate system and reflects the geographic surface projected on a plane.

B) Angles

Angles as measured on the ground are geographic and no correction is needed for use with latitude and longitude coordinates. There is a correction known as the "second term" correction (or "t-T") to make measured angles suitable for use in SPC systems. This correction reduces the angle measured on the ground to a plane surface angle. Since the Mass. SPC system is a relatively small zone, this correction is insignificant for any location in the State and can be disregarded. In states with larger zones, the correction can be significant.

C) Azimuths

Azimuths are similar to bearings and are defined as the angle measured clockwise from the meridian. On the NAD 27 datum this angle was referenced from the south. On NAD 83, azimuths are referenced from the north so that "due east" is 90°, South 180°, and West 270°. The different types of azimuths and their relation to geodetic azimuths are:

1. Compass or Magnetic Azimuth - Needs to be corrected for magnetic declination. Declination in the Boston area is about 16° west of True North (Geographic North). A more precise value can be obtained from commercial software or from the National Geophysical Data Center, (303) 497-6478.

2. Astronomic Azimuth - Azimuths obtained from Polaris or Solar observation have to be modified by the "LaPlace Correction", (3.5" in the Boston area, to 10" in western Mass.). The proper value can be obtained from MHD, NGS, or NGS Deflect 90 and Deflect 93 programs.

3. Grid Azimuth - The correction for grid azimuth is the Convergence Angle. Its value depends on location in the state plane zone and is a function of longitude. In Mass. Mainland Zone, Grid Azimuth and Geodetic Azimuth are the same along 71°-30' West Longitude, the Prime Meridian. To the east of this line, the correction is defined as positive, which means the convergence is added to the grid to get the geodetic azimuth. To the west of this line, the convergence is defined

as negative and is subtracted from the grid to get the Geodetic Azimuth.

Convergence values are given in Appendix A-9, State Plane Coordinate System of 1983, or from coordinate conversion software.

Given a unique state plane coordinate pair in a defined datum and state plane zone, there exists one unique latitude and longitude. One unique latitude/longitude in a defined datum correlates to one unique state plane coordinate pair in the same datum and a defined state plane zone. When referring to coordinates related to a geodetic datum, there is usually no need to discriminate between geodetic coordinates and their state plane equivalents.

3.4.7 Horizontal Control Coordinates and Horizontal Datums

The North American Datum of 1927 (NAD 27) was based on the Clarke Spheroid of 1866 and was the basis for all positions (latitude/longitude) prior to about 1988. At the time of the definition of that datum, state plane zones were set up across the United States in English units.

In the 1980s the reprocessing of the data, which made up NAD 27, along with measurements made subsequent to that time, resulted in the North American Datum of 1983 (NAD 83). WGS 84 and GRS 80, two nearly identical ellipsoids that are associated with NAD 83, can be considered the same for all conventional survey applications. (See Appendix A-9, State Plane Coordinate System of 1983). This reprocessing (a least-squares analysis of the measurement data) produced "readjusted" latitudes and longitudes for all control stations. The shifts in positional information can amount to more than 100 m but are very systematic in magnitude and direction in a localized area. The geodetic direction between two points in a local area stays consistent (the change is normally less than one second) between datums. Unfortunately, an unlabelled latitude and longitude makes it difficult to tell which datum it belongs to, as the change in values between datums is at the level of seconds of arc.

State plane coordinate zones were redefined with different false eastings to make it easier to identify whether the coordinates belonged to NAD 27 or NAD 83, and all coordinates were published in meters instead of feet. A link to existing English units was critical and the U.S. Survey Foot conversion (1 m = 3.280833333333..... ft., or an exact conversion of 1 m = 39.37 in.) has been adopted by Massachusetts as the standard for conversion between units for all survey work. This conversion is required for all NAD 83 coordinates, which are in feet, bench mark elevations in feet, and any horizontal distances or elevation differences in feet.

Since NAD 27 is an English-units based system, all work should be based on the NAD 83 datum.

If NAD 83 equivalents are not available for NAD 27 coordinates, an accepted transformation procedure is NADCON, an NGS program that reads a data base of gridded estimates of coordinate shifts between NAD 27 and NAD 83 and computes weighted shifts for the desired station based on its relative location. The grid was developed from all control stations which had both NAD 27 and NAD 83 coordinates. Coordinates developed by NADCON should always be clearly identified as such.

NADCON only performs the computation in latitude and longitude. If a direct transformation from NAD 27 to NAD 83 state plane coordinates is desired, CORPSCON, a U.S. Army Corps of Engineers program, uses NADCON algorithms in the transformation process and in addition does all the conversions between state plane and latitude/longitude. Both NADCON and CORPSCON are public domain programs because they were developed by government agencies.

NAD 27 coordinates are generally of lower quality than those of NAD 83 because for NAD 83 much more data was available and the computations were performed on computer. It is preferable, therefore, to use control created in NAD 83 rather than NAD 83 control which has been transformed from NAD 27 coordinates.

Massachusetts is part of a New England High Accuracy Regional Network (HARN) based on NAD 83-92. This system was established after publication of NAD 83, so one may find local distortion between existing NAD 83 control and HARN control. If use of HARN is desired, control derived from HARN can be used and all coordinate information generated only from HARN, but the produced coordinates may not "fit" NAD 83 control coordinates in that local area. NAD 83-92 may become the official datum in time, but until it is densified sufficiently to make its use practical it will be used primarily on long range GPS projects.

3.4.8 Bench Mark Elevations and Vertical Datums

The first national definition of a vertical datum was the National Geodetic Vertical Datum of 1929 (NGVD 29), often referred to as "Mean Sea Level of 1929." Bench mark elevations were derived from elevation difference measurements primarily through differential leveling and were published in feet.

For reasons similar to those, which brought about the creation of a new horizontal datum, the North American Vertical Datum of 1988 (NAVD 88) was completed in 1991. A least-squares analysis was made of the vertical leveling control network throughout North America. Final bench mark elevations are published in meters and will probably differ from their NGVD 29 equivalents. The elevation change tends to be consistent in a localized area with variation in changes occurring as the area is expanded.

NGS has prepared VERTCON, a public domain conversion program similar to NADCON, except that it transforms bench mark elevations between NGVD 29 and NAVD 88 by providing an approximate latitude and longitude for the bench mark position. A conversion program is needed if bench marks in the old datum need to be converted to the new datum for a specific project. Version 2 of VERTCON, with refined values, is now available. VERTCON Version 2 is incorporated in CORPSCON Version 4.1.

It is allowable to have a project on NAD 83 and NGVD 29. In this case all bench mark elevations should be converted to meters by the U.S. Survey Foot conversion before computing metric elevations based on the survey measurements.

One should be aware that bench marks can often have systematic error tendencies in a local area. Large projects may require analysis and resolution of these systematic errors.

3.4.9 Reduction of Averaged Raw Survey Data

The first reduction of conventional slope distance/zenith angle/height of instrument/height of reflector values (averaged, if repetition is used) is to horizontal distance and elevation change. Except in precise surveys where correction for earth curvature and atmospheric refraction must occur, the horizontal distance is derived from the slope distance multiplied by the sine of the zenith angle; the elevation change is derived from the height of instrument, plus the slope distance multiplied by the cosine of the zenith angle, minus the reflector height.

The same horizontal distance or elevation change measured on multiple instrument setups (in same direction or in opposite direction via a prism on a backsight) should be averaged. If the separate measurements are of varying quality (e.g., one with a prism mounted on a tribrach and one on a prism pole), a weighted average may be justified where the measurement of higher quality would have more effect in the averaging process.

Horizontal angles, which are repeated, should also be averaged based on the raw horizontal circle readings. If a series of traverse points are being observed, the backsight should generally be the longest line.

Any astronomic observations should be reduced by the standard hour/angle method. Repetitions are critical to assure the quality of this type of measurement. Care should be taken to avoid introducing a systematic error in the time or geodetic position used in the computations. The LaPlace Correction (about 3.5" in the Boston area to about 10" in western Massachusetts) conversion of astronomic to geodetic direction should be applied if significant.

3.4.10 Production of Horizontal Coordinates - State Plane Reductions

If work is in an assumed coordinate system, no factors will be applied to the values derived in the reduction described above. Since most transportation work is related to the state plane grid system, care must be taken to apply realistic scale and elevation factors to all horizontal distances, and convergence angles to any astronomic observations or geodetic azimuth mark values.

Elevation factors are computed by:

$$D_S = D_H \text{ X } [R / (R + h)]$$

where:

D_S is the horizontal distance reduced to the ellipsoid (often called sea level in this reduction), in meters;

D_H is the reduced "ground" horizontal distance in meters;

R is the approximate radius of the earth, 6,372,200 m; and

h is the ellipsoid height derived from the following:

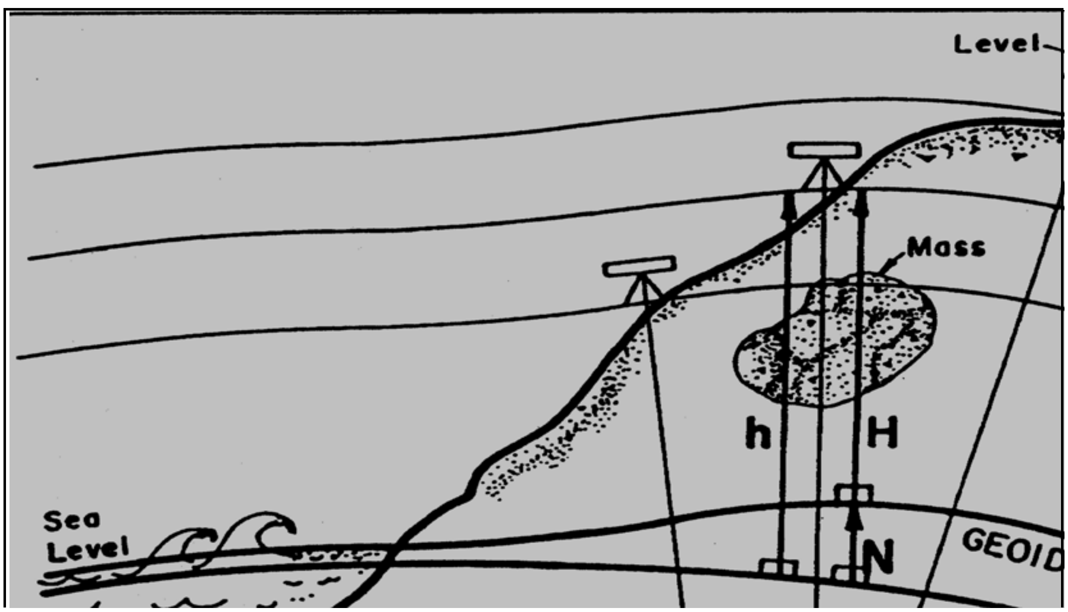
$$h = H + N$$

where:

N is the Geoid Height in meters (a negative number in Massachusetts), - 30 m can be used.

H is the orthometric height (commonly called the elevation), in meters.

While the ellipsoid is the reference surface for horizontal computations, the height of a point above it was rarely known prior to the introduction of GPS. Since the difference between an ellipsoid height and an elevation rarely exceeds 30 m, using elevation in this reduction causes a systematic error of approximately 1/200,000, which is negligible for most survey applications. To avoid the difficulty in obtaining ellipsoid height, elevation can be used.



One elevation "factor" can generally be used for an entire project. The elevation factor for the lowest and highest elevations of the project should be computed and any error introduced in using one project elevation factor should be verified as not significant. If it is significant, multiple or individual elevation factors must be used.

Scale factor is a function of the location of one's position in the zone and changes slowly in an area. Scale factor does not change in an east-west direction in Lambert zones like those in Massachusetts. Scale factors for the corners of a project should be computed and it should be verified whether use of one scale factor will not adversely affect the results. If the scale factor change is significant, multiple or individual scale factors should be used.

The most significant scale factor (difference from 1.000) in a Lambert zone is at the midpoint latitude of the zone. In the Massachusetts Mainland Zone this latitude essentially bisects the State and is thus close to most of the major metropolitan areas. The scale factor at this latitude is 0.9999645 and thus a 4 cm change occurs in 1000 m, which is well within surveying measuring abilities.

The use of software, which automatically calculates all scale factors, elevation factors and convergence angles, is highly recommended. Software, which requires user input of these values, creates the opportunity for transposition and misinterpretation errors.

Software is also available which computes all coordinates in a geodetic fashion although conversion of this to state plane coordinates will invariably be required. Sideshot distances must be reduced to grid prior to coordinate calculations. Grid distance (state plane distance) should not be used for physical distance, i.e. bridge spans, roadways.

3.4.11 Coordinates, Closures, Adjustments, etc.

Any redundant survey measurements produce some type of lack of fit due to random error, usually called closure. In complicated surveys there are many closures which can be calculated as one builds checks by closing traverse routes to, or through, known control points and bench marks, and by creating loop traverses. The least-squares approach is generally recognized as the most suitable adjustment of the random errors in a survey and it has effective reproducibility of results. This method is not required for any survey adjustments but is highly recommended.

Independent of the process used, the amount of adjustment placed on measured distances, angles, astronomic azimuths and elevation differences, generally termed a residual, should be looked at. If a measured horizontal distance reduced to grid is 500.052 m, and after adjustment is 500.073 m, the residual is 0.021 m.

Residuals should be within the random error measuring limits of your equipment and procedures. Generally, horizontal distance residuals should not exceed 0.02 m + 10 ppm, horizontal angle residuals should not exceed 1.5 times the least count of the instrument, except on short lines where the distance multiplied by the sine of the residual should not exceed 0.02 m, and trigonometry-derived elevation change residuals should not exceed 0.03 m + 20 ppm. Trigonometric leveling residual limits will be a function of the desired product, the equipment and procedures used, and the job requirements.

Care must be taken to ensure poor control does not systematically enlarge the magnitude of survey measurement residuals. Control values, which prevent job requirements from being met, must be identified and resolved.

3.4.12 Station-Offset Coordinate Computations

A standard field procedure often involves location by stationing along a centerline and measuring an offset left or right of that point at a right angle to the centerline. Coordinate computations are performed in the same fashion as any other calculations, with consideration for state plane reductions.

3.5 DIGITAL TOPOGRAPHIC MAP INFORMATION

Digital topographic map information is similar to conventional hard copy topographic map information and must meet MHD requirements:

a) Points - represented by a symbol (including color) at a defined scale;

b) Lines - any combination of straight and curved (spline), represented by line type, color and width;

c) Textual information - descriptive information about thepoints and lines, which cannot be represented by symbology.

The biggest difference is that the digital map is truly in ground units (meters), as the points and lines are made up of positions with ground coordinates. These positions can also contain elevation information. A hard copy map requires scaling or digitizing to obtain estimates of ground coordinates. Quality is a function of map quality and scale and the accuracy of the measurements made on the map. Elevation can only be derived from elevation representation, generally from contours or textual data such as spot elevations. Symbology in a digital map is defined in ground units by some map scale ratio to ground units.

3.5.1 Points

A point is represented by a symbol, which should be centered about the position that represents the surveyed point. The center of the symbol, or a mark in the symbol's center as shown on the digital map, should accurately locate the ground survey coordinates of the point.

Symbols are a series of lines (straight or curved) of defined color(s). A feature code (a numeric or alphanumeric label) is the identifier usually assigned in the field as the point data is being collected. Design and survey consultants should ensure that a copy of MHD's list of standard feature codes is obtained before work is commenced. A new feature code should be developed for unanticipated items that do not appear on MHD's standard list, but the MHD Boston Survey Office should be contacted as soon as possible to ensure the coding is satisfactory.

3.5.2 Lines

Lines (often called strings or chains) are a series of points, which are required because the planimetric feature cannot be defined well enough by a simple point. Centerlines, building edges, edge of pavement and shrub outline are all planimetric features which require lines between multiple surveyed points to enable a correct map representation. The points in a line can be defined as curve (spline) or straight. Different software defines different types of splines and how these interact with straight sections of lines.

A centerline is an example of a line with possible straight and curved sections. To define an incoming tangent, circular curve and outgoing tangent, one needs two points (straight) on each tangent and one point on the curve (curve). Software can resolve the entire horizontal curve from this geometry without field location of the PC, PI, or PT.

A building is usually all straight lines. A shrub outline would usually be defined by curves representing the planimetric outline of the feature.

Each line feature code would be represented by a line type and color. Line type might include continuous, dashed, dots, symbols instead of dots, etc. A combination of these types might be used to expand the types of lines where many feature codes are required. Color might be used to distinguish between two related lines types such as centerline and edges of pavement. Line width may also be a definable component of line feature coding.

For a line that closes back upon itself, such as a building, it is desirable to have the same exact coordinates on the end points of the line. Having to re-shoot the initial point as the last point will result in slightly different coordinates and thus not a truly closed figure.

Points in a line may also require point symbology. An example is power poles requiring point symbols but also defining a powerline.

3.5.3 Descriptive Information

Descriptive information about a point or line is often required, as the symbology itself cannot express all of the necessary information. A feature code for tree may require a description of type, diameter, and crown. A manhole may require type, size, etc. A centerline may have a street name as a description. The surveyor must be aware of what descriptive information is required for planimetric features for a particular job before starting the data collection process.

Descriptive information is usually shown at a defined height and offset in a standard fashion from the point or line. In areas where many points are described, a computer screen can become crowded with descriptions. Having software, which shows descriptions for only certain feature codes at one time, can be an extremely useful function.

3.5.4 Other Features of Topographic Information

A break line for Triangulated Irregular Network (TIN) models and production of DTMs could be a special type of feature code or a special on-off attribute associated with all lines. Similarly, points need to be defined for DTM purposes (spot elevations) or simply as a feature, which is not part of a ground model. An on-off "toggle" could designate whether points and lines are part of the "ground" model or that they are "features" whose elevations should not be included in the DTM. Additional attributes (such as cross section point) could also be attached to a point.

Most lines and points are part of the ground model. Feature-only points and lines are usually a small subset of the amount of data, which has been collected.

Another useful way to segregate data is through use of zones or levels. Zones could be by elevation (sub-surface, ground, elevated) or by groups of feature codes (utilities, roads, hypsography, etc.). Some software systems enable the on/off toggling of certain zones or layers of information so a user may concentrate on a smaller amount of data.

3.6 DIGITAL TERRAIN MODEL AND CROSS SECTION DIGITAL DATA

While the conventional cross section approach to roadway design problems related to a ground surface has been a standard for many years, advances in both computer hardware and software have greatly increased the use of Digital Terrain Modeling.

A Digital Terrain Model (DTM) is a ground surface estimator, which is generated from survey data collected on the actual ground surface. The surface data is comprised of spot elevations (with known planimetric location) and "break lines" (known X,Y,Z of points on the lines) which define abrupt changes in the grade of the ground surface. A break line runs along the abrupt change; top of curb, bottom of curb and bottom of ditch are all examples of break lines. An advantage to data collection via DTM is that data does not have to be collected in a specific gridded or cross section fashion. Occasionally, shots taken in cross section fashion are a useful adjunct to DTM spot elevations and break lines.

3.6.1 Development of a Digital Terrain Model (DTM)

A data set of spot elevations and no break lines would imply no abrupt changes in the grade of the ground surface and the following simple example explains how DTMs are generated. Following that is an explanation of how break lines fit into the development of the DTM.

A DTM is generated by initially creating a Triangulated Irregular Network (TIN) of the spot elevations. A TIN is simply a series of interconnected triangles between the spot elevations. No lines cross as the nodes of the triangles are the spot elevation locations. TIN software tries to create the smallest and most equilateral triangles from the spot elevation information. The biggest deviation from these types of triangles will usually be at the edges of the surveyed area.

Since the TIN lines do not cross each other, three survey points, which comprise a triangle in the TIN, have no lines inside of that triangle. The edges of the triangle represent a line along which the DTM would assume a constant grade between the survey points (linear change in elevation). A contour, which enters a triangle based on elevations of the nodes, has its location linearly interpolated between the node points. If a contour line passes through one edge of the triangle it must pass through one and only one other edge of that triangle. A contour line can thus be generated from a TIN by interpolating its location from edge to edge of interconnected triangles in the TIN. Software can smooth the curve of the contour to a user-defined level.

To get an estimate of the elevation of a point inside a triangle, a weighted average (the weight is inversely related to the distance from the point to a triangle node) is used to calculate it from the elevations of the nodes of the triangle that it is contained in.

A profile can be derived from a TIN by defining the desired line and then having software find all of the intersections of that line with lines in the TIN. At each of these intersections an elevation is interpolated between the ends of the triangle edge and from this information the profile can be drawn. Cross sections are drawn similarly but instead are at right angles to the defined line at a defined interval and at a defined extent left and right of the defined line. Centerline and edge of pavement profiles are then derivable from a TIN along with any form of cross section information.

The value of the DTM aspect of the TIN procedure for representing 3-D data is in determining the volume between a design surface and the actual ground. Instead of the historical process of deriving volumes between cross sections, DTM provides a comparison of the two 3-D surfaces. The surfaces are broken up into very small grids and the volume between the two surfaces of each of these grids is computed and totaled. The small trapezoidal areas from the small grids are equivalent to a calculus integration relating the two surfaces.

To incorporate break lines into a DTM, software creates a series of small triangles immediately to the left and right of the break line. These small triangles cause any profile running perpendicular to the break line to show a distinct change in grade at the break line. Break lines allow even vertical features such as top and bottom of curb, or a vertical wall, to be represented in the DTM, allowing more precise volume calculations to be made in the area of the break in grade.

Break lines enhance 3-D perspective viewing of the data. One can envision a computerized view looking down a road and see the top and bottom of curb and a ditch correctly represented. The derivation of a spot elevation from the TIN in areas of the break line is well defined by the inclusion of break line data into the DTM modeling process.

3.7 GLOBAL POSITIONING SYSTEM (GPS) USE IN SURVEY OPERATIONS

GPS is a very useful tool for the collection of high-accuracy positional survey data. This discussion assumes that centimeter level accuracy is desired and that a check on all positions will be made using redundant measurements. Lower accuracy requirements or non-redundant forms of GPS surveying are to be utilized only if appropriate for job requirements. GPS uses for navigation and other comparatively low-accuracy positioning applications are not included in this discussion.

Field procedures described below concern conventional static differential GPS surveys. Other techniques such as rapid static, pseudo-static, pseudo-kinematic, or kinematic may be applicable to certain job requirements.

A more in-depth discussion of GPS uses and Alfred Leick, 1990, Wiley & Sons, Inc, can find procedures, and a list of other sources, in GPS Satellite Surveying.

3.7.1 Global Positioning System (GPS) Field Equipment

Receivers must be capable of centimeter-level accuracy when used in differential mode and the information is processed to a baseline vector.

A) Single vs. Dual Frequency Receivers

On lines less than 20 km in length, single frequency receivers may be used. Dual frequency (L1/L2) receivers may be used but on lines of less than 10 km, processing of the data should be in single frequency mode. When measuring lines longer than 10 km with single frequency receivers, one should occupy points in a session for at least 1.5 hours to enable better ambiguity resolution.

B) Antenna Mounting Systems

The antenna for a GPS receiver should be mounted on a tribrach secured to a standard surveying tripod. The tribrach should have an optical plummet (preferably rotatable) with a cross hair for centering over points that are to be positioned. Tribrachs should be checked at least bi-monthly for proper adjustment by methods such as a tribrach circle, placing two securable tribrachs on top of each other, or checking centering over a point with a plumb bob.

When initially securing a GPS antenna/tribrach combination, one should have assurance from the manufacturer that the center of the GPS antenna and the cross hair of the optical plummet of the tribrach are vertically aligned.

C) Height-of-Instrument Measurement

A mechanism must be provided to precisely measure the height of antenna over the point to 0.001 m without interpolation. A graduated metal alloy (or similar) device provided by the GPS antenna manufacturer should be used for this purpose. The GPS receiver should provide the ability to measure a non-vertical diagonal value to a mark(s), for a check on the antenna with correction by office processing to a vertical component. Because of the importance of a correct height-of-instrument determination, an excellent check is to make the measurement in both feet and meters with two different measuring devices.

In conventional surveying, height-of-instrument and height-of-reflector measurement is unimportant if elevations are not required. This is not true for GPS surveys. Even if only 2-D positions are required, height-of-instrument measurement is critical because GPS measures in a geocentric (earth centered) coordinate system, which is not gravity, based. Any offset from the antenna to the point becomes critical, as it is a shift in all three geocentric coordinates.

3.7.2 GPS Field Procedures

A) Mission Planning

Mission planning before a project is critical to ensure the quality of the output data. Each station should be visited so that the satellite visibility window can be estimated. The extent of any azimuth or elevation angle blockage to the satellite window should be documented. Where obstructions exist, individual site information must be entered so the mission planning software can omit from the solution any satellite information from behind the obstructions.

B) Geometric Accuracy

Accuracy of the data being collected is a function of the number of satellites and the geometry of the satellites relative to the receivers. A variety of accuracy indicators collectively referred to as Dilution Of Precision (DOP) is generated in GPS surveying. Positional DOP (PDOP) is usually the best overall indicator of GPS data quality. Best quality is identified by a low PDOP, and conversely a high PDOP indicates a weak solution due to low number of satellites visible or poor satellite geometry. Data collected when the PDOP is higher than 5.0 is generally considered non-suitable for survey accuracy requirements. If the poor PDOP only exists for 5-10 minutes in a longer (1+ hour) occupation time it will generally not produce adverse results. If poor PDOP data does appear to be contaminating the results, it should be eliminated and reprocessed. If poor results still exist, reoccupation of the line is required. At no time should data be used where less than four satellites are available. PDOP should be evaluated during mission planning and monitored during all field data collection.

C) Elevation Mask

An elevation mask of no less than 15 degrees will be used in all mission planning, data collection and data processing. Satellites within 10 degrees of the horizon generally provide poor geometric solutions, and that data can adversely affect the results from satellites, which are above the mask if incorrectly included in office processing. If office processing is showing questionable results, one may consider masking out data below 20-25 degrees and reprocessing.

D) Occupation Duration

Occupation times for stations should follow FGCC GPS control standards for the level of accuracy required for the project (See Appendix A-11; also available from NGS). As an example, one class and order of survey requiring 60 minute session occupation times for all stations is designed to achieve 20 parts per million or 1/50,000 closures. Even if a 50 minute occupation time yields better accuracy, it is not considered suitable, i.e., field requirements must be met independent of accuracy requirements. Occupation times can vary, but for a particular session the common observation time of all receivers must exceed the minimum value required for the job.

E) Data Collection Interval

GPS data should be collected at no longer than 20 second epochs (time interval between when signals are received). While faster collection rates should not cause less accurate results, neither will faster rates significantly improve results in conventional static mode. Short occupation times may warrant faster collection rates though this practice limits geometry considerations as the satellites will move a shorter distance. Storage requirements and longer data processing times also detract from the use of faster collection rates.

Data collection at Continuously Operated Reference Stations (CORS) is at 30 second epochs. Collection epochs can vary depending on the type of GPS survey.

F) Session Planning

The health of each GPS satellite should be checked when planning a GPS survey, and that status should be monitored during the survey. Unhealthy satellites do not provide useable data and their use should be avoided.

Sessions for a day should be labeled numerically (1, 2, 3, etc.) or alphabetically (A, B, C, etc.) as a standard component of mission planning and station occupation procedures.

Careful planning of station occupation is a critical step prior to GPS field work. Every party member should have a list of the order of station occupations and the times during which those stations will be occupied. If one receiver is collecting data when the other receivers are not, that data is not useable. Likewise, arriving at a station 10 minutes late may cause that station's occupation time to be less than the job requirements. Each station should be assigned a unique four character name/number and that station designation should be included in the document describing the field sessions.

G) Station Occupation Requirements

In occupying a station in the field, the surveyor should first confirm that the correct station has been found by verifying an existing field sketch and ties to objects. A sketch of the point and its surrounding topographic features should be made in a bound fieldbook. Any disturbance of the monument should be documented and if possible a fieldbook rubbing of the monument should be taken. Extreme care should be made to secure tripod legs and to ensure that the GPS antenna is secure in the tribrach and precisely centered over the station. The receiver ID number, height of instrument and temperature and barometric pressure should be measured and always recorded in a fieldbook and also into the field computer, if possible, before and after the session. The 4-character station name and the session label should be entered into the fieldbook and, if possible, into the field computer. The fieldbook should also contain the session start and stop times and any other information, which will ensure accurate and efficient office processing.

Any malfunctioning satellites should be omitted from the data collection process. If a surveyor does not become aware that a satellite is malfunctioning until after returning to the office, the satellite must then be tagged as omitted.

H) Building Checks into the GPS Network

The reoccupation of a station in the next session does not mean simply leaving the instrument setup exactly as the previous session. The tripod must be reset and the antenna repositioned exactly over the station. This will require re-measurement of the height of instrument, providing a check on that value in both sessions, and a check on the centering of the antenna over the station. The second session should have the same fieldbook and computer information reentered.

In control densification projects every station in the project must be occupied in at least two sessions. For most projects, if a station is only occupied for two sessions, those two sessions cannot contain the same combination of other stations. This means if Stations 0001, 0002 and 0003 are occupied during Session A, occupation of the same stations again during Session B does not constitute two occupations of those stations.

An exception to the previous rule is made for a small project such as the establishment of control for a bridge survey. In such a case, two control points will be positioned in proximity to the bridge from two existing control points. With four receivers, two sessions on the same points is a reasonable approach. Between the two sessions each setup should be taken down and reestablished as a check on the height of instrument and the positioning of the antenna over the point. Fewer than four receivers could be used, but more sessions and some movement of receivers would be required.

3.7.3 GPS Office Processing Procedures

Traditional differential GPS baseline processing should be used in the conversion of raw GPS data to GPS vectors (measured difference in geocentric coordinates between two stations).

A) Checking Field-Entered Data

Before beginning the mathematical processing for a session, all station names, heights of instruments, temperatures and barometric pressures should be verified as correctly entered. If the data was not entered in the field, it should be entered before processing. Data found to be incorrectly entered by checking fieldbooks should be corrected before processing is begun. If it is known that certain satellites were not operating properly (health of satellites is available through electronic bulletin board services), they should be tagged and omitted from baseline processing if this was not already done in the field.

B) Baseline Processing

There are three types of standard baseline processing usually performed: double difference fixed solution, double difference float solution, and triple difference solution. Since the processing algorithms are slightly different, each solution produces a slightly different value for the vector. Noisy GPS data, short occupation times, poor satellite geometry or long measured lines can preclude generation of a fixed solution. If a fixed solution is obtained, it is recognized as the "best" of the three solutions and should be utilized. On lines of less than 10 km in length, the inability to produce a fixed solution often indicates the vector produced by the other methods is of suspect quality. Measuring lines longer than 20 km in length with single frequency receivers is not recommended because it often fails to produce fixed solutions. Dual frequency data, processed using L1/L2 frequencies, will be more successful at producing fixed solutions on long lines. Use of a float or triple difference solution in absence of a fixed solution is also not recommended. The correction for failure to reach a fixed solution is reoccupation of the line.

C) Evaluating Vector Processing Quality

Quality control indicators in baseline processing output are of three types:

1. Error estimate of the vector:

   A "plus or minus" in the quality of the DX, DY, DZ (change in geocentric X, Y, Z) between the two endpoints of the vector is estimated from the baseline processing results. It is derived from how the individual data epochs fit one another. The error estimates are in meters and need to be considered relative to the length of the line: an error estimate DX of 0.100 m on a 100 m line would not generally be considered acceptable as 0.1/100 = 1/1000 for a relative error estimate. By comparison, an error of 0.1 m on a 10 km line, or 1/100,000, is typical of standard GPS control surveying.

   If 1/50,000 results are desired, an error estimate, which produces a relative error of 1/20,000, should be considered suspect, but it should not be considered unsuitable until the "fit" with other measured vectors through loop closures and least-squares analysis are checked.

   The root-mean-square (rms) error estimate of the vector is a cumulative error estimate derived from the DX, DY, DZ error estimate values, and can be used in lieu of the individual error estimates of DX, DY, DZ.

2. Number of epochs used vs. total number of epochs:

   This reflects the number of satellite epochs (measurements), which were considered by the software as realistically "fitting" the other epochs. Experience has shown that if less than 95% of the total number of epochs was used (5% or more were rejected) then the vector may be of unsuitable quality. As in (1) above, it should only be considered as suspect at this point.

3. Ratio error:

   This is a quick indicator of the strength of the solution. The optimal solution results in 100.0 for a ratio indicative of excellent resolve of solution ambiguities. The optimal ratio will normally be obtained under good satellite geometry and sufficient occupation time. Ratio errors of less than 98 should be considered unsuitable. Any ratio less than 100 should be considered suspect and closely reviewed in subsequent network analysis.

   The true test of the quality of the GPS results is not achieved by baseline processing. Incorrect instrument height, failure to center over a point, incorrect control coordinates and misidentified stations are examples of errors not resolved at this point. The next step is examining the internal quality of all GPS vector information and the integration of all sessions' vectors.

3.7.4 Non-Trivial vs. Trivial GPS Vectors

Before data quality can be examined, the principal measurements produced from baseline processing must be defined. One session with two GPS receivers will produce one vector measurement between the two stations occupied. The DX, DY, DZ values from station A to B are equivalent but opposite in sign to the values from B to A. In this case A to B and B to A are not two measurements. A reoccupation of A and B in a subsequent session would produce a second vector measurement between these stations to compare to the value obtained from the first session.

If three GPS receivers are being used per session, baseline processing will produce three vectors. For a given session, the third vector is always the product of the first two. The DX, DY, DZ values add up to zero, minus round off. For three receivers in a session, two non-trivial and one trivial vector is produced; only two vectors have been measured, and a third vector is generated from the other two.

Having "n" GPS receivers in a session produces "n-1" non-trivial vectors, most easily visualized as all vectors in a session being from one of the stations to the other "n-1" stations. What station is selected as the "from" station for a session should be defined during mission planning to ensure at least two occupations of all stations. Baseline processing of all combinations (trivial and non-trivial) vectors does no harm, but does require longer computation time. It is important to realize a loop closure using all vectors in a session will produce perfect results minus round off because of the nature of the computations. Likewise, using all six vectors (3 non-trivial, 3 trivial) in a four receiver session as part of a network analysis provides misleading results as the network will appear to have more redundant checks than it truly does. The solution is to never use trivial vectors in any analysis of GPS solutions. The rest of this discussion refers to analysis of only the non-trivial vectors.

3.7.5 GPS Loop Closures

The simplest procedure for evaluating network results is a loop closure with the sum of the DX, DY, DZ components in the loop. Due to the random errors in the measuring system, a misclosure in DX, DY, DZ will be produced in a loop. The square root of the sum of the squares of the misclosures in DX, DY, DZ is the linear misclosure. The sum of the vector lengths is the total distance covered, and thus the linear error of closure divided by the total distance in the loop produces the traditional ratio error of closure. While in conventional surveying this value is usually in 1/x form, in GPS this value is usually given as a ppm (10 ppm = 1/100,000). Any loop closure should yield a ratio closure or ppm better than the accuracy requirements of the job.
Every vector should be included on at least one loop closure computation. If a closure indicates an error, performing different loop closures can help identify the vector which is the most logical source of the problem.

Propagating the error estimates in the vectors along the loop enables computation of the estimate of the closure. This will be a one standard deviation (one sigma) estimate, and form a normal distribution (bell shaped) curve. The closure can only be expected to be better than the error estimate 67% of the time. This is why the outer limit for a sure method of error detection is considered three times the propagated value; 3 sigma is approximately 98.6% confidence.

In the case of small loop distances, the probability of obtaining a very small ppm closure is limited by the short distance. Closure for a small loop of better than 0.01 m can be considered suitable for most control work, independent of the resulting ppm.

3.7.6 Least-Squares Analysis and Error Estimation

Least squares is the most accepted process for analyzing a GPS network. The limit of the loop closure is that it can only look at a portion of the data. Least squares uses the error estimates of the measurements in evaluating where misclosures on vectors, called residuals, should exist. For example, a vector with a DX error estimate of 0.1 m will generally have a larger residual than a vector with a DX error estimate of 0.01 m. It is possible that the first DX may have a smaller residual simply because it fits the rest of the data better. A residual can thus be considered the difference between the measured and the adjusted quantity based on the produced final coordinates.

Error estimates for vectors being subjected to the least squares are usually generated by one of the following methods:

a) Error estimate results from baseline processing are used, or

b) The error estimates are similar to electronic distance measurement errors, which are usually a constant plus a ppm. A line receives an error estimate no matter how short it is (constant error) and the error estimate grows for longer measured lines (ppm error).

Each method of error estimation is considered valid, each is suitable for least-squares analysis and each is used by commercially available software. MHD recommends method (a) above as those error estimates are related to the conditions that existed at the time of the data collection, e.g. noise, atmosphere, window blockage, cycle slips, etc.

3.7.7 Minimally Constrained Least-Squares Analysis

The first goal in network analysis (as opposed to individual loop analysis) is verification of the internal fit of the GPS vectors among themselves. In this analysis the final coordinates are not important and the standard procedure is to hold one of the stations fixed in 3-D position based on its approximate coordinate values from single point positioning data generated during vector processing. Holding only one point fixed (minimally constrained) ensures the residuals of the GPS vectors will not be influenced by control coordinates. Mathematically there is no check on control coordinates in a network with only one fixed station and likewise any error in control coordinates will not be propagated into the residuals in the GPS vectors.

The minimally constrained adjustment is an indicator of the quality of the GPS independent of the control which was occupied and is analogous to a conventional traverse with one known horizontal station, one bench mark and one fixed bearing. The value of residuals in DX, DY, DZ components should be close to their error estimates and if it is more than three times the error estimate the measurement is considered suspect. A ratio error of closure can be calculated for the vector by dividing the square root of the sum of the squares of the DX, DY, DZ residuals by the length of the line and that can be converted to a ppm equivalent. No 1/x or ppm derived from the residuals should exceed the tolerances for the job based on the survey accuracy required. The exclusion is short lines in which independent of 1/x, a linear residual of less than 0.01 m is acceptable for most survey applications.

3.7.8 Constrained Least-Squares Analysis

When suitable results from the minimally constrained analysis are confirmed, the next step is to see how the GPS vectors fit the existing control stations, which are part of the network. One difficulty in GPS surveying is that the measurements can often be of better quality than the control coordinates. Holding less precise control coordinates fixed will result in GPS vectors with larger residuals than reflected by the minimally constrained analysis.

A) Control Issues

Control can be 3-D, 2-D or 1-D in nature. While in route projects the 3rd dimension is elevation relative to the Geoid, often approximated and referred to as mean sea level, GPS does not directly measure elevation differences. The third dimension in GPS is distance up or down from the ellipsoid (Clarke 1866 in NAD 27 and GRS 80 in NAD 83).

B) Relation of the Ellipsoid to the Geoid

In a local area (no larger than 10 km in any direction) where no gravity anomalies are present, an ellipsoid height difference and an elevation difference between two stations will be within 10 cm. The direction of gravity and a perpendicular to the ellipsoid will also be near coincident and the ellipsoid height difference between two stations can be assumed to be very near the elevation difference in magnitude for GPS networks in a local area. This assumption is valid for surveys where precise elevations are not required and is within the accuracy limits of trigonometric leveling. The exact difference between ellipsoid and orthometric heights can be determined by comparing ellipsoid height difference obtained by GPS with orthometric height difference obtained by leveling.

In a local area, the elevations can be considered ellipsoid heights in what could be called a local ellipsoid solution. In a least-squares adjustment to determine 3-D position, the correct elevation should always be the ellipsoid height. To test the reliability of elevations being produced, withhold some of the bench mark elevations as control and see how close the least-squares results are to their published values.

The solution to resolving the ellipsoid height to elevation issue is to occupy three or more bench marks in every GPS network. At least three horizontal control points should also be included in every GPS network, because if one of the GPS points is in error, having only two control points can identify the problem but not resolve it. While lack of window due to obstructions or time constraints may prevent some control points from being occupied, preference is that all control of suitable quality in an area should be part of a GPS survey to ensure its consistency.

It is highly recommended that all GPS surveys be tied into the HARN (High Accuracy Regional Network). The HARN is the basis for the NAD 83-92 adjustment. At this time, NGS is publishing only the NAD 83-92 values for its horizontal positions. Another important reason for using HARN is that it provides an accurate ellipsoid height. To obtain good orthometric heights for a GPS network, several things are required:

1. At least one, and preferably more, accurate ellipsoid heights;

2. At least three accurate elevations of GPS points;

3. A method of determining Geoidal separation at all points within the network.

With the introduction of Geoid 96, determination of orthometric heights with GPS has become easier and much more accurate. Geoid 96 has an accuracy of +/- 0.01 m and its use in a network with known ellipsoid and orthometric heights allows the determination of elevations to within +/- 0.01 m. Unless stated otherwise by MHD, differential leveling will produce all control elevations.

C) Biased Constraints

The constrained least-squares analysis normally allows "biased constraints" to be defined, which allows rotation of the GPS vectors to better fix the control points in the area. This is especially useful when using elevations directly because the Geoid/ellipsoid separation changes slowly over an area, which amounts to a rotation. The biased constraints should be used with extreme caution as "rotating" the GPS survey to fit unreliable control coordinates is an incorrect process. The unreliable coordinates should be identified and not used.

D) Control Evaluation Process

The most logical way to start the constrained least-squares evaluation process is to add control in sequentially. Start with the best horizontal and vertical control station(s). One 2-D and one 1-D as control is the equivalent of a minimally constrained analysis. Run the least squares and examine the "closeness" of the produced coordinates of control, which was withheld to the published values. If the results appear in close agreement, those control coordinates can probably be added to the adjustment without adverse effect. Control coordinates that don't match well should be added last or not at all due to lack of fit with the other control coordinates in the network.

As you add more control the least-squares analysis, statistics may worsen, due to imperfections in control coordinates. If the data becomes dramatically worse, a suspect control coordinate has been entered and must be eliminated or otherwise resolved (e.g., the coordinate may have been entered in the computer incorrectly).

E) Identifying a Realistic Analysis Result

A realistic constrained analysis will produce vector residuals, which are no worse than the job accuracy requirement. If desired accuracy is 1/50,000 (20 ppm) no vector should have a residual greater than 20 ppm except for extremely short lines where the 0.01 m residual can be used. If the minimally constrained analysis produces reliable results, and if with all control entered the constrained analysis shows results, which do not meet, desired accuracy, the surveyor must determine which control points are suspect and they should be removed from the constrained analysis.

All control coordinates must be in the same datum. HARN network coordinates may not match some local control performed by conventional traverse in even fairly recent years. Two vertical datums exist in North America and they cannot be mixed. Final results must produce coordinates, which are precise relative to one another and from which a route project can realistically be developed.

3.7.9 GPS Control Survey Final Products

A completed GPS control survey should include the following:

a) Mission planning results for all sessions showing satellite visibility (including any obstructions present), PDOP plot and a plot of the number of healthy satellites available.

b) A copy of all fieldbook information, which should include descriptions, sketches and rubbings of all points for each session. The height of instrument (measured twice by independent methods), temperature and pressure should be included in the fieldbook information for each occupation.

c) All raw receiver file information should be provided in RINEX format or such other format approved by MHD.

d) Any edited raw receiver files should have changes noted.

e) All output files from vector processing should be included. Any solutions used which were not fixed solution, or a ratio of 100, should be highlighted and detailed as to why they were used.

f) A listing of the minimally constrained least squares network analysis where the "best" control point's coordinates were held as fixed. Loop closures should be provided if deemed appropriate.

g) A listing of the constrained adjustment highlighting the control that was used. If some occupied control coordinates were not used, the cause for rejection should be explained.

h) Any elevations produced should be clearly identified as GPS-derived and not given any class or order. The procedure for obtaining elevation (GEOID 93, GEOID 96, published Geoid/ellipsoid separation, etc.) should be described in detail.

3.8 PHOTOGRAMMETRY

Photogrammetry is the use of aerial photographs for measurement and mapping purposes. Traditional applications for most route projects involve:

a) Large-format calibrated aerial cameras with a specific mount in an airplane;

b) Processing of photographs in one-to-one production of images on film or glass diapositives specifically designed for minimization of distortion due to temperature, pressure, and humidity changes during production;

c) Adequate ground control, which is either targeted, or photo identifiable;

d) A measurement and ensuing least-squares analysis process called aerotriangulation, which validates the ground control and densifies it as needed for use in stereoplotter orientation and map compilation;

e) Stereoplotter orientation based on the densified ground control which resolves the relation of the photos to each other and the ground at the time of exposures and provides a check on the quality of the aerotriangulation;

f) Compilation of the desired features, which could include planimetric features (line and point symbology), contours, cross sections, profiles, break lines, spot elevations, and text information;

g) Clean-up of compiled information to make it topologically pleasing, e.g., an edge of driveway should not extend past the outline of a house;

h) Addition of field survey data that could not be collected photogrammetrically due to obstructions, cover, shadow, etc.;

i) Production of final products, which could include translation to other digital map formats and/or hard copy output.

3.8.1 Camera

The typical camera used for photogrammetry is metric with a format of about 227 mm x 227 mm and a focal length of 152.4 mm (nominally 6 inches). The exposure system has a film flattening mechanism so that during exposure the film is flat and perpendicular to the line from the camera principal point to the rear nodal point of the lens. A large magazine for film storage is required to accommodate the large number of exposures, which can occur during a mission. The camera should contain at least eight fiducial marks in the corners and sides of the picture area for measurement of film shrinkage or expansion. A mechanism to correct for airplane movement during exposure is highly desired.

The camera should be calibrated by the U.S. Geological Survey approximately every two years. Calibration reports provide calibrated focal length, fiducial mark coordinates, principal point location, resolution, and radial lens distortion.

3.8.2 Photogrammetric Flight Mission

Photography is taken at a flying height, which is a function of the accuracy of the desired product and should be within 5% of the desired scale.

Since adverse weather will most likely affect picture quality, marginal days for aerial photography should be avoided. Low sun angle, a function of time of day and season, should also be avoided so that the image quality should not be adversely affected by shadows or reflection. Obstruction by leaf cover, a function of season, should also be avoided.

A suitably equipped airplane flying as straight as possible along a flight line captures aerial photography. Since stereo coverage (each point on at least two photos) is required, overlap (endlap) between successive photos along the flight line should generally be between 60-65% with deviation of more than 5% considered unacceptable. To ensure proper stereo viewing, the camera axis should be as near vertical as possible, with tilts along the flight line or perpendicular from it of more than three degrees usually considered unacceptable. Rotation of the flight line between successive photos, or from the overall intended flight line direction, of more than five degrees is considered unacceptable.

Substitute photography, obtained in place of unacceptable photography, shall comply with the original requirements. Overlap to the original accepted photographs must meet the minimum specified limits but can overlap more extensively at the ends to ensure no gaps in the data. Numbering of substitute photography shall continue in an unbroken sequence from the last number used in the original photography.

Some projects require multiple flight lines to satisfy job requirements. The flight lines are usually parallel and overlap between ground coverage of adjacent flight lines is generally 20 to 35 percent. Multiple route geometries for the same project (intersecting roads) may require multiple flight lines, which are non-parallel. To ensure no gaps in coverage, standard practice is to have 100% overlap of these lines in the intersection area.

3.8.3 Film Production

Film production should result in so-called diapositives on either glass or film. All prints should be contact prints, produced at the same size as the negative. The diapositive image should be distinct and clear so that all desired planimetric and topographic features are easily measured. The processing should minimize any shrinkage or expansion of the film.

To ensure easy management of the information, a photography index should be prepared. Consecutive overlapping photographic prints are edge-matched near the center of their overlap area and attached together using staples, glue, etc. The photos may be trimmed in the overlap area to ensure neat edges. The photos should be labeled with project number, date of flight mission, job-specific photo identification and any other pertinent information deemed appropriate. The labeling must be in a consistent location on all photos.

3.8.4 Ground control

Horizontal and vertical ground control is required in the coverage areas for any photogrammetry project. MHD experience with photogrammetry operations indicates a need for horizontal control every 5-6 photos along a flight line, with vertical control every 3-4 photos. The need for vertical control is due to the weaker geometry of aerotriangulation and analytical stereoplotter orientation for mathematical solutions in the Z dimension. If aerotriangulation is not being used, a minimum of three horizontal and four vertical control points are required in every stereomodel to provide the required geometry and a redundant check. The ends of flight lines should contain both horizontal and vertical control.

Since route projects are often single flight lines, all of the exposure stations are in a near straight line. This geometry requires that some of the vertical control be located at the outer edge of the photography, perpendicular to the direction of the flight line. These points are often called wing points and require ground survey to be performed off the project right-of-way as the route is usually parallel to the flight line and near the middle of the photography.

Ground survey of the control points should tie in all control stations in one contiguous survey network, or ensure only high-order control stations are used. Utilizing independent surveys based on different control does not confirm the reliability of control coordinates to each other, so it is important to create a survey "network." Except in unusual circumstances, all surveys should tie to current standard horizontal and vertical datums and all coordinates produced should be based on those datums. Datums are discussed in Section 3.4, Conventional Field Data Processing.

Care must be taken to ensure that control will be visible on the photographs at the time of the flight mission. Shadows from trees or other structures can easily make control unidentifiable, as can vehicles moving over control points located on a paved surface.

Targeting of control points is highly recommended to ensure easier identification on the photographs. Target size is a function of the flying height of the photography. On suitable surfaces (pavement, sidewalk, etc.) targets are painted in a color that contrasts with the surrounding area; targets of durable material (wood, fiberglass, strong cloth, etc.) are used on grass or dirt to ensure that grass will not grow to obscure the target or rain will not cause runoff to cover the target in the interval between targeting and flying. Targets are usually in the shape of a "+" or a "T" with the intersection precisely positioned over the control point. In the case of a vertical control point where the target is elevated over the survey point, one must ensure that the mapping is based on the target elevation corrected for its offset to the survey point.

Control points should be explicitly described in a fieldbook so the position is uniquely defined. The location of control points should be pin pricked on prints and highlighted with grease pen. A circle with a marker should be made on the back side of the print around the pin prick location, along with a point identification name or number, and short description if needed.

The control survey should follow standard survey procedures as described in Section 3.2, Field Operations With Conventional Survey Equipment. GPS is especially suited for such use since both it and control targeting require open areas. If GPS is used for vertical control, one must ensure that the derived elevations from ellipsoidal height differences are of suitable quality for the required accuracy for the project. If trigonometric leveling is used for elevation determination, care also must be taken to ensure it is of suitable quality for the project. Differential leveling for vertical control will be required for many MHD projects to ensure required accuracy.

Airborne kinematic GPS can be used in the measurement of exposure station positions as control to reduce the amount of ground control needed. However, there must be sufficient ground control so that aerotriangulation can be processed and used as a check on the kinematic GPS positions and to ensure required accuracies are maintained.

3.8.5 Aerotriangulation

Aerotriangulation is a procedure, which validates the quality of the ground control, coordinates and densifies that control to where it is suitable for the collection of the positional and attribute information desired for the project. The quality of the aerotriangulation results is not only dependent on the quality of the ground control, but also on the quality of the photogrammetric measurements, which are part of the aerotriangulation process. Photogrammetric measurements in this case are precise measurements of the x,y photocoordinates of points based upon a coordinate system aligned to the fiducial marks, which places its origin near the center of the photograph and whose x axis is between the side fiducials which are along the flight line axis.

Measurement of photocoordinates is usually performed on an analytical stereoplotter but could be performed on an instrument called a comparator whose sole purpose is coordinate measurement and not map compilation. Resolution of measurements is generally in the 0.001 mm range with accuracy tending to be in the 0.003-0.010 mm range. Instruments of lesser measuring quality should only be used if results are obtainable that meet the accuracy requirements of the project. Coordinates are usually measured in stereo as it is a quicker measuring process but can be measured monoscopically if all points are monoscopically photoidentifiable.

To obtain photocoordinates, fiducial marks are first measured. This allows the arbitrary coordinate measuring system of the stereoplotter to be transformed to a photocoordinate system. Since the calibrated fiducial coordinates can be compared to their measured values during that process, a correction for shrinkage and expansion of the diapositive can be applied. Control extension is performed through identification and measurement of "pass points". These must be user-picked photoidentifiable points, but in many cases are marked positions using a process called "pugging". A pug is an artificial mark in the emulsion of the photograph usually made by a very small drill bit on a pug machine. Pug points are usually picked by stereo viewing on the pug machine, at locations which ensure the necessary geometry for a mathematical solution in aerotriangulation and are also easily viewed in stereo for precise measuring purposes. If the points will be measured in stereo the pug is only made on one photo as the mark will "appear" in an adjacent photo through stereo viewing.

Aerotriangulation requires at least six points to be measured in every stereomodel, positioned approximately as shown below.

1	2
3	4
5	6

Points 1, 3, and 5 should be measurable in the adjacent left stereomodel, and likewise points 2, 4, and 6 should be measurable in the adjacent right stereomodel. The ability to perform this operation is ensured by the 60-65% endlap rule. If the job has multiple flight lines that are adjacent, exterior points (1, 2, 5, and 6) should be measurable. A control point near one of these locations can be substituted as a pass point in lieu of identifying an additional point.

Points 1, 3, and 5 are near the center of the left photo in the X direction and would be pugged (if this process is used) on that photo. They would then be able to be viewed in stereo in the two models that it is part of. Similarly points 2, 4, and 6 would be pugged on the right photo.

The first check on the quality of the measurements is that, except for end photos on a flight line, a point is measured in at least two stereomodels, i.e. measured twice so coordinates can be compared and generally averaged if deemed acceptable.

The mathematical component of the aerotriangulation is a three step process though some processing systems take only two steps.

A) Stereomodel Joining

During the collection process, each stereomodel produces 3-D coordinates of measured points called model coordinates, which are derived from photocoordinates. The model XYZ system is simply an assumed system necessary for processing, but not used in the final product.

Independent stereomodels along a flight line are "joined" together based on common points and 3-D coordinate transformations to make one uniform coordinate system. The fit of the coordinate transformation residuals is an indicator of the quality of the photogrammetry measurements.

B) Strip and Block Adjustment

The flight line's contiguous model coordinates for control points are compared to their ground control coordinates permitting a 3-D coordinate transformation of all model coordinates into the ground coordinate system. To account for systematic errors in the control and the systematic error buildup of independent model coordinate joining, polynomial corrections (usually no higher than second degree) are applied as part of the 3-D coordinate transformation. Since the control should be redundant, residuals in ground coordinate units indicate the "fit" of the process. As part of this process, usually called strip adjustment, errors in control coordinates can usually be identified.

If multiple flight lines exist, the tie points between flight lines are used in 3-D coordinate transformations with polynomial corrections to indicate the fit of the separate strip adjustments and attempt to produce a more uniform result for the ground coordinates of all measured points.

C) Bundle Adjustment

This procedure is a simultaneous least-squares analysis of all photogrammetric measurements and control coordinates, and theoretically, is the most effective means for analyzing final results and producing the best possible final coordinates. The processes detailed in (A) and (B) above serve as an information check and set up some necessary intermediate information required for this step. In addition to solving for all final ground coordinates, this procedure solves for all camera exposure positions and the angular orientation of the camera axis at the time of each exposure relative to the ground coordinate system. These results validate that appropriate flying heights and camera orientations existed during the flight mission.

Because of the high precision of the photogrammetric measurements (0.003-0.010 mm), holding ground coordinates (which contain random error) fixed can affect the output accuracy indicators of the photocoordinates. Suitable error estimates should be placed on the photocoordinates and on the ground control coordinates to produce suitable least-squares analysis statistical indicators from the bundle adjustment. Final results should place the overall photocoordinate root-mean-square (rms) error in the 0.003-0.010 mm range, while the ground coordinate rms errors should reflect the expected reliability of their values based on the survey procedures used in their derivation.

Based on suitable aerotriangulation results, all points measured during the aerotriangulation process will have ground X,Y,Z coordinates resulting from a simultaneous analysis of all ground control and photogrammetric measurement data. Due to the location of pass points in stereomodels, at least six points have ground coordinates, a redundant solution for the stereoplotter orientation required for mapping. Since the control also exists in multiple stereomodels and possibly flight lines, there is assurance that in map compilation the X,Y,Zs of collected data in one stereomodel will closely match (called edge match) their corresponding positions in adjacent stereomodels.

3.8.6 Types of Stereoplotters

An analog stereoplotter without encoding has no computer communication. All orientations are performed manually and all information must be compiled in hard copy format. Since this type of compiling dramatically limits the accuracy of the produced information, and it is inaccessible to computer systems, its use is not recommended.

An analog stereoplotter with encoding enables limited communication between the computer and the stereoplotter. Computer software can enhance the orientation process, although it is still performed through a series of operator-controlled dials. Data can be entered directly into the computer with the limitation that the computer generally cannot drive the plates which the photographs lie on. Aerotriangulation is usually not performed on these instruments. While this process could be used if it is verified that the encoding is precise enough to meet job requirements, its use is not recommended.

An analytical stereoplotter is one in which all orientations are computer driven, the computer can drive the stereoplotter plates, and the data collection is completely digital though all measurements are made by the operator. It must be verified that the measuring resolution of the stereoplotter is precise enough to meet the job requirements.

With a digital stereoplotter ("soft copy") photographic diapositives are replaced by computer images. While the image could be produced by some non-film based raster collection system, most digital images are produced using a scanning process of the image from the diapositives. One needs to ensure the scanning resolution is of a size that permits the required accuracy requirements to be achieved, or that any systematic errors in the scanning process do not cause similar problems. Digital systems can usually replace some of the human operations through image recognition, infinite zooming capability, and by allowing the collected data to be superimposed on the raster image. Due to the large file size of scanned images at small resolution, and the need for efficient management of the data files, large computer drives and rapid computer processing speeds are required.

3.8.7 Stereoplotter Orientation

Stereoplotter orientation involves recreating the camera exposure geometry (position and altitude) and the creation of a ground coordinate system derived from the stereomodel.

Orientation is a three step process, though the last two are often integrated into one measuring process.

a) Inner Orientation - The process of placing the photographs on the plates in the stereoplotter and measuring the fiducial mark locations. A 2-D transformation (unique to the left and right photos) produces photocoordinates and residuals since the transformation is a redundant solution.

b) Relative Orientation - The same point in the left and right photo is measured for six distinct points usually in the general locations of the six standard pass points. More than six points can be used for a more redundant, and hopefully better solution. This solution resolves how the exposures were angularly related to each other at the time of exposure and on computer based systems results in the production of model coordinates. Residuals again indicate the quality of the measurements of this procedure.

c) Absolute Orientation - All control points in the stereomodel are measured and a 3-D conformal transformation converts model coordinates into ground coordinates, with residuals if the solution is redundant in nature. On analog systems this is commonly referred to as a scaling, rotation, and leveling procedure. The residuals (in ground units) should be of sufficient quality to meet the job accuracy requirements.

   A measuring mark is moved through the stereomodel. After absolute orientation, the X,Y,Z position of the mark changes as it moves. To measure a position, a mark is placed on a point's image. The position is in 3-D since it is viewed in stereo.

The merging of relative and absolute orientation, usually called outer orientation, is performed by measuring control points as part of the relative orientation process. If aerotriangulation has been used, analytical and digital stereoplotters can perform automatic relative and absolute orientation from the aerotriangulation results and only redoing inner orientation will be required prior to mapping. On some digital stereoplotters, inner orientation does not have to be reperformed when aerotriangulation is complete since there is no placing of diapositives back into an instrument.

Inner and relative orientation residuals are in photocoordinate units and thus should rarely exceed 0.010 mm. Absolute orientation is in ground units and a general rule is that these residuals should not exceed 50% of the expected accuracy of the final product. For example, if a DTM is expected to be within 0.20 m relative to a field-checked position, the vertical residuals in absolute orientation should not exceed 0.10 m.

3.8.8 Topographic Data Collection via Photogrammetry

Once stereoplotter orientation has been completed, any form of topographic information can be entered. Such topographic map information generally refers to items such as planimetric features (line and point symbology), contours, cross sections, profiles, break lines, spot elevations, and text information. If a job requires a digital product (usually in addition to hard copy output), MHD file format should be followed.

A) Planimetric Features

Job requirements will determine what planimetric features need to be collected and MHD symbology standards should be adhered to. Some job specifications may define what symbology will be used to define points and lines, while for other projects it may be left up to the photogrammetrist to come up with a standard symbology and convey the linkage of the symbology of the data to what it actually represents.

Symbology for points (manholes, power poles, trees, etc.) are at a given size based on the desired scale of the final product. Both color and line type make up a symbol. Likewise, textual information is generally of a size which is realistic for the product's map scale.

Lines can be either straight, curved, or a combination of the two. In most cases "curved" implies a spline representation of the point information. The curve can imply circular, such as the case where only three points make up a curved line, or when two straight tangents are each defined by two points, and intermediate to them is a curve point. This case produces a circular arc between the two tangents.

Map information should be seamless across map sheets and across stereomodels. When compiling a stereomodel, which overlaps one previously compiled, it is important to verify that no significant misclosure in planimetric position or in elevation exists for the same line or feature. The same is true for a spot elevation. If a misfit between stereomodels is well within random error limits, common practice is to "fit" the latter to the data already collected. A misfit exceeding random error limits is generally caught in the aerotriangulation and/or orientation process. If a large misfit is found during map collection, mathematical solutions in that region should immediately be checked.

B) Contours

Collection of contours by photogrammetric means has traditionally been an actual 3-D (stereo) view tracing of the contour line. The contour line is made topologically correct when it meets items such as roads, buildings, etc.

Prior to the digital map era, the stereoplotter operator ended contours at planimetric features like buildings. Digital mapping allows the creation of many different types of maps, one of which is only contours. Such contours would be continuous, even passing through a building location. Software allows continuous contours at features like buildings to be "clipped" if that type of map is desired.

Many contours are now derived from DTMs, which are a combination of spot elevations and break lines. DTMs afford a better 3-D analysis of data and output of various elevation information. An internal check of the quality of contours derived from a DTM is made by comparing those to contours traced conventionally. An external check would involve comparing field cross sections to those derived from the DTM, or comparing contours derived from a field survey with those developed photogrammetrically. Accuracy requirements would be a function of the intended purpose of the data.

C) Cross Sections

In open areas, the collection of cross section information by photogrammetry can dramatically speed up tedious field data collection while at the same time eliminating danger due to traffic in the field collection process.

Photogrammetric cross section software requires the identification of stationing along a defined baseline. The desired interval for cross sections and the distance to be cross sectioned left and right of centerline is also input by the user. Software will utilize this information on analytical and digital stereoplotters to move the operator to the proper location to be measured.

The operator can set up the software to drive the measuring mark along the cross section line as the mark is manually set to the correct elevation, or the software can limit the operator to moving the mark only perpendicular to the centerline. When obstructions or shadows prevent measurement directly along the cross section, software permits the operator to move left or right of it to collect an elevation, which will be "moved" back to the cross section line.

Software generally allows the operator to define more cross sections at important intermediate positions such as intersections, high points, low points, etc. The cross sections should be able to be output in hard copy at a desired scale, or translated to a desired format for engineering design purposes.

D) Profiles

A cross section is a profile at a defined station perpendicular to the centerline. A profile is simply a generic form of elevation representation along a defined line. Profiles are compiled photogrammetrically in the same way as cross sections.

Typical profiles required on route projects may include centerline, edge of pavement, top or bottom of curb, bottom of ditch, etc. It is usually important to identify important planimetric information along a profile such as edge of driveway, manhole, etc.

E) Break Lines and Spot Elevations

While a more thorough discussion of this topic is found in Section 3.2, Field Operations With Conventional Survey Equipment, break lines and spot elevations are the type of data included in creation of a DTM. A DTM which is developed strictly from spot elevations infers that no sharp breaks in grade occur anywhere in the ground surface. Since this is rarely the case on route projects, sharp breaks which should be collected as break lines occur at bottoms and tops of ditches, at other abrupt changes in the ground surface, and at man-made breaks in grade such as top of curb, etc. The DTM surface will noticeably "break" at these lines, while in areas where only spot elevations exist the surface will look smoother.

Photogrammetric collection of spot elevations (X, Y, Z mass points) is usually performed on analytical or digital systems by having software dictate a user-defined interval throughout the stereomodel in grid form. The density of the grid is a function of the required accuracy and the undulation level of the terrain. The user can ignore a point or move a point slightly for measurement purposes if a grid intersection is obscured due to trees, shadows, or other reasons. The selected points are usually displayed graphically. Break lines are simply a special line type and are collected as any other line type, though special care must be taken to measure precise elevations during the process.

Most DTMs are generated from creation of a triangulated irregular network (TIN) of the spot elevations and the break lines. Contours, perspective views, profiles, and cross sections can then be derived from the DTM. As an internal check to verify the internal and external quality of the DTM, it is common to collect some profiles and/or cross sections from the same information that the DTM was created from and compare them to those derived from the DTM. To check externally, collect profiles and/or cross sections by conventional field procedures and compare them to those derived from the DTM. The accuracy requirements will be a function of the intended use of the data.

F) Text Information

As with any ground-based topographic survey, descriptive information is often required in addition to map symbology and positional information. The textual information should have consistent placement relative to the positional information, with a size compatible with the desired hard copy scale. MHD specifications should be strictly adhered to.

Information required for cross sections, stationing, and profiles tends to be fairly standard. The difficulty lies in how much textual information is required to augment planimetric data which cannot be conveyed through symbology. Consistency for a given project is required, and the type of job will dictate the amount of textual information required.

3.8.9 Map Clean-up

The stereoplotter operator is usually not responsible for final-product topographic information. It is not considered time and cost effective for the operator to spend much time making map data topologically pleasing. For example, for compilation of a driveway added to an already compiled building, the end of the driveway will most likely not end precisely at a building edge line. Similarly, the end line for a building outline may fail to close exactly with the initial point. A more difficult problem is to ensure that a contour line crosses the edge of pavement at that line's elevation.

Manual map cleanup is usually performed at a digitizing tablet, where problems are visually searched for and corrected. It can be a time consuming process and conditions that need correcting can be easily missed.

Software can be used to perform map cleanup automatically, and can work well if effective feature coding is utilized. For example, all driveway feature codes whose end-of-lines come within X m of a line designated by a house feature code should intersect perfectly. All house feature code end-of-lines which do not intersect or connect with another line should be intersected with another end if within Y m. The values for X and Y would be determined by the accuracy at which the data is being collected. While such software will not find every possible type of clean-up required, it highly automates the process for the most common types of clean-up.

3.8.10 Integration with Field Survey Information

It is rare that all required information can be collected photogrammetrically. As examples, an edge of pavement may be obscured by tree cover, underground utilities will not be seen from an aerial photo, an object may be difficult to measure because of high reflectance and the distance from bottom to top of curb is often too small to see photogrammetrically. This type of data must be collected by conventional ground survey and added to the photogrammetry data. In such cases, the same control must be utilized to integrate the data correctly.

Field survey should also be used to check both the positional and attribute photogrammetric data. Positional checks can include horizontal position of planimetric features, spot elevations of distinct points, profiles, and cross sections. It is most important that the same control used for the photogrammetry is used for the field survey and that field-checked spot elevations, profiles and cross sections are correctly located. An improperly located field check cannot match the photogrammetric data since each set of data will represent a different ground position.

Confirming attribute information involves field checking that the plotted information for a point or line is correct. For example, a stone wall may have been inadvertently coded as a wooden fence. Also, the surveyor needs to check for missing point or line information.

3.8.11 Final Products

Hard copy output of collected data is an important job product, since the end user may not have the necessary hardware or software to work the data digitally.

The scale, type of map media (paper or mylar) and the type of output are important needs, which must be defined before the project begins. The end user should also define layout size, title positioning, scale representation, north arrow, etc. MHD has specifications for these parameters.

When digital (computer based) products are desired, the photogrammetry system's digital output may not be in the same format as the software utilized by the end user for purposes such as planning, engineering design, traffic, right-of-way, etc., and translation between formats is necessary. Accurate and efficient translation that provides a correct end product must be ensured.

APPENDIX A-1

* MASSACHUSETTS GENERAL LAWS

The Massachusetts General Laws, Annotated, are in a conveniently arranged group of volumes accompanied by the addendae to update them. These books contain all of the General Laws and amendments (to a particular date), legal data and comments, references, tables of contents, indexes, etc. The Chapters are numbered 1 through 281, and are grouped under superior titles of TITLE and PART.

On the following pages there will be presented a partial list of the General Laws which are of interest to surveying. Those marked with an asterisk will be further detailed at the end of this section. This presentation should be considered informative, not legally authoritative, as it was compiled by a surveyor (not a lawyer).

"IGNORANCE OF THE LAW IS NO EXCUSE" is a statement chiseled in granite over the entrance of a courthouse in Massachusetts.

PART I ADMINISTRATION OF THE GOVERNMENT (Chap. 1 - 182)

TITLE II EXECUTIVE AND ADMINISTRATIVE OFFICERS OF THE COMMONWEALTH (Chap. 6 - 28A)

- Chap. 13 Department of Civil Service and Registration
- Chap. 26 Department of Public Works

TITLE III LAWS RELATING TO STATE OFFICERS (Chap. 29 - 30A)

TITLE IV CIVIL SERVICE, RETIREMENTS, AND PENSIONS (Chap. 31 - 32B)

TITLE VI COUNTIES AND COUNTY OFFICERS (Chap. 34 - 38)

- Chap. 36 Register of Deeds
  (Other county functions also, as Probate)

TITLE VII CITIES, TOWNS, AND DISTRICTS (Chap. 39 - 49A)

- Chap. 40A Zoning Regulations
- Chap. 42 Boundaries of cities and towns
  (Town boundaries are fixed by statute.)
  (Selectmen must erect permanent stone monuments at every angle in a town line and where highways cross. Note: Surveyors find many highway stones to be inaccurate.)
  (Changes in boundaries: Vote of towns, review by Mass. D.P.W., ratification by General Court.)
  (Disputes are referred to the Land Court.)
- Chap. 49 Fence Viewers, Pounds, and Field Drivers
  (Laws on fences, walls, and fence viewer authority. Note: The fence viewer can make judgement and can order a survey to be made due to disputes.)

TITLE X PUBLIC RECORDS (Chap. 66 only)

TITLE XIII EMINENT DOMAIN AND BETTERMENTS (Chap. 79-80A)

TITLE XIV PUBLIC WAYS AND WORKS (Chap. 81 - 92)

- Chap. 81 State Highways
- Chap. 82 The Laying out, Alteration, Relocation and Discontinuance of Public Ways and Specific Repairs Thereon
- Chap. 83 Sewers, Drains, and Sidewalks
- Chap. 84 Ways and Bridges
- Chap. 85 Regulations and By-laws Relative to Ways and Bridges
- Chap. 86 Boundaries of Highways and other Public Places and Encroachment Thereon
- Chap. 87 Shade trees
- Chap. 88 Ferries, Canals, and Public Landings
- Chap. 89 Law of the Road
- Chap. 90 Motor Vehicles and Aircraft
- Chap. 90A The Highway Safety Act
- Chap. 90B Motorboats and other Vessels
- Chap. 91 Waterways
- Chap. 91A Port of Boston Commission
- Chap. 92 Metropolitan Sewers, Waters, and Parks

TITLE XV REGULATION OF TRADE (Chap. 93 - 110B)

- Chap. 97 Surveying of Land
  (Several sections not generally known)
- Chap. 99 The Metric System of Weights and Measures

TITLE XVI PUBLIC HEALTH (Chap. 111 – 114)
Chap. 112 Registration of Certain Professions and Occupations (includes engineers and surveyors)
Chap. 113 (Willful cutting of timber, etc., punishable)
PART II REAL AND PERSONAL PROPERTY AND DOMESTIC RELATIONS (Chap. 183 – 210)
TITLE I TITLE TO REAL PROPERTY (Chap. 183 – 189)
Chap. 185 The Land Court and Registration of Title to Land
TITLE II DESCENT AND DISTRIBUTION, WILLS, ESTATES OF DECEASED PERSONS, ETC. (Chap. 211 – 262)
PART III COURTS, JUDICIAL OFFICERS, AND PROCEEDINGS IN CRIMINAL CASES (Chap. 211 – 262)
PART IV CRIMES, PUNISHMENTS, AND PROCEEDINGS IN CRIMINAL CASES (Chap. 263 – 281)
TITLE I CRIMES AND PUNISHMENTS
Chap. 266 Crimes against Property
Sec. 120 Private property; entry after being forbidden; penalty; arrest
Sec. 120A Motor vehicle; parking in private way; prosecution; evidence
Sec. 120B Entry on land by abutting property owner not constituting trespass (Note: under certain conditions)
Sec. 120C Entry upon adjoining lands by surveyors not constituting trespass (Note: N/A to surveyors working for public authority, public utility, or railroad.)

Chapter 266 of the General Law is hereby amended by inserting after Section 120B the following section:

Section 120C: Whenever a land surveyor registered under chapter one hundred and twelve deems it reasonably necessary to enter upon adjoining lands to make surveys of any description included under "Practice of land surveying", as defined in section eighty-one D of said chapter one hundred and twelve, for any private person, excluding any public authority, public utility or railroad, the land surveyor or his authorized agents or employees may, after reasonable notice, enter upon lands, waters, and premises, not including buildings, in the Commonwealth, within a reasonable distance from the property line, of the land being surveyed, and such entry shall not deem a trespass. Nothing in this act shall relieve a land surveyor of liability for damage caused by entry to adjoining property, by himself or his agents or employees.

Chapter 81 Section #7F.

Entry on private land for purpose of surveys, soundings and drillings.

Whenever the department deems it necessary to make surveys, soundings, drillings or examinations to obtain information for or to expedite the construction of state highways or other projects under its jurisdiction, the department, its authorized agents or employees may, after due notice by registered or certified mail, enter upon any lands, waters and premises, not including buildings, in the commonwealth for the purpose of making surveys, soundings, drillings, and examinations as they may deem necessary or convenient for the purposes of this act, and such entry shall not be deemed a trespass nor shall an entry for such purposes be deemed an entry under any condemnation proceedings which may be then pending. The department shall make reimbursement for any injury or actual damage resulting to such lands, waters and premises caused by any act of its authorized agents or employees and shall so far as possible restore such lands to the same condition as prior to the making of such surveys, soundings, drillings, or examinations.

Chapter 86 Section #1

Erection of monuments. The county commissioners, aldermen, selectmen or road commissioners shall cause permanent bounds to be erected at the termini and angles of all ways laid out by them. Such bounds shall be of stone, Portland cement or concrete not less than three feet long, two feet of which at least shall be set in the ground, or of stone not less than three feet long with holes drilled therein and filled with lead placed a few inches below the travelled part of the way, or if stone, Portland cement or concrete bounds are impractical, a heap of stones, a living tree, a permanent rock, or the corner of a building, or such other permanent bounds as said officers may determine. If they neglect to establish such monuments after being notified so to do by an owner of land abutting on such way, the county or city, if it is a highway, or the town if it is a town road, shall forfeit to him fifty dollars for each month during which such neglect continues.

Chapter 86 Section #2

Buildings or fences as boundaries. If buildings or fences have been erected and continued for more than twenty years, fronting upon or against a highway, town way, private way, training field, burying place, landing place, street, lane or alley, or other land appropriated for the general use or convenience of the inhabitants of the commonwealth, or of a county, city, town or parish, and from the length of time or otherwise the boundaries thereof are not known and cannot be made certain by the records or by monuments, such buildings or fences shall be taken to be the true boundaries thereof.

Chapter 97

Section 1

The posts or pillars erected pursuant to chapter two hundred and eighty-six of the acts of eighteen hundred and seventy to indicate the true meridian lines shall remain the property of the respective counties in which they are situated, under the care of the county commissioners, and shall be accessible to any surveyor or civil engineer residing in the same county or engaged in surveying therein, for the purposes of testing the variation of the compass for the time being. Whoever willfully displaces, alters, defaces, destroys, or otherwise injures any of the aforesaid posts or the fixtures thereof, shall be punished by a fine of not more than two hundred dollars, to be divided between the complainant and the county.

Section 2

Every land surveyor shall, at least once in every year, adjust and verify his compass by the meridian line so established in the county in which his surveys are to be made, insert in his field notes both the true and magnetic bearings of the lines of his surveys and the days on which such lines were run and shall enter in a book open to public inspection, to be provided by the commissioners of each county and kept by the clerk of courts, or by a person appointed by the commissioners therefor, the variation of his compass from the true meridian, whether east or west, and shall sign and make oath to such entry.

Section 3

Every land surveyor shall use only a tape that has been standardized against a tape that has been examined by the United States Bureau of Standards and has had its exact length established, and shall maintain in his office or principal place of business a record of the date, the name of the person making the comparison, and the number of the United States Bureau of Standards test which established the length of the standardizing tape, or the number of the tape. Such comparisons shall be made as frequently as required by good practice, but at least once every year. Said record shall be exhibited upon demand to the sealer of weights and measures for the town where such surveyor has his office or principal place of business. Whoever violates any provision of this section shall be punished by a fine of not more than twenty dollars.

Section 7

A surveyor who violates the provision of section two, if he used his compass to measure an angle in surveying within the year preceding, shall be punished by a fine of ten dollars, to be divided between the complainant and the county.

Chapter 248

AN ACT FURTHER REGULATING THE SURVEYING OF LAND.

Be it enacted, etc., as follows:

Chapter 97 of the General Laws is hereby amended by striking out sections 8, 9, 10, 11, 12, and 13 and inserting in place thereof the following ten sections:

Section 8

The system of plane rectangular coordinates shall be the North American Datum of 1983 as established by the National Ocean Services / National Geodetic Survey (NOS/NGS), or its successors for defining and stating the geographic positions or locations of points on the surface of the earth within the commonwealth and shall be known as the Massachusetts Coordinate System North American Datum (NAD), 1983.

For the purposes of the use of said system the commonwealth is hereby divided into a mainland zone and an island zone.

The area now included in the following counties shall constitute the mainland zone: Barnstable, Berkshire, Bristol, Essex, Franklin, Hampden, Hampshire, Middlesex, Norfolk, Plymouth, Suffolk, and Worcester.

The area now included in the counties of Dukes County and Nantucket shall constitute the island zone.

Section 9

As established for use in the mainland zone, said system shall be named, and in any land description in which it is used it shall be designated, the "Massachusetts Coordinate System, NAD 1983, Mainland Zone".

As established for use in the island zone, said system shall be named, and in any land description in which it is used it shall be designated, the "Massachusetts Coordinate System, NAD 1983, Island Zone".

Section 10

The plane rectangular coordinates of a point on the earth's surface, shall be used in expressing the position or location of such point in the appropriate zone of said system, and shall consist of two distances, either expressed in meters and decimals of meters or in the United States survey feet and decimals of a foot. One of these distances, to be known as the easting or "X" coordinate shall give the position in an east-west direction; the other to be known as the northing or "Y" coordinate shall give the position in a north-south direction. Such coordinates shall be made to depend upon and conform to the plane rectangular metric coordinates values for the horizontal control stations of the North American Horizontal, Geodetic Control Network as published by said National Ocean Service, National Geodetic Survey or its successors, and whose plane coordinates have been computed on the systems defined in this chapter.

Section 11

Nothing contained in this chapter shall prohibit the conversion of the coordinates on the metric system to the United States Survey Foot nor shall the use of distances in feet and decimals of a foot be prohibited from any map, report of survey, deed, or other document. For conversion of meters to United States survey feet, the meters shall be multiplied by 39.37 and divided by 12 which results in a constant multiplier having a value of 3.28083333333 to 12 significant figures.

Section 12

For the purpose of more precisely defining said system:

1. The Massachusetts Coordinate System, NAD, 1983, Mainland Zone, Federal Zone Code M2001, is a Lambert conformal conic projection of the North American Datum of 1983, having standard parallels at north 41 degrees 43 minutes and 42 degrees 41 minutes, along which parallels the scale shall be exact. The origin of coordinates is at the intersection of meridian 71 degrees 30 minutes west of Greenwich and the parallel 41 degrees 00 minutes north latitude. The origin shall have the coordinates North (Y) = 750,000 meters and East (X) = 200,000 meters.

2. The Massachusetts Coordinate System, NAD, 1983, Island Zone, Federal Zone Code M2002, is a Lambert conformal conic projection of the North American Datum of 1983, having standard parallels at north 41 degrees 17 minutes and 41 degrees 29 minutes, along which parallels the scale shall be exact. The origin of coordinates is at the intersection of meridian 70 degrees 30 minutes west of Greenwich and the parallel 41 degrees 00 minutes north latitude. The origin shall have the coordinates North (Y) = 0 meters and East (X) = 500,000 meters.

Section 13

The use of the term "Massachusetts Coordinate System" on any map, report of survey, or deed or other document, shall be limited to coordinates based on the Massachusetts State Plane Coordinate System as defined in sections eight to thirteen, inclusive.

Section 14

The coordinate system based upon the North American Datum of 1927 which prior to the enactment of this statute was defined as the Massachusetts Coordinate System shall now be designated the Coordinate System, North American Datum of 1927, (NAD 1927), and can continue in use for the preparation of any maps, reports of survey, or other documents, until such time as the Board of Registration of Professional Engineers and Land Surveyors declares, with the concurrence of the chief engineer of the Massachusetts department of public works, by regulation that said Coordinate System, (North American Datum of 1927), shall no longer be used except for interpreting existing maps, reports of surveys, deeds or other documents. For the Coordinate System, North American Datum, 1927, the commonwealth is divided into the same two zones defined in section eight.

Section 15

The plane rectangular coordinates of a point on the earth's surface, to be used in expressing the position or location of such point in the appropriate zone of said system, shall consist of two distances, expressed feet and decimals of a foot. One of these distances, to be known as the "X" coordinate, shall give the position in an east-west direction; the other to be known as the Y" coordinate shall give the position in a north-south direction. Such coordinates shall be referred to the appropriate origin as defined in section sixteen, and shall be made to depend upon and conform to the plane rectangular coordinates of the triangulation and traverse stations of the former United States Coast and Geodetic Survey within the commonwealth, as these coordinates had been determined by said survey.

Section 16

The Coordinate System, North American Datum of 1927, Mainland Zone, consists of a Lambert conformal projection of the Clarke spheroid of eighteen hundred and sixty-six, having standard parallels at north 41 degrees 43 minutes and 42 degrees 41 minutes, along which parallels the scale shall be exact. The origin of coordinates for this zone is at the intersection of the meridian 71 degrees 30 minutes west longitude and the parallel 41 degrees 00 minutes north latitude. The origin is given the coordinates X=600,000 feet; Y=0 feet.

The NAD 1927 Coordinate System, North American Datum of 1927, Island Zone, consists of a Lambert conformal projection of the Clarke spheroid of eighteen hundred and sixty-six, having the standard parallels at north latitudes 41 degrees 17 minutes and 41 degrees 29 minutes, along which parallels the scale shall be exact. The origin of coordinates for this zone is at the intersection of meridian 70 degrees 30 minutes west longitude and the parallel 41 degrees 00 minutes north latitude. The origin is given the coordinates: X=200,000 feet; Y=0 feet.

The position of said system shall be as marked on the ground by triangulation or traverse stations established in conformity with the standards adopted by the former United States Coast and Geodetic Survey for first-order and second-order work, whose geodetic positions had been rigidly adjusted on the North American Datum of nineteen hundred and twenty-seven, and whose plane coordinates have been computed on the system herein defined. Any such station may be used for establishing a survey connection with said NAD 1927 coordinate system.

Section 17

For the purpose of describing the location of any survey station or land boundary point in the commonwealth it shall be a complete, legal and satisfactory description of such location to give the position of said survey station or land boundary point on the Massachusetts Coordinate System. Nothing contained in sections eight to seventeen, inclusive, shall be interpreted as requiring any purchaser or mortgagee to rely exclusively on a description based on said system.

Approved October 8, 1991

APPENDIX A-2

* RELATION OF DATUM PLANES NGVD 29 (METERS)
* RELATION OF DATUM PLANES NAVD 88 (METERS)

TOWN OF ORANGE	+132.747
TOWN OF NORTH ATTLEBOROUGH	+52.197
NEW ENGLAND POWER CO. (WESTBOROUGH)	+32.266
WESTERN MASS ELECTRIC CO. (GREENFIELD)	+21.110
CITY OF LOWELL	+16.825
LOCKS AND CANALS CORP. (LOWELL)	+1.585
CITY OF LAWRENCE	+1.548
MEAN HIGH WATER*	+1.536
TOWN OF FRAMINGHAM	+1.238
CITY OF FALL RIVER	+0.829
CITY OF TAUNTON	+0.789
CITY OF NEW BEDFORD	+0.777
CITY OF ATTLEBORO	+0.689
CITY OF LEOMINSTER	+0.494
CITY OF WORCESTER	+0.235
MEAN TIDE LEVEL*	+0.082
NGVD 29 DATUM** =	0.000 METERS
TOWN OF NATICK	-0.012
CITY OF CHICOPEE	-0.110
CITY OF SPRINGFIELD	-0.140
TOWN OF GREENFIELD	-0.253
CITY OF HAVERHILL	-0.436
CITY OF BROCKTON	-0.479
HOLYOKE WATER POWER CO.	-0.771
CITY OF HOLYOKE	-0.792
CITY OF SALEM	-1.329
SOUTH ESSEX SEWAGE DISTRICT (SALEM)	-1.359
TOWN OF MANCHESTER	-1.362
MEAN LOW WATER*	-1.375
CITY OF PEABODY	-1.469
BOSTON LOW WATER	-1.490
LOGAN AIRPORT (WATERWAYS)	-1.609
CITY OF LYNN	-1.612
LOGAN AIRPORT (HIGHWAYS)	-1.631
CITY OF SOMERVILLE	-1.643
TOWN OF WELLESLEY	-1.676
CITY OF BEVERLY	-1.686
TOWN OF DEDHAM	-1.719
BOSTON CITY BASE & WALTHAM CITY BASE	-1.722
CITY OF NEWTON	-1.743
TOWN OF STOUGHTON	-1.759
TOWN OF BROOKLINE	-1.762
CITY OF QUINCY	-1.774
CITY OF REVERE	-1.780
CITY OF EVERETT	-1.798
CITY OF CHELSEA	-1.829
TOWN OF NORWOOD	-1.838
CITY OF CAMBRIDGE	-3.304
TOWN OF WALPOLE	-7.096
THIRD HARBOR TUNNEL	-30.480
U.S. ARMY ENGINEERS (BOSTON)	-30.587
BOSTON NAVY YARD (BASIC BENCH)	-32.028
MASS. WATER RESOURCES AUTHORITY & BOSTON TRANSIT COMMISSION	-32.193
TOWN OF NEEDHAM	-32.193
MASS. BAY TRANSPORTATION AUTHORITY	-32.202
MASS. BAY TRANSPORTATION AUTHORITY REDLINE (BOSTON)	-32.269

1.536 m

0.082 m

1.375m

AS OF 1978

* Tidal Bench Marks located at Appraiser's Stores Building, U.S. Custom House, Purchase and High Streets, Boston.

** Formerly referred to as MEAN SEA LEVEL.

REVISED – Nov. 2001

MASS HIGHWAY SURVEY MANUAL

RELATION OF DATUM PLANES NGVD 29 (METERS)

DATE OF ISSUE SEPTEMBER 1996

APPENDIX A-2

TOWN OF ORANGE	+132.560
TOWN OF NORTH ATTLEBOROUGH	+51.962
NEW ENGLAND POWER CO. (WESTBOROUGH)	+32.051
WESTERN MASS ELECTRIC CO. (GREENFIELD)	+20.942
CITY OF LOWELL	+16.579
LOCKS AND CANALS CORP. (LOWELL)	+1.339
CITY OF LAWRENCE	+1.306
MEAN HIGH WATER*	+1.290
TOWN OF FRAMINGHAM	+1.006
CITY OF FALL RIVER	+0.571
CITY OF TAUNTON	+0.541
CITY OF NEW BEDFORD	+0.526
CITY OF ATTLEBORO	+0.451
CITY OF LEOMINSTER	+0.261
CITY OF WORCESTER	+0.045
NAVD 88 DATUM =	0.000 METERS
MEAN TIDE LEVEL*	−0.164
TOWN OF NATICK	−0.247
CITY OF CHICOPEE	−0.328
CITY OF SPRINGFIELD	−0.361
TOWN OF GREENFIELD	−0.421
CITY OF HAVERHILL	−0.681
CITY OF BROCKTON	−0.721
HOLYOKE WATER POWER CO.	−0.984
CITY OF HOLYOKE	−1.005
CITY OF SALEM	−1.576
SOUTH ESSEX SEWAGE DISTRICT (SALEM)	−1.606
TOWN OF MANCHESTER	−1.607
MEAN LOW WATER*	−1.621
CITY OF PEABODY	−1.715
BOSTON LOW WATER	−1.736
LOGAN AIRPORT (WATERWAYS)	−1.854
CITY OF LYNN	−1.857
LOGAN AIRPORT (HIGHWAYS)	−1.876
CITY OF SOMERVILLE	−1.890
TOWN OF WELLESLEY	−1.916
CITY OF BEVERLY	−1.933
TOWN OF DEDHAM	−1.959
BOSTON CITY BASE & WALTHAM CITY BASE	−1.968
CITY OF NEWTON	−1.990
TOWN OF STOUGHTON	−1.996
TOWN OF BROOKLINE	−2.011
CITY OF QUINCY	−2.020
CITY OF REVERE	−2.023
CITY OF EVERETT	−2.039
CITY OF CHELSEA	−2.072
TOWN OF NORWOOD	−2.078
CITY OF CAMBRIDGE	−3.553
TOWN OF WALPOLE	−7.334
THIRD HARBOR TUNNEL	−30.726
U.S. ARMY ENGINEERS (BOSTON)	−30.833
BOSTON NAVY YARD (BASIC BENCH)	−32.274
TOWN OF NEEDHAM	−32.433
MASS. WATER RESOURCES AUTHORITY & BOSTON TRANSIT COMMISSION	−32.439
MASS. BAY TRANSPORTATION AUTHORITY	−32.448
MASS. BAY TRANSPORTATION AUTHORITY REDLINE (BOSTON)	−32.515

1.290 m

0.164 m

1.621 m

AS OF 1978

* Tidal Bench Marks located at Appraiser's Stores Building, U.S. Custom House, Purchase and High Streets, Boston

NOTE: Elevations shown are derived from NGS VERTCON PROGRAM, VERSION 2.0, for NGVD 29 to NAVD 88 conversions. Tests of the predictive capability of the physical model show a 2.0 cm RMS agreement. The VERTCON model can be considered accurate at the 2 cm level nationwide, with better accuracy in the eastern United States.

APPENDIX A-3

* STANDARD SURVEY ABBREVIATIONS
* STANDARD RIGHT-OF-WAY ABBREVIATIONS

The following is a list of common abbreviations used in surveying by the Massachusetts Highway Department.

Accm	Asphalt coated corrugated metal
Ask	Askew
Aux.	Auxiliary
AZ or az	Azimuth
AZ. MK.	Azimuth mark
BA or ba	Back azimuth
Bc	Bottom of curb
Bd or Bnd	Bound
Bit Conc	Bituminous concrete
BL	Baseline
Bldg	Building
BM	Benchmark
BMA	Bituminous macadam, asphalt
Brg.	Bearing
B.S.	(1) Bottom of slope (2) Backsight
B.W.	Back wall
b. wire	Barbed wire
c	Chord of part of horizontal circular curve
C	(1) Correction (2) Long chord (P.C. to P.T.) of horizontal circular curve
C-Factor	Correction for precise leveling
C.B.	Catch basin
C.B.	Concrete bound
C.B.C.I.	Catch basin, curb inlet
cc	Center of curve (on the arc)
cf	Crows-foot (mark)
ch or chisel	Chiseled as "chiseled square"
C.I.	Cast iron
C.L.	Centerline
C.L. or Ch.Lk	Chain link
Co.	County
Conc.	Concrete
Const.	Construction
Coord.	Coordinate
Cor	Corner
Culv.	Culvert
d or dist.	Distance
D	Direct theodolite position
D	Degree of curve
D.D.	Decimal degrees
D.E.	Difference in elevation
Def. or Defl.	Deflection
Deg.	Degree
D.H. or dh	Drilled hole
D.I.	Drop inlet
Diff.	Difference, as elevation difference
diff. elev.	Difference in elevation
Diam.	Diameter
Dist.	Distance

D.M.H.	Drainage manhole
DMS	Degrees, minute, seconds
E	Edge
E.	External, horizontal circular
Ecc.	Eccentric
EDM	Electronic distance measurement
El. or Elev.	Elevation
E.P.	Escutcheon pin (as "e-pin in lead")
E-pin	Escutcheon pin (as "e-pin in lead")
Esc. pin	Escutcheon pin (as "e-pin in lead")
Fd. or Fnd	Found
Fed.	Federal
F/L	Flowline
For.	Forward as "For. run"
F.S.	Forward sight, or foresight
G. or Grd.	Ground
Gal.	Galvanized
Gar.	Garage
Gd. Rail	Guard rail
G.G.	Gas gate
Gr. az.	Grid azimuth
Gran	Granite
Grano.	Granolithic
H.B.	Highway bound
Hdr.	Header
Hdwl.	Headwall
H.I.	Height of instrument (above ground)
H.I.	Elevation of instrument above datum
Ho.	House
Hor. or Horz.	Horizontal
H.P.	High pressure
H.W.	High water
Hwy.	Highway
Hyd.	Hydrant
I	Iron
I	Central angle of horizontal circular curve
I.D.	Inside diameter
Inst.	Instrument
Int.	Intersection
Inv.	Invert (low point of inverted arch)
I.P.	Iron pipe
I.T.	Intersection of slopes or profile grade lines
jnct.	Junction
l	length
l	link, 1/25th of a rod (1 rod = 16.5 ft. = 5.029 m.)
L	Length of curve
L	Ledge
Lat.	Latitude
L.B.	Leaching basin
L.C.B.	Land court bound
L.O.	Layout
Long.	Longitude

M	Middle ordinate
Mac.	Macadam
NAD 27	North American datum of 1927
NAD 83	North American datum of 1983
Mag.	Magnetic
Maint.	Maintenance
M.H.	Manhole
M.H.B.	Massachusetts highway bound
M.H.W.	Mean high water
M.L.W.	Mean low water
Mon.	Monument
MP	Measuring point
M.P.	Mile post
M.S.L.	Mean sea level
N.B.	(1) survey notebook (2) North–bound
N.F. or nf	not found
N.G.S.	National geodetic survey
N.G.V.D. 29	National geodetic vertical datum of 1929
N.A.V.D. 88	North American vertical datum of 1988
Nl.	Nail
O.	Origin (center of circle)
Obs.	Observed or observation
O.D.	Outside diameter
Off.	Offset
P.C.	Point of curvature (beginning of curve)
P.C.C.	Point of compound curvature
Perm.	Permanent
P.I.	Point of intersection of curve tangents
P–K	P–K nail
P.L.	Property line
P.O.C.	Point of curve
P.O.L.	Point on line
P.O.T.	Point on tangent
p.p.	Prick punch mark
P.R.C.	Point of reverse curve
Prelim.	Preliminary
Proj.	Project (a noun)
Prop.	(1) Property (2) Proposed
P.T.	Point of tangency (end of curve)
Pt.	Point
PTH	"Planimeter to here"
Pvt or Pvmt	Pavement
P. WW	Paved waterway
R	Radius
Rad.	Radian (sometimes radius)
R.C.	Reinforced concrete
Rd.	Road
Rdwy	Roadway
Ref.	Refer or reference
Reg.	Regular or reference
Reg. Land	Registered in land court
Rec.	Reconnaissance

Ret. W.	Retaining wall
R.L.S.	Registered land surveyor
R.M.	Reference mark (sometimes Ref. Mk.)
R.O.W. or R/W	Right-of-way
R.R.	Railroad
Rt. or Rte.	Route
S.B.	(1) Stone bound (2) South-bound
S & S	Stake and stones
Sec.	Section
S.L.	Sideline
S/L	State line (sometimes S.L. or St.L.)
S.M.H.	Sewer manhole
Spk.	Spike
S.S.	Sanitary sewer
S.T.	Surface treatment
Sty	Story (as 2-Sty house)
Sta.	Station
Sur.	Survey
T	(1) Tangent (2) Temperature (3) Town
Tang.	Tangent
T.B.M.	(1) Tidal bench mark (2) Temporary benchmark
Tel.	(1) Telephone (2) Telegraph
Temp.	(1) Temperature (2) Temporary
T/L or T.L.	Town line
Tope	Topography or topographic
T.P.	Turning point (differential leveling)
T.S.	Top of slope
USC & GS	U.S. Coast and Geodetic Survey (now NGS)
USGS	U.S. Geological Survey
V.A.	Vertical angle
V.C.	(1) Vertical curve (2) Vitrified clay
Vert.	Vertical
Wd	Wood
W.G.	Watergate
W.M.	Water main
X	Cross, as X-road or X-section
Zen.	Zenith

Symbols for Fee Takings

I	Taken in fee in behalf of the Commonwealth
I–C	Taken in fee in behalf of the City
I–T	Taken in fee in behalf of the Town
I–RR	Taken in fee in behalf of the Railroad
I–U	Taken in fee, ordinarily conveyed to utility
A5–I	Roadside facility (Section A, site 5)
I X	Excess land
M–I	Maintenance area
D–I–F	Drainage taking in fee

Symbols for Easement Takings

AT–I	Access taking
B–I	Bridge
BA–I	Bridge Abutment
C–I	Channel
CD–I	Channel drainage
D–I	Drainage
DS–I	Drainage and slope
E–I	Highway easement (portion of right-of-way)
E–RR–I	Easement on behalf of railroad
FB–I	Footbridge
FS–I	Flight of stairs
GD–I	Gravel dike
GR–I	Guard rail
HS–I	Highway sign
PL–I	Power line
R–I	Right-of-way taken in behalf of owner of land whose rights of access thereto and egress therefrom would otherwise be inoperative due to the limited access provisions
R–B–I	Road and bridge
R–B–S–I	Road, bridge, and slope
RD–I	Right-of-way drainage
RR–I	Railroad bypass
R–RR–I	Road and railroad bypass
RS–I	Right-of-way slope
RT–I	Temporary easement for removal or demolition of certain structures
S–I	Slope
SS–I	Sanitary sewer
SW–I	Sidewalk
SW–S–I	Sidewalk and slope
TE–I	Temporary easement for various purposes
TR–I	Temporary road
U–I	Utility easement (ordinarily conveyed to a utility co.)
W–I	Wall
WM–I	Watermain
WHD–I	Watermain and drainage
WS–I	Wall and slope

Notes:

(1) Temporary easements are preceded by the letter "T"
(TD-I, TWM-I, etc.)

(2) Easements in behalf of town, city, railroad, or the M.D.C. are followed by the letters "T", "C", "R", and "MDC" respectively.
(D-I-T, D-I-C, D-I-RR, D-I-MDC, etc.)

(3) EG-I is a symbol used to delineate an area comprising a portion of the right-of-way in which an easement is to be granted.

(4) The symbols listed and described above may be preceded by a number (1-I, 1-D-I, 2-I, 2-D-I, etc.).

(5) The symbols A, B, C, etc. designate "spot" takings in fee. The symbols B 11-1 B 11-2, etc. designate "block" takings fee.

APPENDIX A-4

* HORIZONTAL CURVE AND BASIC SURVEY FORMULAS

HORIZONTAL CURVE COMPUTATION
INTRODUCTION

THE MASSACHUSETTS HIGHWAY DEPARTMENT USES SIMPLE HORIZONTAL CURVES IN ITS DESIGN. A SIMPLE CURVE CONSISTING OF A CIRCULAR ARC JOINING TWO TANGENTS. IN THIS SYSTEM THE LENGTH OF THE CURVE IS MEASURED (COMPUTED) ALONG THE ARC, AS IS COMMONLY DONE IN OTHER STATES. MASSACHUSETTS DOES DIFFER IN DEFINING THE CURVE. MASSACHUSETTS AND CALIFORNIA (AND POSSIBLY A FEW OTHER STATES) DEFINE CURVES BY RADIUS, AS "A CURVE OF 1500 METER RADIUS," INSTEAD OF BY DEGREE OF CURVE, AS "A 3 DEGREE CURVE." MOST CURVE TABLES ARE BASED UPON DEGREE OF CURVE, USING EVEN DEGREES AND MINUTES, AND NOT BY EVEN RADIUS, AND THUS ARE SOMEWHAT LIMITED IN USE IN MASSACHUSETTS. **

SIMPLE CURVES MAY BE COMBINED TO FORM COMPOUND OR REVERSE CURVES, OCCASIONALLY FOUND IN ALL HIGHWAY WORK (INCLUDING FREEWAY DESIGN), WHERE COMPOUND CURVES MAY BE USED FOR TRANSITIONS ON BOTH THE OPEN HIGHWAY AND ON RAMPS, AND REVERSE CURVES ON RAMPS IN TIGHT AREAS.

* DEGREE OF CURVE, D, IS DEFINED AS THE ANGLE AT THE ORIGIN OF THE CURVE THAT SUBTENDS BY RADII A 100 METER CHORD (RAILROAD DEFINITION) OR A 100 METER ARC (HIGHWAY DEFINITION).

** MOST SURVEYORS PREFER TO FIGURE CURVES BY FORMULA, AND HAVING CALCULATORS AND COMPUTERS HELPS ELIMINATE THE NEED FOR TABLES. THERE IS STILL VALUABLE INFORMATION IN CURVE HANDBOOKS.

LAYING OUT A HORIZONTAL CURVE

HOLD: R = 400 m. I = 25–30–25 PC STA. = 4 + 47.00

CALCULATED: T = 90.536 E = 10.118
L = 178.072 d / meter = 00–04–18

THE CURVE IS TO BE STAKED AT 20 m INTERVALS.

DEFLECTION TABLE

	STATION	DEFLECTION ANGLE (dd–mm–ss)
P.C.	4+47.00	00–00–00
	4+60.00	00–55–52
	4+80.00	02–21–48
	5+00.00	03–47–45
	5+20.00	05–13–42
C.C.	5+36.036	06–22–36
	5+40.00	06–39–38
	5+60.00	08–05–35
	5+80.00	09–31–32
	6+00.00	10–57–28
	6+20.00	12–23–25
P.T.	6+25.072	12–45–13

LAYING OUT A HORIZONTAL CURVE

TO LAY OUT CURVE FROM PC:

SET ZERO ON PI, OR FULL DEFLECTION ON PT, OR ZERO ON BACK TANGENT WITH SCOPE IN REVERSE POSITION, OR TO DEFLECTION OF ANY KNOWN STATION. NOTE THE CC HAS ONE-HALF OF THE FULL DEFLECTION.

TO LAY OUT CURVE FROM PT:

SET ZERO ON PC, OR FULL DEFLECTION ON PI, OR FULL DEFLECTION ON FORWARD TANGENT WITH SCOPE IN REVERSE POSITION. NOTE THE CC HAS ONE-HALF OF THE FULL DEFLECTION.

TO LAY OUT CURVE FROM CC:

SET ZERO ON PC, OR FULL DEFLECTION ON PT, OR DEFLECTION OF ANY KNOWN STATION, OR 90 DEGREES PLUS DEFLECTION OF CC ON THE PT.

TO LAY OUT CURVE FROM ANY STATION ON THE CURVE:

SET ZERO ON PC, OR FULL DEFLECTION ON PT, OR GIVEN DEFLECTION TO ANY OTHER STATION ON THE CURVE.

TO FIND TANGENT LINE AT ANY POINT ON CURVE:

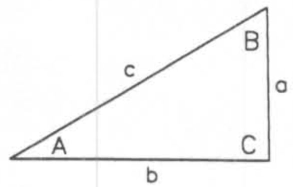
SET DEFLECTION TO ANY OTHER KNOWN POINT, TURN ANGLE TO DEFLECTION OF POINT SET UPON.

TO FIND RADIAL LINE:

TURN 90 DEGREES FROM THE TANGENT LINE. TO SET AN ACCURATE RADIAL LINE, THE POINT ON THE CURVE MUST BE MORE PRECISELY SET THAN OTHER POINTS, OR THE LINE SHOULD BE COMPUTED FROM AHEAD ON THE TANGENT.

BASIC SURVEY FORMULAS

Right Triangle

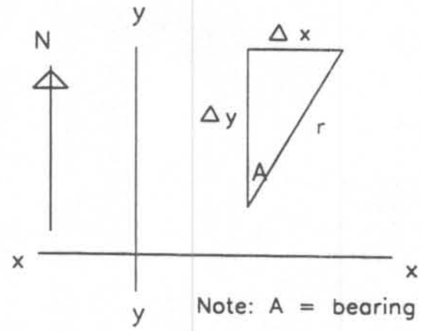


$\sin A = a/c$ $\qquad c^2 = a^2 + b^2$

$\cos A = b/c$

$\tan A = a/b$ $\qquad a^2 = (c - b)/(c + b)$

Coordinate Computation



Note: A = bearing

$\Delta x \; \Delta y$ = change in coor.

$\Delta y = r \cos A$

$\Delta x = r \sin A$

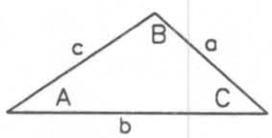
Inverse solution:

$\tan A = \Delta x / \Delta y \qquad r = \sqrt{\Delta x^2 + \Delta y^2}$

Suggested form for coordinates

STA	BEARING	DISTANCE	y (or N)	x (or E)

Oblique Triangle



Law of sines

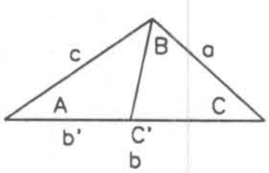
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

Ambiguous Case



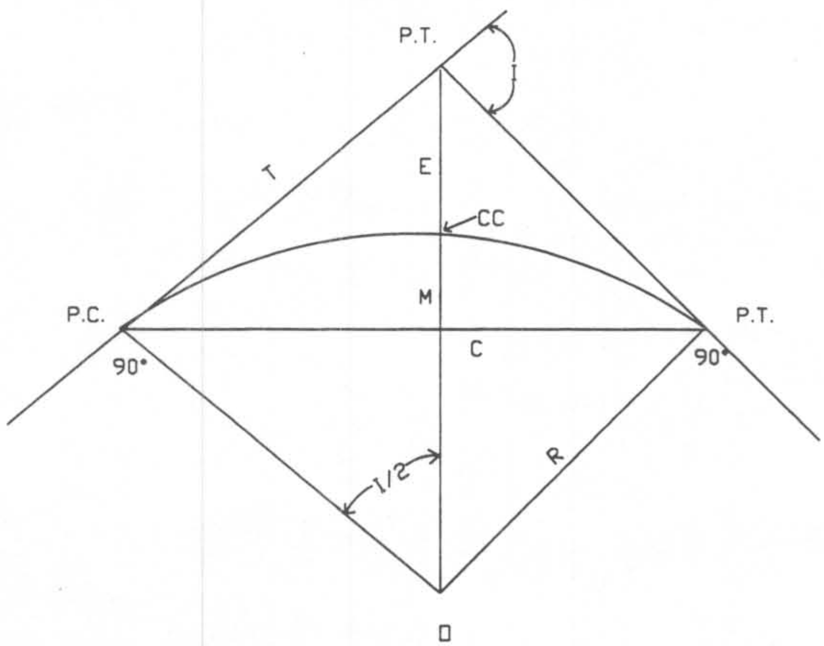
Known: A, a, c

Two possible solutions when $c > a$.

Example: $A = 30°$, $c = 100.00$, $a = 77.79$

$\sin C = (c \sin A)/a \qquad C = 40°$ or $140°$

HORIZONTAL CURVE DEFINITIONS AND FORMULAS



P.C. = Point of Curvature

P.T. = Point of Tangency

R = Radius

C = Long Chord

O = Origin

M = Middle Ordinate

P.I. = Point of Intersection (of Tangents)

I = Central Angle (also know as $\triangle$)

L = Length of Curve

T = Tangent

E = External

cc = Center of Curve (Midpoint of Arc)

x = value of Central Angle in radians

$T = R (\tan I/2)$

$E = T (\tan I/4)$

$L = Rx$

$L = R (I * \pi / 180)$ (I in decimal degrees)

D = I/2 (Deflection Angle of Curve)

d per meter = 1718.873/R (d is deflection in decimal minutes)

d per meter = D/L (D and d in decimal degrees)

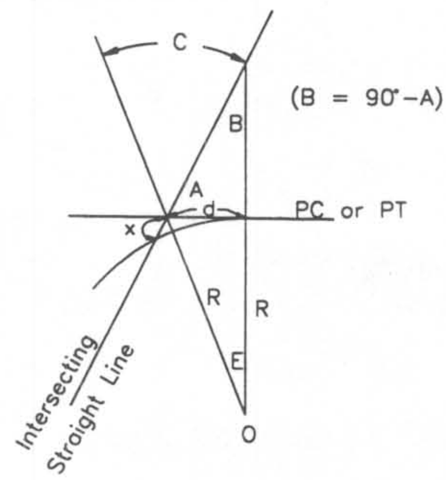
$C = 2R (\sin I/2)$

HORIZONTAL CURVE COMPUTATION

Intersection of Curve and Straight Line

CASE I

Angle 'A' less than 90°



Known: Radius R   Distance d   Angle A

Needed: x and E (to set intersection)

Formulas: $\sin C = \cos A + d/(R \sin A)$

$E = C - B$

$x = (R \sin E - d)/\cos A$

In Case I, the straight line intersects the line from O (origin) to PC (or PT) extended. This situation occurs when angle A is less than 90°.

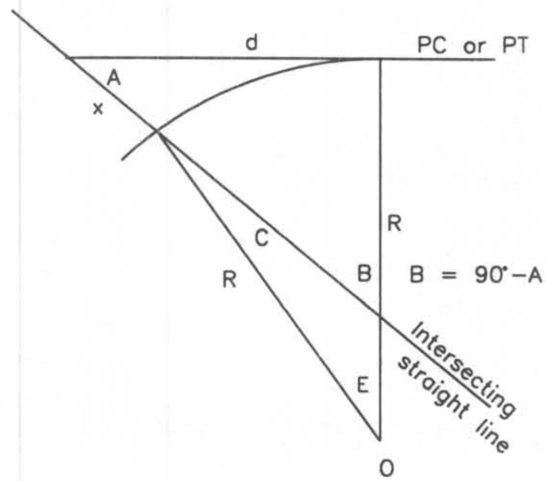
The required information d and A, if not readily available from field conditions, can be calculated geometrically or from coordinates

HORIZONTAL CURVE COMPUTATION

Intersection of Curve and Straight Line

CASE II

Angle 'A' less than 90° and d/R sin A less than cos A



Known: Radius R Distance d Angle A

Formulas: $\sin C = \cos A - d/(R \sin A)$

$E = B - C$

$x = (d - R \sin E)/\cos A$

The straight line intersects the radius between O (origin) and the PC (or PT).

If d/R sin A = cos A (or tan A = r/d) the straight line is radial and runs through O.

Bound points are often radial from the layout station on the baseline curve to the bound.

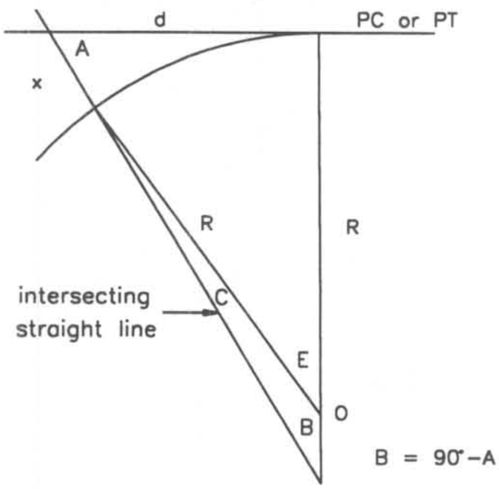
As in Case I, required information can be calculated from accessible field data.

HORIZONTAL CURVE COMPUTATION

Intersection of Curve and Straight Line

CASE III

Angle 'A' less than 90° and d/R sin A greater than cos A.



Known: Radius R Distance d Angle A

Formulas: $\sin C = d/(R\sin A) - \cos A$

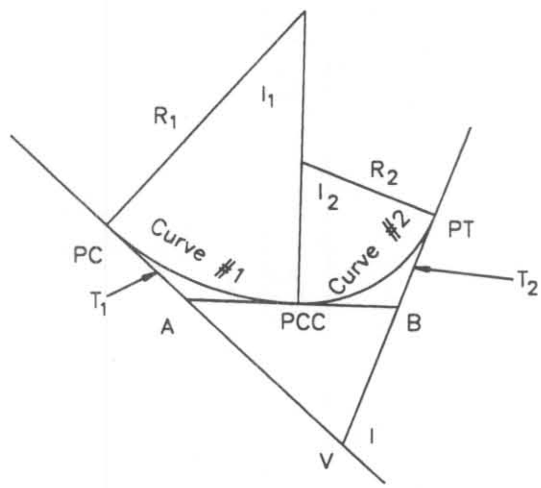
$E = B + C$

$x = (d - R \sin E)/\cos A$

The straight line intersects the radius extended PC − O.

As in Cases I & II, information required to calculate the intersection may be easier to obtain than would field derived values of d and A.

Compound Curve



A compound curve is two connected simple horizontal curves.

A = PI Curve #1, B = PI Curve #2
PCC = PT Curve #1 & PC Curve #2, AB = Common tangent
$I = I_1 + I_2$, Usually two of these are known – other can be found.

Radius of each curve is also usually known.

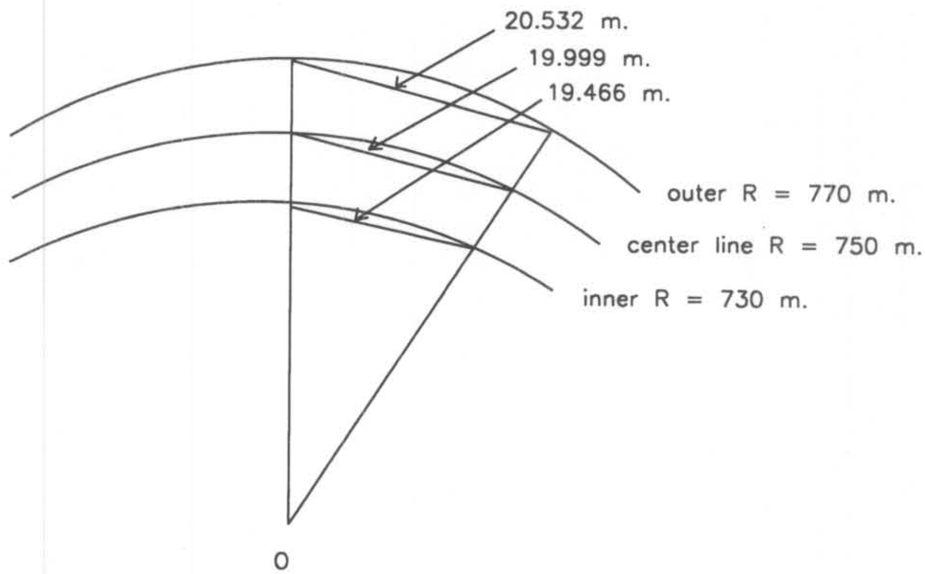
The tangent (T) of each simple curve can be computed.

In triangle ABV, AB is the sum of the two tangents.
The triangle can be solved using that distance and the 'I' angles.
PC to V = T_1 + AV   V to PT = VB + T_2

In most compound curves, key to solutions is in solving triangle ABV.

HORIZONTAL CURVE COMPUTATION

Offset Curve



Example: 20 meter offset curves are to be laid out. Lengths of chords between radial lines needed at 20 m. stationing.

Chords and radii are proportional so:

$$\frac{\text{chord on offset}}{\text{chord on C.L.}} = \frac{\text{radius on offset}}{\text{radius on C.L.}}$$

Inside chord = (730 * 19.999)/750 = 19.466

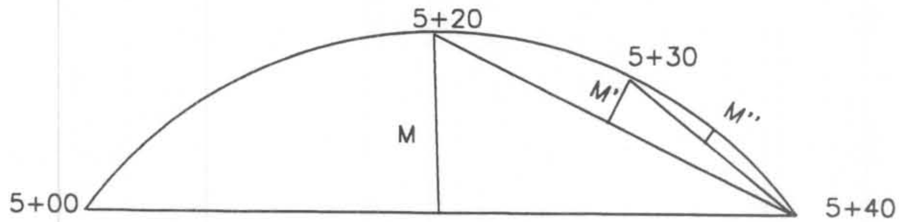
Outside chord = (770 * 19.999)/750 = 20.532

HORIZONTAL CURVE COMPUTATION

Other methods of locating curve points.

The recommended procedure for curve layout is usually by deflection angle.

Middle Ordinate Method



This method enables quick location of a destroyed point, or an efficient procedure for placing extra points on the curve.

The middle ordinate M on the 40 meter curve is about four times the middle ordinate of the 20 meter curve, which in turn is about four times that of the 10 meter curve (M''). Thus, if M were 4.0,

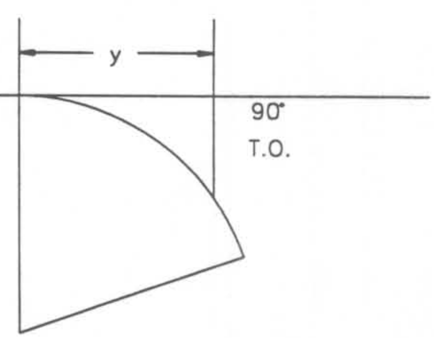
M' and M'' are 1.0 and 0.25 meters respectively.

Tangent Offset Method

This method is useful for laying out short radius curves without a theodolite. Right angles are laid out by the 3–4–5 method.

Exact: $T.O. = R - \sqrt{R^2 - y^2}$

Approx. $T.O. = y^2/(2R)$



APPENDIX A-5

* ENGLISH – METRIC CONVERSION FACTORS
* TEMPERATURE TAPE CORRECTIONS
* STATE PLANE PROJECTION TABLES
* HORIZONTAL CURVE DEFLECTION ANGLE AND SUBCHORD TABLE

Length			
	U.S. survey ft.	meter (m)	12/39.37
	mile	kilometer (km)	1.609344
	inch	millimeter (mm)	25.4
Area			
	acre	hectare (ha)	0.04046856
	acre	square meters (m)	4046.856
	square foot	square meters (m^2)	0.09290304
	square inch	square millimeters (mm^2)	645.16
Volume			
	cubic yard	cubic meters (m^3)	0.764555
	cubic foot	cubic meters (m^3)	0.0283168
	cubic inch	cubic millimeters (mm^3)	16387.064
Mass			
	pound	kilogram (kg)	0.453592
Mass/unit length			
	pound/linear foot	kilogram/meter (kg/m)	1.48816
Mass/unit area			
	pound/square foot	kilogram/square meter (kg/m)	4.88243
Mass density			
	pound/cubic foot	kilogram/cubic meter (kg/m^3)	16.0185
Force			
	pound	Newton (N)	4.44822
Force/unit length			
	pound/linear foot	Newton/meter (N/m)	14.5939
Pressure, stress, modulus of elasticity			
	pound/square foot	Pascal (Pa)	47.8803
	pound/square inch	kiloPascal (kPa)	6.89476
Bending moment, torque, moment of force			
	foot*pound	Newton*meter (N*m)	1.35582
Temperature			
	degree Fahrenheit	degree Celsius	(deg. F −32)/1.8
Energy			
	foot*pound	Joule (J)	1.35582

Tapes assumed calibrated at 20 degrees Celcius.

Deg C	Corr. for 30 m	Corr. for 50 m	Deg C	Corr. for 30 m	Corr. for 50 m	Deg C	Corr. for 30 m	Corr. for 50 m
−18	−.0132	−.0221	2	−.0063	−.0104	22	+.0007	+.0012
−17	−.0129	−.0215	3	−.0059	−.0099	23	+.0010	+.0017
−16	−.0125	−.0209	4	−.0056	−.0093	24	+.0014	+.0023
−15	−.0122	−.0203	5	−.0052	−.0087	25	+.0017	+.0029
−14	−.0118	−.0197	6	−.0049	−.0081	26	+.0021	+.0035
−13	−.0115	−.0192	7	−.0045	−.0075	27	+.0024	+.0041
−12	−.0111	−.0186	8	−.0042	−.0070	28	+.0028	+.0046
−11	−.0108	−.0180	9	−.0038	−.0064	29	+.0031	+.0052
−10	−.0104	−.0174	10	−.0035	−.0058	30	+.0035	+.0058
−9	−.0101	−.0168	11	−.0031	−.0052	31	+.0038	+.0064
−8	−.0098	−.0163	12	−.0028	−.0046	32	+.0042	+.0070
−7	−.0094	−.0157	13	−.0024	−.0041	33	+.0045	+.0075
−6	−.0091	−.0151	14	−.0021	−.0035	34	+.0049	+.0081
−5	−.0087	−.0145	15	−.0017	−.0029	35	+.0052	+.0087
−4	−.0084	−.0139	16	−.0014	−.0023	36	+.0056	+.0093
−3	−.0080	−.0134	17	−.0010	−.0017	37	+.0059	+.0099
−2	−.0077	−.0128	18	−.0007	−.0012	38	+.0063	+.0104
−1	−.0073	−.0122	19	−.0003	−.0006	39	+.0066	+.0110
0	−.0070	−.0116	20	+.0000	+.0000	40	+.0070	+.0116
1	−.0066	−.0110	21	+.0003	+.0006	41	+.0073	+.0122

Massachusetts Mainland Lambert Conformal Projection

Lat dd mm	Scale Factor	Lat dd mm	Scale Factor	Lat dd mm	Scale Factor	Lat dd mm	Scale Factor
41 00	1.00018194	41 30	1.00003876	42 00	0.99997065	42 30	0.99997817
41 01	1.00017596	41 31	1.00003528	42 01	0.99996968	42 31	0.99997973
41 02	1.00017007	41 32	1.00003188	42 02	0.99996879	42 32	0.99998138
41 03	1.00016426	41 33	1.00002857	42 03	0.99996799	42 33	0.99998311
41 04	1.00015853	41 34	1.00002533	42 04	0.99996727	42 34	0.99998492
41 05	1.00015289	41 35	1.00002218	42 05	0.99996664	42 35	0.99998682
41 06	1.00014733	41 36	1.00001912	42 06	0.99996609	42 36	0.99998881
41 07	1.00014185	41 37	1.00001614	42 07	0.99996562	42 37	0.99999087
41 08	1.00013645	41 38	1.00001324	42 08	0.99996524	42 38	0.99999303
41 09	1.00013114	41 39	1.00001042	42 09	0.99996494	42 39	0.99999527
41 10	1.00012591	41 40	1.00000769	42 10	0.99996473	42 40	0.99999759
41 11	1.00012076	41 41	1.00000504	42 11	0.99996460	42 41	1.00000000
41 12	1.00011570	41 42	1.00000248	42 12	0.99996455	42 42	1.00000249
41 13	1.00011072	41 43	1.00000000	42 13	0.99996459	42 43	1.00000507
41 14	1.00010582	41 44	0.99999760	42 14	0.99996471	42 44	1.00000774
41 15	1.00010100	41 45	0.99999529	42 15	0.99996492	42 45	1.00001049
41 16	1.00009627	41 46	0.99999306	42 16	0.99996521	42 46	1.00001332
41 17	1.00009162	41 47	0.99999092	42 17	0.99996559	42 47	1.00001624
41 18	1.00008706	41 48	0.99998885	42 18	0.99996605	42 48	1.00001924
41 19	1.00008258	41 49	0.99998688	42 19	0.99996659	42 49	1.00002233
41 20	1.00007818	41 50	0.99998498	42 20	0.99996722	42 50	1.00002551
41 21	1.00007386	41 51	0.99998317	42 21	0.99996794	42 51	1.00002877
41 22	1.00006963	41 52	0.99998144	42 22	0.99996874	42 52	1.00003211
41 23	1.00006548	41 53	0.99997980	42 23	0.99996962	42 53	1.00003554
41 24	1.00006141	41 54	0.99997824	42 24	0.99997059	42 54	1.00003906
41 25	1.00005743	41 55	0.99997677	42 25	0.99997164	42 55	1.00004266
41 26	1.00005353	41 56	0.99997538	42 26	0.99997278	42 56	1.00004634
41 27	1.00004971	41 57	0.99997407	42 27	0.99997400	42 57	1.00005011
41 28	1.00004598	41 58	0.99997285	42 28	0.99997531	42 58	1.00005397
41 29	1.00004233	41 59	0.99997171	42 29	0.99997670	42 59	1.00005791
						43 00	1.00006194

Massachusetts Mainland Lambert Conformal Projection

Long dd mm	Conv. Angle d mm ss.s	Long dd mm	Conv. Angle d mm ss.s	Long dd mm	Conv. Angle d mm ss.s	Long dd mm	Conv. Angle d mm ss.s
69 50	+1 07 10.4	70 48	+0 28 12.8	71 46	−0 10 44.9	72 44	−0 49 42.5
69 51	+1 06 30.1	70 49	+0 27 32.5	71 47	−0 11 25.2	72 45	−0 50 22.8
69 52	+1 05 49.8	70 50	+0 26 52.1	71 48	−0 12 05.5	72 46	−0 51 03.1
69 53	+1 05 09.5	70 51	+0 26 11.8	71 49	−0 12 45.8	72 47	−0 51 43.4
69 54	+1 04 29.2	70 52	+0 25 31.5	71 50	−0 13 26.1	72 48	−0 52 23.7
69 55	+1 03 48.9	70 53	+0 24 51.2	71 51	−0 14 06.4	72 49	−0 53 04.0
69 56	+1 03 08.5	70 54	+0 24 10.9	71 52	−0 14 46.7	72 50	−0 53 44.3
69 57	+1 02 28.2	70 55	+0 23 30.6	71 53	−0 15 27.0	72 51	−0 54 24.6
69 58	+1 01 47.9	70 56	+0 22 50.3	71 54	−0 16 07.3	72 52	−0 55 04.9
69 59	+1 01 07.6	70 57	+0 22 10.0	71 55	−0 16 47.6	72 53	−0 55 45.2
70 00	+1 00 27.3	70 58	+0 21 29.7	71 56	−0 17 27.9	72 54	−0 56 25.5
70 01	+0 59 47.0	70 59	+0 20 49.4	71 57	−0 18 08.2	72 55	−0 57 05.8
70 02	+0 59 06.7	71 00	+0 20 09.1	71 58	−0 18 48.5	72 56	−0 57 46.1
70 03	+0 58 26.4	71 01	+0 19 28.8	71 59	−0 19 28.8	72 57	−0 58 26.4
70 04	+0 57 46.1	71 02	+0 18 48.5	72 00	−0 20 09.1	72 58	−0 59 06.7
70 05	+0 57 05.8	71 03	+0 18 08.2	72 01	−0 20 49.4	72 59	−0 59 47.0
70 06	+0 56 25.5	71 04	+0 17 27.9	72 02	−0 21 29.7	73 00	−1 00 27.3
70 07	+0 55 45.2	71 05	+0 16 47.6	72 03	−0 22 10.0	73 01	−1 01 07.6
70 08	+0 55 04.9	71 06	+0 16 07.3	72 04	−0 22 50.3	73 02	−1 01 47.9
70 09	+0 54 24.6	71 07	+0 15 27.0	72 05	−0 23 30.6	73 03	−1 02 28.2
70 10	+0 53 44.3	71 08	+0 14 46.7	72 06	−0 24 10.9	73 04	−1 03 08.5
70 11	+0 53 04.0	71 09	+0 14 06.4	72 07	−0 24 51.2	73 05	−1 03 48.9
70 12	+0 52 23.7	71 10	+0 13 26.1	72 08	−0 25 31.5	73 06	−1 04 29.2
70 13	+0 51 43.4	71 11	+0 12 45.8	72 09	−0 26 11.8	73 07	−1 05 09.5
70 14	+0 51 03.1	71 12	+0 12 05.5	72 10	−0 26 52.1	73 08	−1 05 49.8
70 15	+0 50 22.8	71 13	+0 11 25.2	72 11	−0 27 32.5	73 09	−1 06 30.1
70 16	+0 49 42.5	71 14	+0 10 44.9	72 12	−0 28 12.8	73 10	−1 07 10.4
70 17	+0 49 02.2	71 15	+0 10 04.6	72 13	−0 28 53.1	73 11	−1 07 50.7
70 18	+0 48 21.9	71 16	+0 09 24.3	72 14	−0 29 33.4	73 12	−1 08 31.0
70 19	+0 47 41.6	71 17	+0 08 43.9	72 15	−0 30 13.7	73 13	−1 09 11.3
70 20	+0 47 01.3	71 18	+0 08 03.6	72 16	−0 30 54.0	73 14	−1 09 51.6
70 21	+0 46 21.0	71 19	+0 07 23.3	72 17	−0 31 34.3	73 15	−1 10 31.9
70 22	+0 45 40.7	71 20	+0 06 43.0	72 18	−0 32 14.6	73 16	−1 11 12.2
70 23	+0 45 00.3	71 21	+0 06 02.7	72 19	−0 32 54.9	73 17	−1 11 52.5
70 24	+0 44 20.0	71 22	+0 05 22.4	72 20	−0 33 35.2	73 18	−1 12 32.8
70 25	+0 43 39.7	71 23	+0 04 42.1	72 21	−0 34 15.5	73 19	−1 13 13.1
70 26	+0 42 59.4	71 24	+0 04 01.8	72 22	−0 34 55.8	73 20	−1 13 53.4
70 27	+0 42 19.1	71 25	+0 03 21.5	72 23	−0 35 36.1	73 21	−1 14 33.7
70 28	+0 41 38.8	71 26	+0 02 41.2	72 24	−0 36 16.4	73 22	−1 15 14.0
70 29	+0 40 58.5	71 27	+0 02 00.9	72 25	−0 36 56.7	73 23	−1 15 54.3
70 30	+0 40 18.2	71 28	+0 01 20.6	72 26	−0 37 37.0	73 24	−1 16 34.6
70 31	+0 39 37.9	71 29	+0 00 40.3	72 27	−0 38 17.3	73 25	−1 17 14.9
70 32	+0 38 57.6	71 30	+0 00 00.0	72 28	−0 38 57.6	73 26	−1 17 55.2
70 33	+0 38 17.3	71 31	−0 00 40.3	72 29	−0 39 37.9	73 27	−1 18 35.5
70 34	+0 37 37.0	71 32	−0 01 20.6	72 30	−0 40 18.2	73 28	−1 19 15.8
70 35	+0 36 56.7	71 33	−0 02 00.9	72 31	−0 40 58.5	73 29	−1 19 56.1
70 36	+0 36 16.4	71 34	−0 02 41.2	72 32	−0 41 38.8	73 30	−1 20 36.4
70 37	+0 35 36.1	71 35	−0 03 21.5	72 33	−0 42 19.1	73 31	−1 21 16.8
70 38	+0 34 55.8	71 36	−0 04 01.8	72 34	−0 42 59.4	73 32	−1 21 57.1
70 39	+0 34 15.5	71 37	−0 04 42.1	72 35	−0 43 39.7	73 33	−1 22 37.4
70 40	+0 33 35.2	71 38	−0 05 22.4	72 36	−0 44 20.0	73 34	−1 23 17.7
70 41	+0 32 54.9	71 39	−0 06 02.7	72 37	−0 45 00.3	73 35	−1 23 58.0
70 42	+0 32 14.6	71 40	−0 06 43.0	72 38	−0 45 40.7	73 36	−1 24 38.3
70 43	+0 31 34.3	71 41	−0 07 23.3	72 39	−0 46 21.0	73 37	−1 25 18.6
70 44	+0 30 54.0	71 42	−0 08 03.6	72 40	−0 47 01.3	73 38	−1 25 58.9
70 45	+0 30 13.7	71 43	−0 08 43.9	72 41	−0 47 41.6	73 39	−1 26 39.2
70 46	+0 29 33.4	71 44	−0 09 24.3	72 42	−0 48 21.9	73 40	−1 27 19.5
70 47	+0 28 53.1	71 45	−0 10 04.6	72 43	−0 49 02.2		

Massachusetts Island Lambert Conformal Projection

Lat dd mm	Scale Factor
41 00	1.00002074
41 01	1.00001885
41 02	1.00001704
41 03	1.00001532
41 04	1.00001368
41 05	1.00001212
41 06	1.00001065
41 07	1.00000926
41 08	1.00000796
41 09	1.00000674
41 10	1.00000560
41 11	1.00000455
41 12	1.00000358
41 13	1.00000270
41 14	1.00000190
41 15	1.00000118
41 16	1.00000055
41 17	1.00000000
41 18	0.99999954
41 19	0.99999916
41 20	0.99999886
41 21	0.99999865
41 22	0.99999852
41 23	0.99999848
41 24	0.99999852
41 25	0.99999865
41 26	0.99999886
41 27	0.99999916
41 28	0.99999954
41 29	1.00000000
41 30	1.00000055
41 31	1.00000118
41 32	1.00000190
41 33	1.00000270
41 34	1.00000359
41 35	1.00000456
41 36	1.00000561
41 37	1.00000675
41 38	1.00000798
41 39	1.00000929
41 40	1.00001068

Long dd mm	Conv. Angle d mm ss.s
69 50	+0 26 26.6
69 51	+0 25 47.0
69 52	+0 25 07.3
69 53	+0 24 27.6
69 54	+0 23 48.0
69 55	+0 23 08.3
69 56	+0 22 28.6
69 57	+0 21 49.0
69 58	+0 21 09.3
69 59	+0 20 29.6
70 00	+0 19 50.0
70 01	+0 19 10.3
70 02	+0 18 30.6
70 03	+0 17 51.0
70 04	+0 17 11.3
70 05	+0 16 31.6
70 06	+0 15 52.0
70 07	+0 15 12.3
70 08	+0 14 32.6
70 09	+0 13 53.0
70 10	+0 13 13.3
70 11	+0 12 33.6
70 12	+0 11 54.0
70 13	+0 11 14.3
70 14	+0 10 34.7
70 15	+0 09 55.0
70 16	+0 09 15.3
70 17	+0 08 35.7
70 18	+0 07 56.0
70 19	+0 07 16.3
70 20	+0 06 36.7
70 21	+0 05 57.0
70 22	+0 05 17.3
70 23	+0 04 37.7
70 24	+0 03 58.0
70 25	+0 03 18.3
70 26	+0 02 38.7
70 27	+0 01 59.0
70 28	+0 01 19.3
70 29	+0 00 39.7
70 30	+0 00 00.0
70 31	−0 00 39.7
70 32	−0 01 19.3
70 33	−0 01 59.0
70 34	−0 02 38.7
70 35	−0 03 18.3
70 36	−0 03 58.0
70 37	−0 04 37.7
70 38	−0 05 17.3
70 39	−0 05 57.0
70 40	−0 06 36.7
70 41	−0 07 16.3
70 42	−0 07 56.0
70 43	−0 08 35.7
70 44	−0 09 15.3
70 45	−0 09 55.0
70 46	−0 10 34.7
70 47	−0 11 14.3
70 48	−0 11 54.0
70 49	−0 12 33.6
70 50	−0 13 13.3
70 51	−0 13 53.0
70 52	−0 14 32.6
70 53	−0 15 12.3
70 54	−0 15 52.0
70 55	−0 16 31.6
70 56	−0 17 11.3
70 57	−0 17 51.0
70 58	−0 18 30.6
70 59	−0 19 10.3
71 00	−0 19 50.0
71 01	−0 20 29.6
71 02	−0 21 09.3
71 03	−0 21 49.0
71 04	−0 22 28.6
71 05	−0 23 08.3
71 06	−0 23 48.0
71 07	−0 24 27.6
71 08	−0 25 07.3
71 09	−0 25 47.0
71 10	−0 26 26.6

Rad. m	Deflec. per m	Deflection Angle for 10 m dd-mm-ss	20 m dd-mm-ss	30 m dd-mm-ss	Chord Length for 10 m	20 m	30 m
50	00-34-23	05-43-46	01-23-85	17-11-19	9.983	19.867	29.552
75	00-22-55	03-49-11	07-38-22	01-23-85	9.993	19.941	29.800
100	00-17-11	02-51-53	05-43-46	08-35-40	9.996	19.967	29.888
125	00-13-45	01-13-71	04-35-01	06-52-32	9.997	19.979	29.928
150	00-11-28	01-54-35	03-49-11	05-43-46	9.998	19.985	29.950
175	00-09-49	01-38-13	00-99-37	04-54-40	9.999	19.989	29.963
200	00-08-36	02-08-45	02-51-53	01-83-70	9.999	19.992	29.972
250	00-06-53	01-64-45	01-13-71	04-24-55	9.999	19.995	29.982
300	00-05-44	00-57-18	01-54-35	02-51-53	10.000	19.996	29.988
350	00-04-55	00-49-07	01-38-13	00-73-63	10.000	19.997	29.991
400	00-04-18	00-42-58	02-08-45	02-01-28	10.000	19.998	29.993
450	00-03-49	00-38-12	00-87-82	01-54-35	10.000	19.998	29.994
500	00-03-26	00-34-23	01-64-45	01-43-08	10.000	19.999	29.996
550	00-03-08	00-31-15	01-09-60	01-33-45	10.000	19.999	29.996
600	00-02-52	00-28-39	00-57-18	02-08-45	10.000	19.999	29.997
650	00-02-39	00-26-27	00-52-53	00-73-24	10.000	19.999	29.997
700	00-02-27	00-24-33	00-49-07	01-46-23	10.000	19.999	29.998
750	00-02-18	00-22-55	00-45-50	01-64-45	10.000	19.999	29.998
800	00-02-09	00-21-29	00-42-58	00-98-66	10.000	19.999	29.998
850	00-02-01	00-20-13	00-40-27	01-00-40	10.000	20.000	29.998
900	00-01-55	00-19-06	00-38-12	00-57-18	10.000	20.000	29.999
950	00-01-49	00-18-06	00-36-11	00-54-17	10.000	20.000	29.999
1000	00-01-43	00-17-11	00-34-23	00-51-34	10.000	20.000	29.999
1100	00-01-34	00-15-38	00-31-15	00-46-53	10.000	20.000	29.999
1200	00-01-26	00-14-19	00-28-39	00-42-58	10.000	20.000	29.999
1300	00-01-19	00-13-13	00-26-27	00-39-40	10.000	20.000	29.999
1400	00-01-14	00-12-17	00-24-33	00-36-50	10.000	20.000	29.999
1500	00-01-09	00-11-28	00-22-55	00-34-23	10.000	20.000	30.000
1600	00-01-04	00-10-45	00-21-29	00-32-14	10.000	20.000	30.000
1700	00-01-01	00-10-07	00-20-13	00-30-20	10.000	20.000	30.000
1800	00-00-57	00-09-33	00-19-06	00-28-39	10.000	20.000	30.000
1900	00-00-54	00-09-03	00-18-06	00-27-08	10.000	20.000	30.000
2000	00-00-52	00-08-36	00-17-11	00-25-47	10.000	20.000	30.000
2200	00-00-47	00-07-49	00-15-38	00-23-26	10.000	20.000	30.000
2400	00-00-43	00-07-10	00-14-19	00-21-29	10.000	20.000	30.000
2500	00-00-41	00-06-53	00-13-45	00-20-38	10.000	20.000	30.000
2600	00-00-40	00-06-37	00-13-13	00-19-50	10.000	20.000	30.000
2800	00-00-37	00-06-08	00-12-17	00-18-25	10.000	20.000	30.000
3000	00-00-34	00-05-44	00-11-28	00-17-11	10.000	20.000	30.000

APPENDIX A-6

* SURVEY NOTEKEEPING
* RE-ESTABLISHING LAYOUT AND BASELINE
* PROPERTY SURVEY NOTES
* BORING STAKEOUT AND NOTES
* FUNDAMENTAL BRIDGE NOMENCLATURE
* EXAMPLE OF TRAVERSE
* EXAMPLE OF PEG TEST
* EXAMPLE OF MICROMETER DOUBLE SCALE LEVELING
* BENCH LEVEL RUN SINGLE HAIR
* EXAMPLE OF 3-WIRE LEVELS
* DETAIL NOTES TAKEN MANUALLY
* EXAMPLE OF DRAINAGE NOTES FOR ELECTRONIC FIELD BOOKS
* DETAIL NOTES TAKEN ELECTRONICALLY
* EXAMPLES OF PRELIMINARY CROSS-SECTION NOTES
* EXAMPLE OF GRADE STAKE NOTES

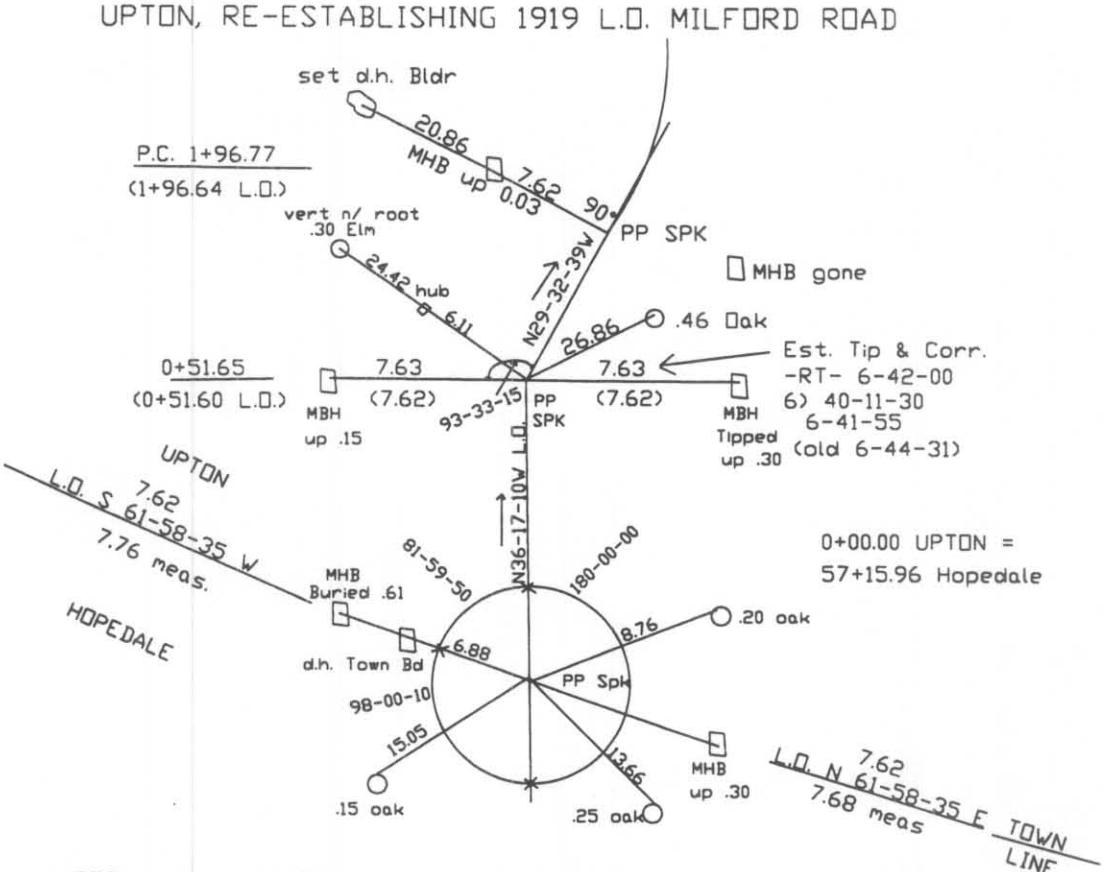
Survey Notekeeping

The following survey notes are descriptive of our normal procedure and have been derived from the previous survey manual and from actual notes.

The intent of the recorder should be to present clear and complete information. His sketches and printing should be plain, neat and understandable, not necessarily artistic, but readable.

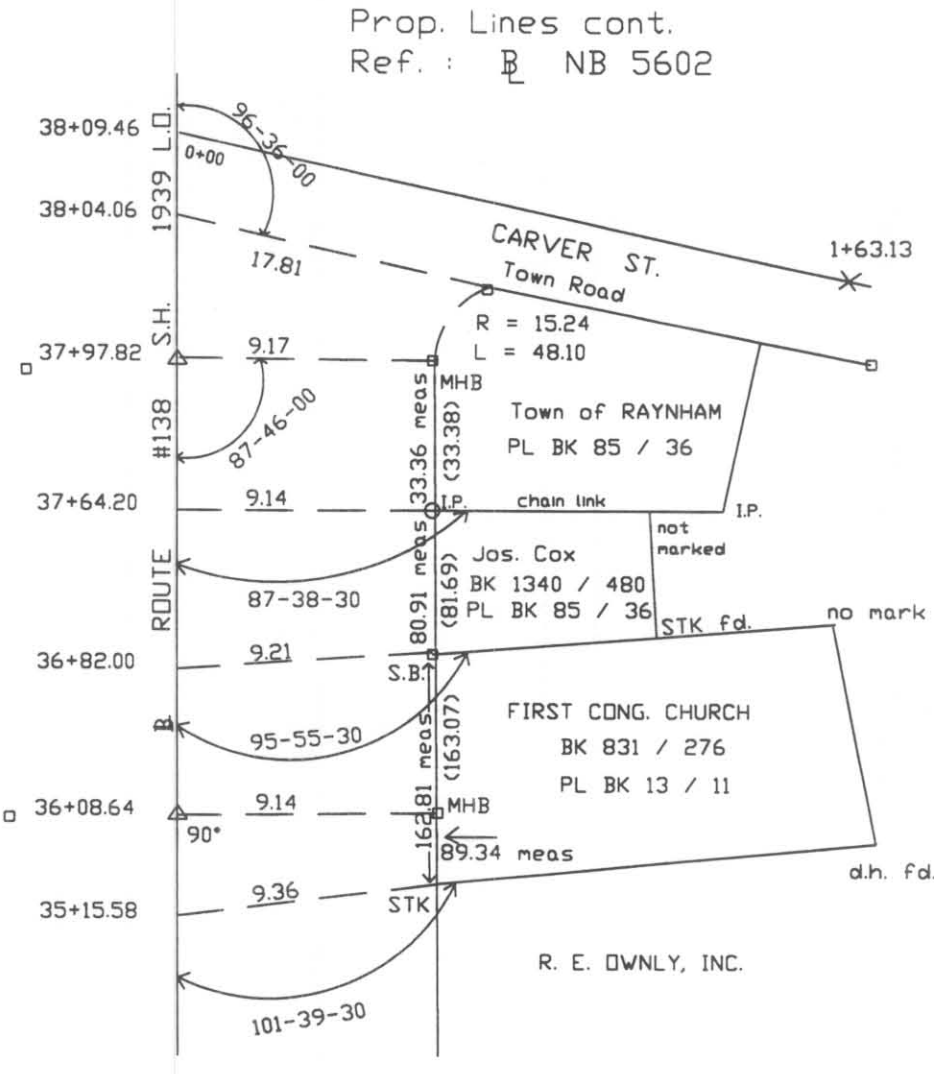
Notebooks are very important to those who use them: they may be the only source of information available. Survey notes are often invaluable both from engineering and legal standpoints. It is important that they be original and show no sign of erasures. The chief's name, name of members of the party, and date, should appear on each page. The book should be well indexed so that its contents may be readily ascertained.

Good notekeeping skills are difficult to acquire, but a person with engineering skill should be able to put out acceptable notes after practice, even if that person has limited drawing ability. (Some notekeepers take "pretty" notes which are pretty inaccurate, some take plain but excellent notes.)



NOTE: Note on MBH's, split backs for points, old type CB's, no dh's

ANGLE DATA, TAPING, CURVES, ETC., CAN BE PLACED ON FACING PAGE IN ORDER TO AVOID CROWDING NOTES


Prop. Lines cont.
Ref. : B NB 5602
38+09.46
38+04.06
37+97.82
37+64.20
36+82.00
36+08.64
35+15.58
1939 L.O.
S.H.
ROUTE #138
96-36-00
0+00
17.81
CARVER ST.
Town Road
1+63.13
R = 15.24
L = 48.10
9.17
87-46-00
MHB
33.36 meas
(33.38)
Town of RAYNHAM
PL BK 85 / 36
9.14
I.P.
chain link
I.P.
not marked
87-38-30
80.91 meas
(81.69)
Jos. Cox
BK 1340 / 480
PL BK 85 / 36
STK fd.
no mark
9.21
S.B.
95-55-30
162.81 meas
(163.07)
FIRST CONG. CHURCH
BK 831 / 276
PL BK 13 / 11
9.14
MHB
90°
89.34 meas
d.h. fd.
9.36
STK
R. E. OWNLY, INC.
101-39-30

100 LEVELS

BORING LOCATIONS

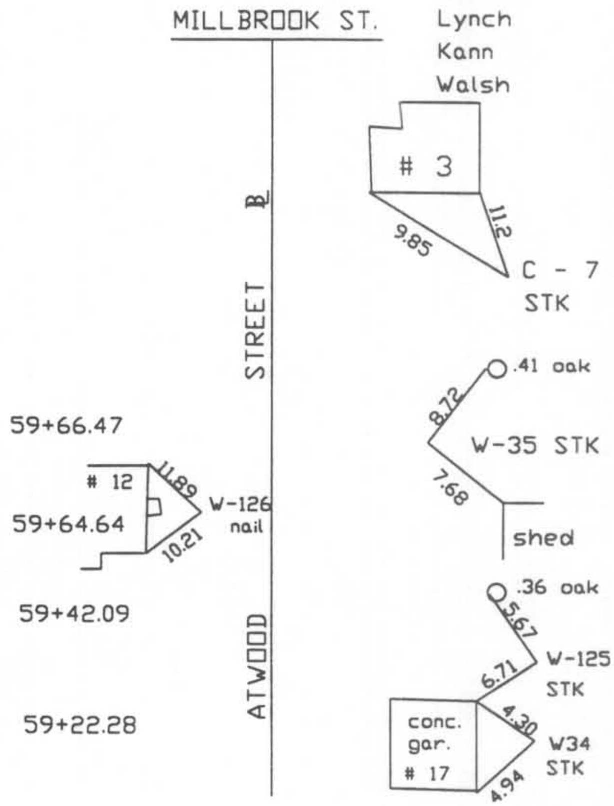
Atwood St.

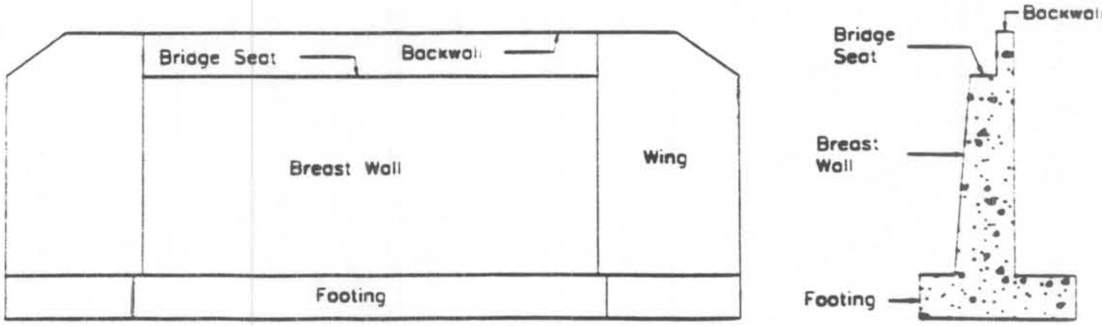
Located fr. main B.L. N 23020 / 88 - 90

BM 15 spk .71 oak 59+25.32	RT 18.3	2.10	157.16		155.06
W - 34 59+22.28	RT 18.9			2.38	154.78
W - 125 59+42.09	RT 18.3			1.10	156.06
W - 126 59+64.64	LT 9.14			1.01	156.15
W - 35 59+66.47	RT 15.5			2.29	154.87
C - 7 59+89.03	RT 12.8			1.71	155.45
TP nl .20 ash 60+46.02	LT 7.62			0.59	156.57

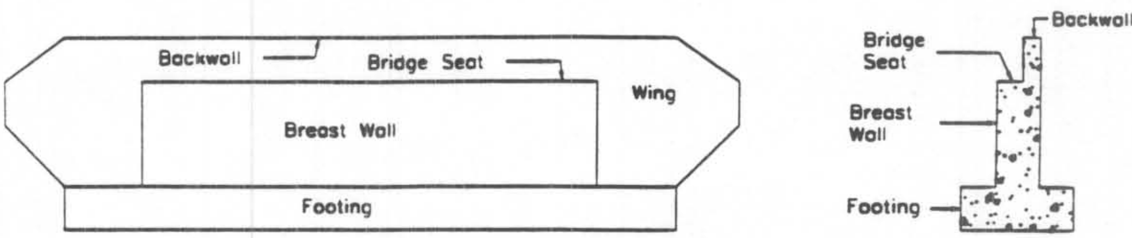
4-18-94 101

Kalloch
Lynch
Kann
Walsh

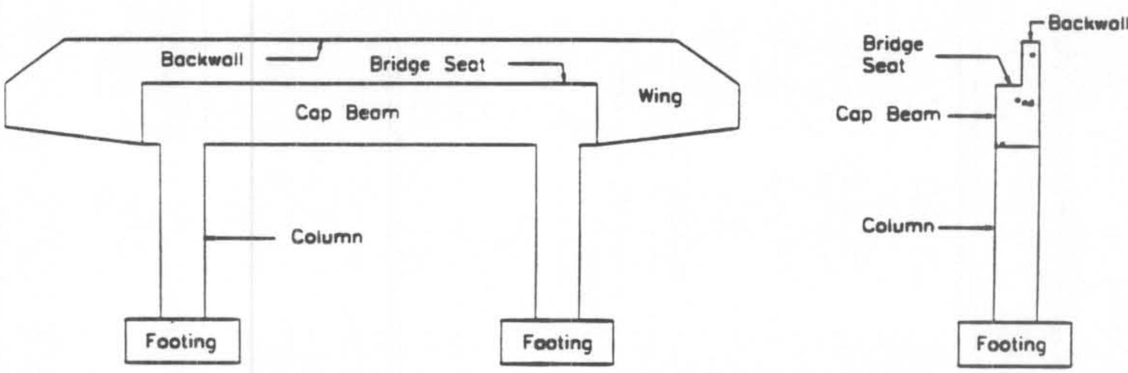




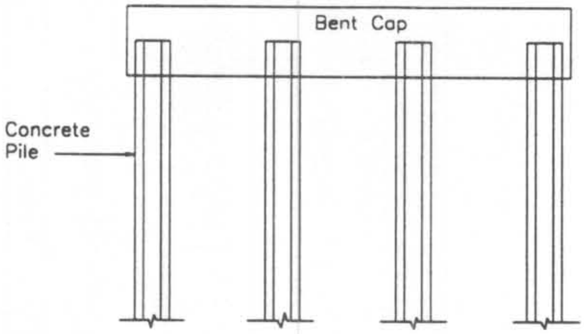
FULL HEIGHT ABUTMENT



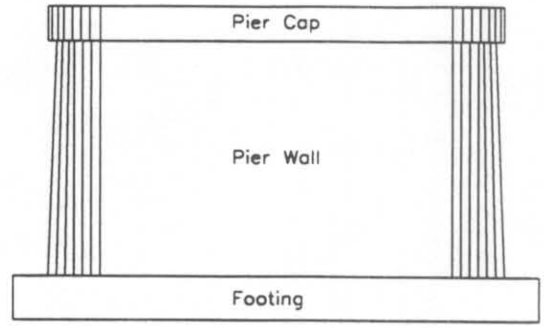
STUB ABUTMENT



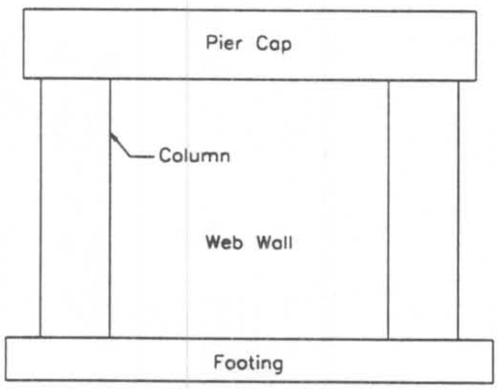
OPEN ABUTMENT



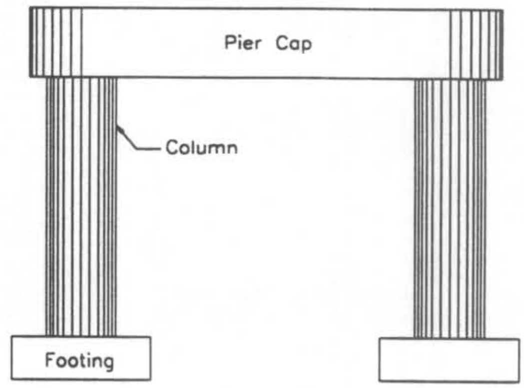
PILE BENT



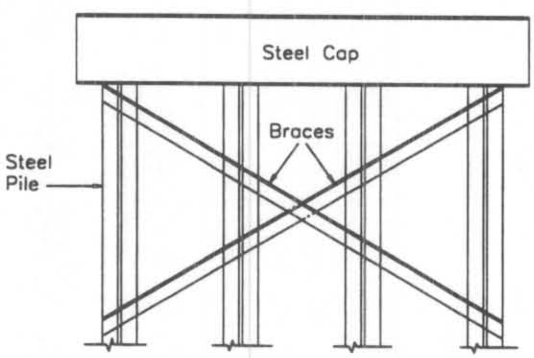
SOLID PIER



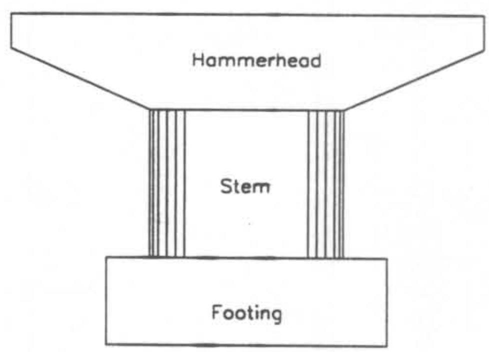
COLUMN PIER WITH SOLID WEB WALL



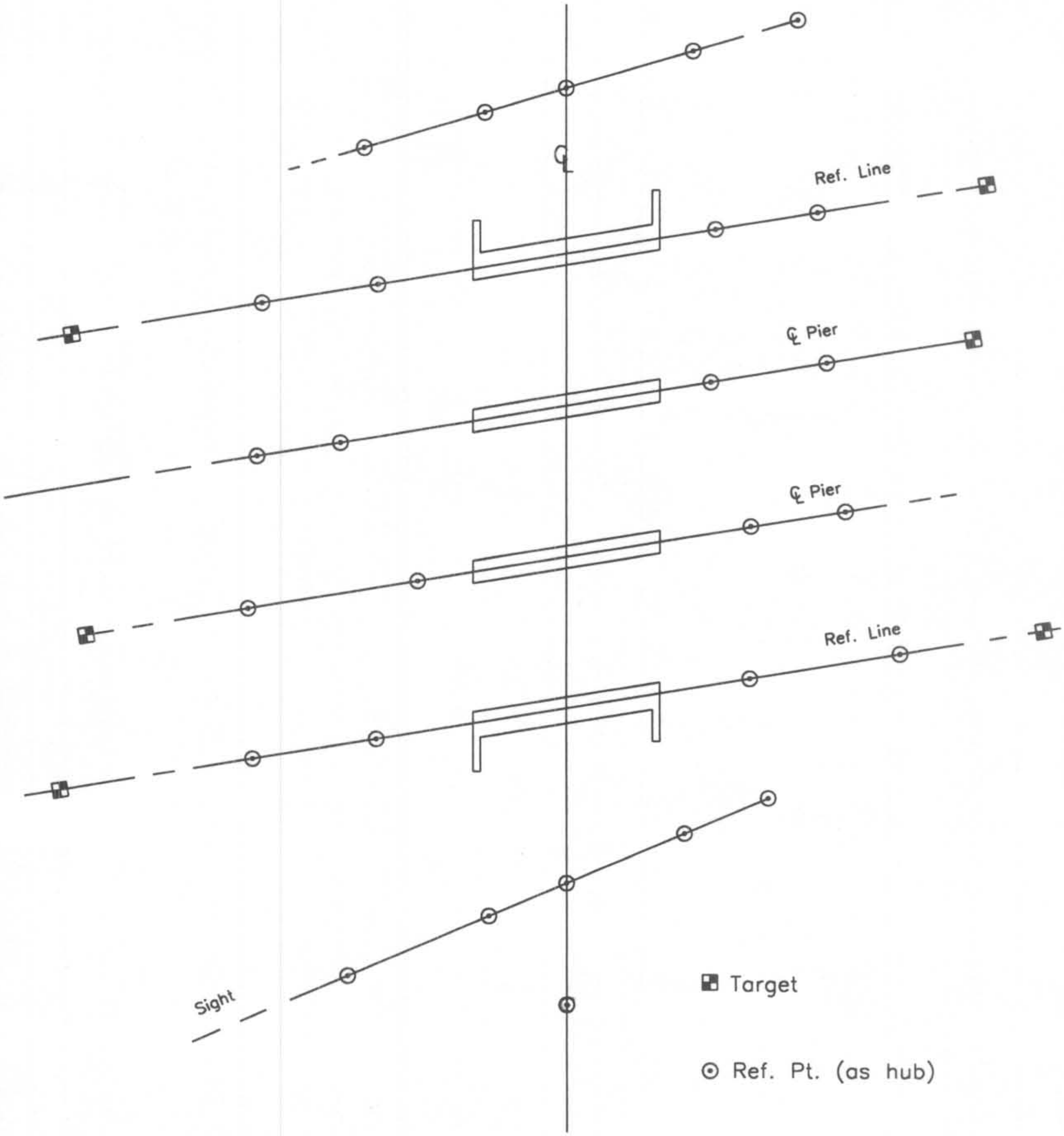
COLUMN BENT OR OPEN PIER



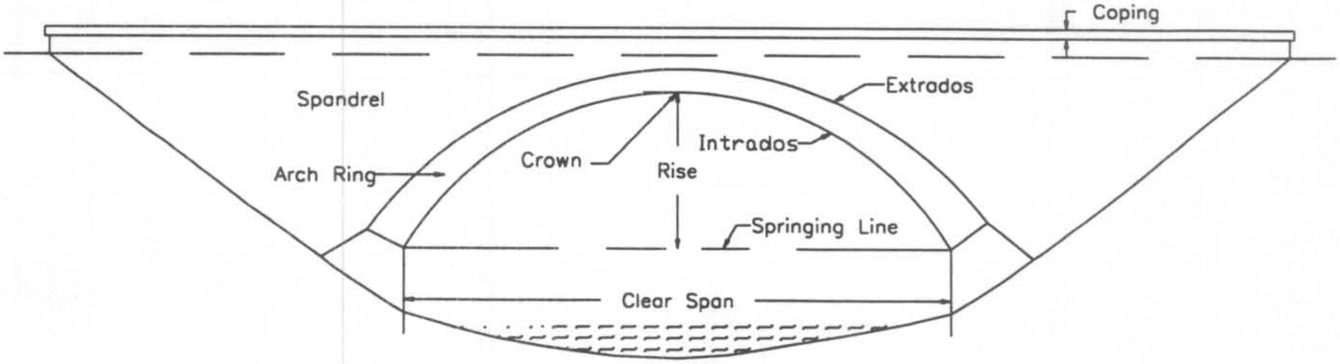
STEEL BENT



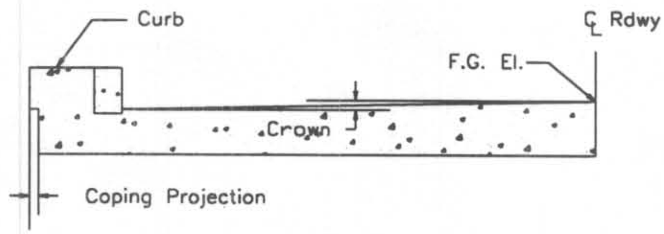
CANTILEVER PIER OR HAMMERHEAD PIER

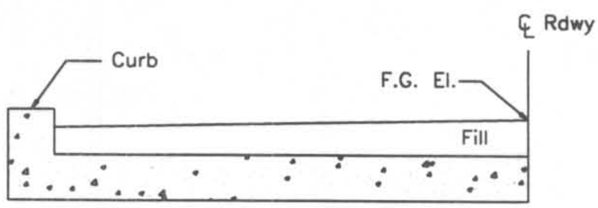


TYPICAL BRIDGE LAYOUT

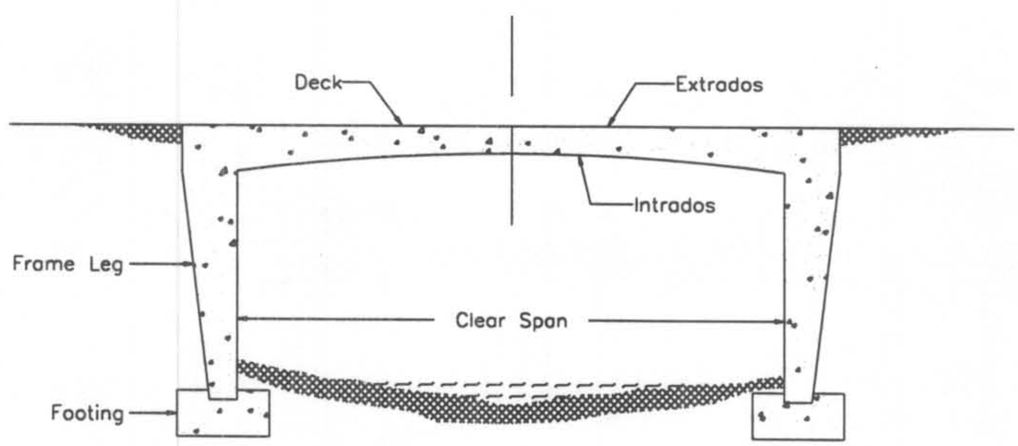


ARCH





CONCRETE DECKS



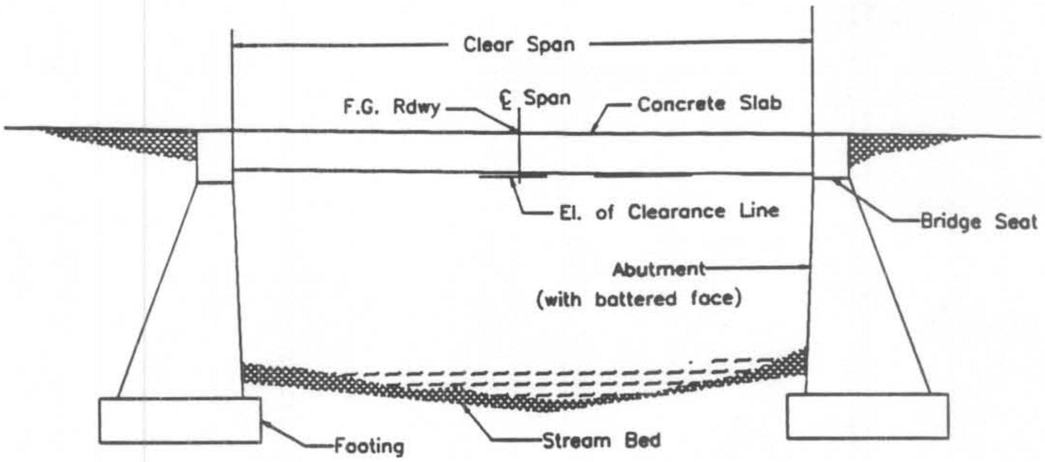
CONCRETE FRAME

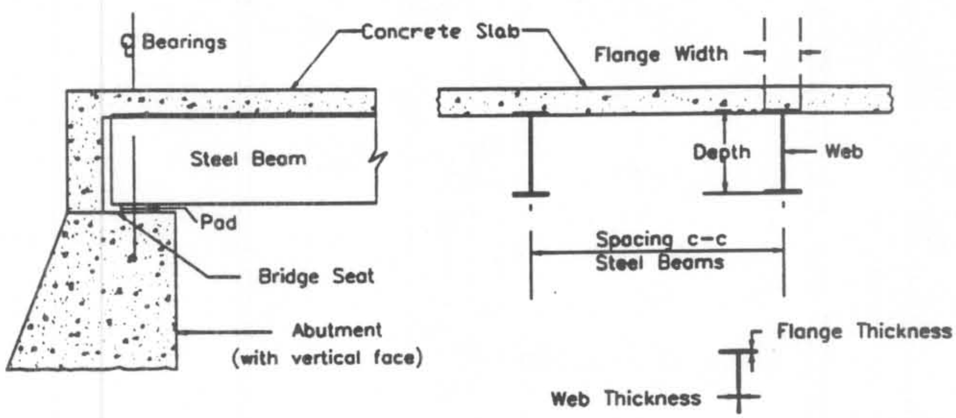
MASS HIGHWAY SURVEY MANUAL

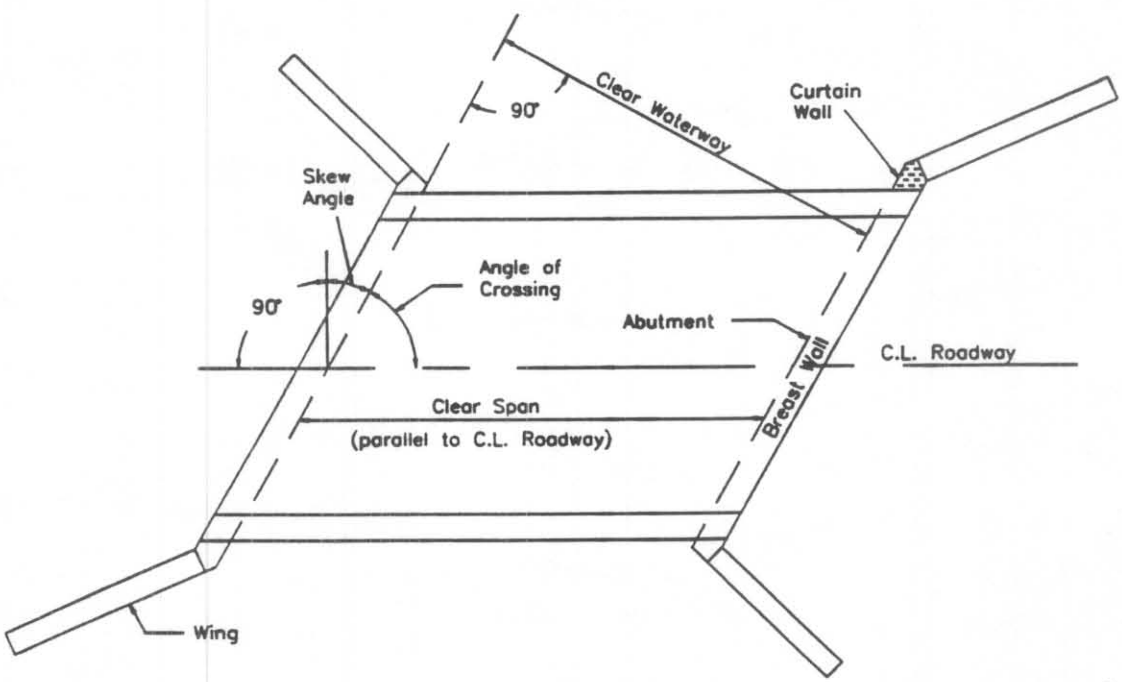
FUNDAMENTAL BRIDGE NOMENCLATURE

DATE OF ISSUE SEPTEMBER 1996

APPENDIX **A-6**







MASS HIGHWAY SURVEY MANUAL

FUNDAMENTAL BRIDGE NOMENCLATURE

DATE OF ISSUE
SEPTEMBER 1996

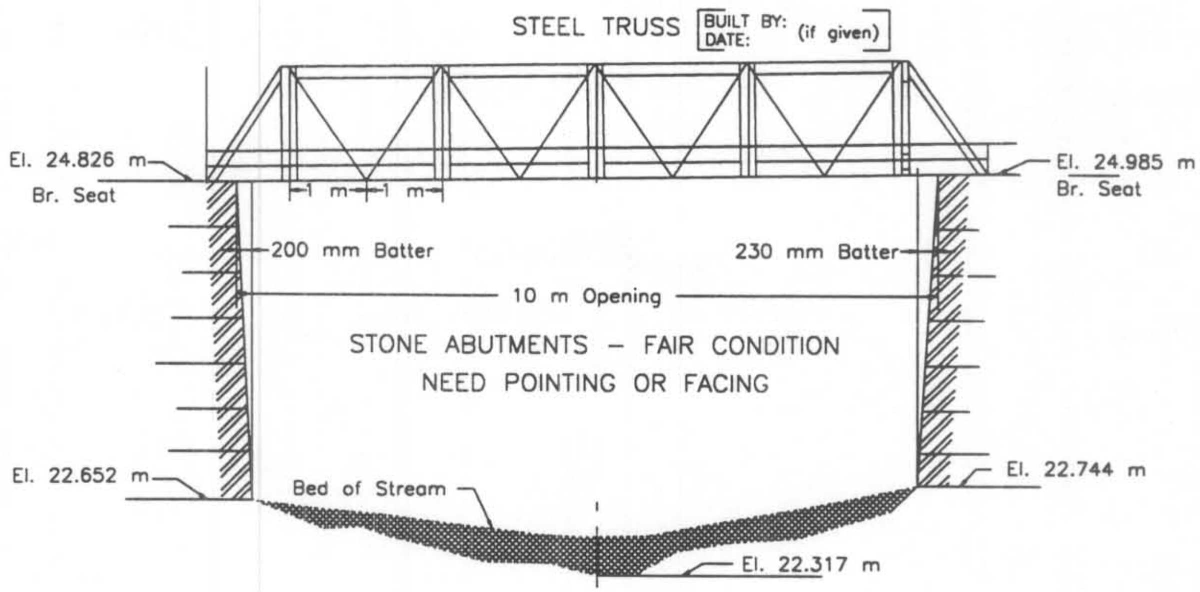
Sufficient data to be secured to plot longitudinal and transverse sections of bridge, particularly showing clear width, height and area of waterway, as well as width, length, and elevations of bridge seats, parapets, wing walls, and piers.

Note character and materials of abutments, wing walls and piers, and type and material in superstructure.

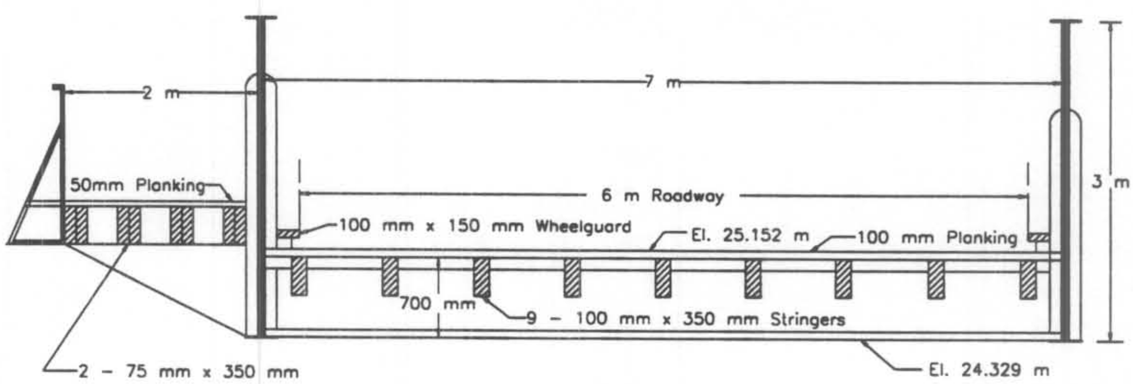
All locations of points by angle distances shall have check measurements between the points.

Water elevations to be given as follows:

Ex. High Water   Av. High Water   Low Water   Ex. Low Water



TYPICAL LONGITUDINAL SECTION



TYPICAL TRANSVERSE SECTION

WILD T-2
168883

19 JUL 88
OVERCAST-COOL

CHIEF OF PARTY
TRANSITMAN
RODMAN

HORIZ. ⊼ @ STATION RICHDALE

STA.		°	'	"	MEAN	DIR.
CHURCH SPIRE (1ST BAPTIST)	D	0	00	10	07.0	
	R	180	00	04		
CHINESE REST.		166	05	34	32.0	25.0
		346	05	30		
CROSSWALK		228	03	23	21.5	14.5
		48	03	20		
SUMMIT		335	37	55	53.5	46.0
		155	37	51		
CHURCH SPIRE	R	45	02	21	23.0	
	D	225	02	25		
CHINESE REST		211	07	46	48.0	25.0
		31	07	50		
CROSSWALK		273	05	33	35.5	12.5
		93	05	38		
SUMMIT		20	40	06	07.5	44.5
		200	40	09		

∡ TO CHINESE = 166° 05' 25.0"

∡ TO CROSSWALK = 228° 03' 13.5"

∡ TO SUMMIT = 335° 37' 45.25"

VERTICAL ⊼ @ STATION RICHDALE HI = 1.576m

STA.		°	'	"	MEAN	DIR.
CHINESE	CL	90	22	41	+44	
HI = 1.670m	CR	269	38	03		19.0
	CL	90	22	40	+42	19.0
	CR	269	38	02		
					Σ	38.0
					MN	19.0

V∡ = 90° 22' 19.0"

STA.		°	'	"	MEAN	DIR.
CROSSWALK	CL	89	21	58	+52	
HI = 1.740m	CR	270	38	54		32.0
	CL	89	21	58	+51	32.5
	CR	270	38	53		
						Σ 64.5
						MN 32.25

V∡ - 89° 21' 32.25"

STA.		°	'	"	MEAN	DIR.
SUMMIT	CL	89	58	00	+60	
HI = 1.603m	CR	270	03	00		30.0
	CL	89	58	02	+62	
	CR	270	03	00		31.0
						Σ 61.0
						MN = 30.5

V∡ = 89° 57' 30.5"

WILD - DI 3000

P = 1018 mb
T = 68°F 20°C
∴ ΔD_1 = +7 ppm

DIST. — ⊼ @ STATION RICHDALE — EDM HI = 1.667 m

	CHINESE V4 90°22'19.0"	CROSSWALK V4 = 89°21'32.3"	SUMMIT V4 = 89°57'30.5"
HI	1.670 m	1.740 m	1.603 m
SLOPE @ +7 PPM & Ø T.O.C.	183.945 m	26.297 m	540.507 m
MN OF S	183.945 m	26.297 m	540.507 m
HORIZ. (AD. + V4)	183.941 m	26.295 m	540.507 m
Δh	−1.192 m	+0.294 m	+0.410 m

78 CHIEF OF PARTY
TRANSITMAN
RODMAN

12/2/94

PEG TEST

NEAR ROD

	3 WIRE	MEAN	INT	DIST
1	1573 1558 1543	15580	15 15	30
2	1527 1511 1495	15110	16 16	32
		30690		62

ZEISS #33309 WX-00000 79
WILD ROD - 5129 A&B

FAR ROD

3 WIRE	MEAN	INT	DIST
1601 1538 1474	15377	63 64	127
1596 1532 1471	15330	64 61	125
	30707		252

30707
- 4
30703
- 30690
13 cm
13 mm ÷ 63.33 m
= 0.020 mm/m

252
- 62
190 = 63.33m

* NOTE *
THESE VALUES COMPUTED FOR THE ZEISS LEVEL, WHICH HAS A STADIA VALUE OF 0.33, MOST LEVELS HAVE A VALUE OF 1.0.

MASSHIGHWAY SURVEY MANUAL	EXAMPLE OF A PEG TEST	DATE OF ISSUE SEPTEMBER 1996
		APPENDIX A-6

6 BM 5949 TO BM 5950 FORWARD   START 11:11 - 70° END 11:56 - 76°   SUNNY-WINDY 40+ M.P.H. WARM   7

BM 5949	STADIA BACKWARD	Σ STADIA	STADIA FORWARD	Σ STADIA	LOW SCALE	Δ LOW SCALE	HIGH SCALE	Δ HIGH SCALE	Δhh-ΔhL
23 'A'	137	23	288	21	B 13717	− 15137	B 43871	− 15139	+02
	1347		2859		F 28854		F 59010		
24	064	9	213	9	B 06452	− 14913	B 36609	− 14913	00
	0631	32	2121	30	F 21365	− 30050	F 51522	− 30052	+02
25	119	159	211	152	B 11901	− 9267	B 42054	− 9268	+01
	1031	191	1958	182	F 21168	− 39317	F 51322	− 39320	+03
26	069	104	299	114	B 06911	− 23029	B 37065	− 23027	−02
	0586	295	2876	296	F 29940	− 62346	F 60092	− 62347	+01
27	061	60	249	61	B 06176	− 18814	B 36331	− 18814	00
	0550	355	2429	357	F 24990	− 81160	F 55145	− 81161	+01
28	052	70	235	74	B 05264	− 18251	B 35420	− 18249	−02
	0450	425	2276	431	F 23515	− 99411	F 53669	− 99410	−01
29 BM 5950	130	249	166	250	B 13070	− 3555	B 43224	− 3553	−02
	1051	674	1410	681	F 16625	− 102966	F 46777	− 102963	−03
30 'A'	168	106	059	100	B 16890	+ 10920	B 47045	+ 10920	00
	1574	780	0490	781	F 05970	− 92046	F 36125	− 92043	−03

√ By D2 3/27/91

For mean = 9.2044^5 Meter
Back mean = 9.2047^1 See P. 14

14 BM 5949 TO BM 5950 BACKWARD

START 9:35 – 41° Cloudy
END 10:25 – 40° COOL

ATF-135
5812
5812

BM / Station	STADIA BACKWARD	Σ STADIA	STADIA FORWARD	Σ STADIA	LOW SCALE	Δ LOW SCALE	HIGH SCALE	Δ HIGH SCALE	Δhh-ΔhL
BM 5950 ③	057	109	166	107	B 05770	− 10833	B 35923	− 10834	+01
'A'	0461		1553		F 16603		F 46757		
④	165	275	080	271	B 16565	+ 8515	B 46715	+ 8513	+02
	1375	384	0529	378	F 08050	− 2318	F 38202	− 2321	+03
⑤	276	67	057	79	B 27678	+ 21902	B 57833	+ 21902	00
	2693	451	0491	457	F 05776	+ 19584	F 35931	+ 19581	+03
⑥	289	79	086	70	B 28996	+ 20358	B 59150	+ 20358	00
	2811	530	0790	527	F 08638	+ 39942	F 38792	+ 39939	+03
⑦	250	76	112	79	B 25076	+ 13864	B 55231	+ 13865	−01
	2424	606	1041	606	F 11212	+ 53806	F 41366	+ 53804	+02
⑧	206	141	127	145	B 20619	+ 7848	B 50775	+ 7850	−02
	1919	747	1125	751	F 12771	+ 61654	F 42925	+ 61654	00
⑨	239	15	051	9	B 23900	+ 18760	B 54054	+ 18760	00
	2375	762	0501	760	F 05140	+ 80414	F 35294	+ 80414	00
BM 5949 ⑩	255	21	138	22	B 25522	+ 11635	B 55675	+ 11632	+02
'A'	2529	783	1358	782	F 13887	+ 92049	F 44043	+ 92046	+02

Checked: GB
4/2/91

ALLOW (2 x 156) = 0.312 km.

$4\,mm\sqrt{0.312} = 2.23$

− 92044⁵ MEAN FOR.
+ 92047 MEAN BACK
.02⁵ CM
0.2⁵ MM

− 30.1987 Ft.

9.2046 M

15

Zeiss 33309 1/1/94

BENCH LEVELS, FOR RUN, CONT'D.

MILL RD

STA.	BS.	HI	F.S.	ELEV.
BM #6	0.731	83.492		82.761 m
TP 16	0.863	82.008	2.347	81.145
TP 17	1.022	79.880	3.150	78.858
TP 18	0.921	78.648	2.153	77.727
TP 19	0.821	77.472	1.997	76.651
TP 20	1.034	76.026	2.480	74.992
BM #7			2.271	73.755 m
	+5.392		-14.398	
			+5.392	
			-9.006	
			+82.761	
			73.755 m	

CLOUDY 48°

CHIEF OF PARTY
TRANSITMAN
RODMAN

COMMENTS

SEE PG. 18

HOR SPK - SMLD Ø35 6+40 7m LT - UP .2m

HOR SPK NET Ø27 7+10 6m LT. UP .2m

VERT SPK - ROOT 0.3m MAPLE 7+82 13m LT.

TOP OF LEDGE 8+43 14m LT

TOP OF BLDR 9+24 13m RT.

□ CUT LEDGE HIGH POINT 10+20 14m RT

7AC-SESD 6 BACK 12/2/94

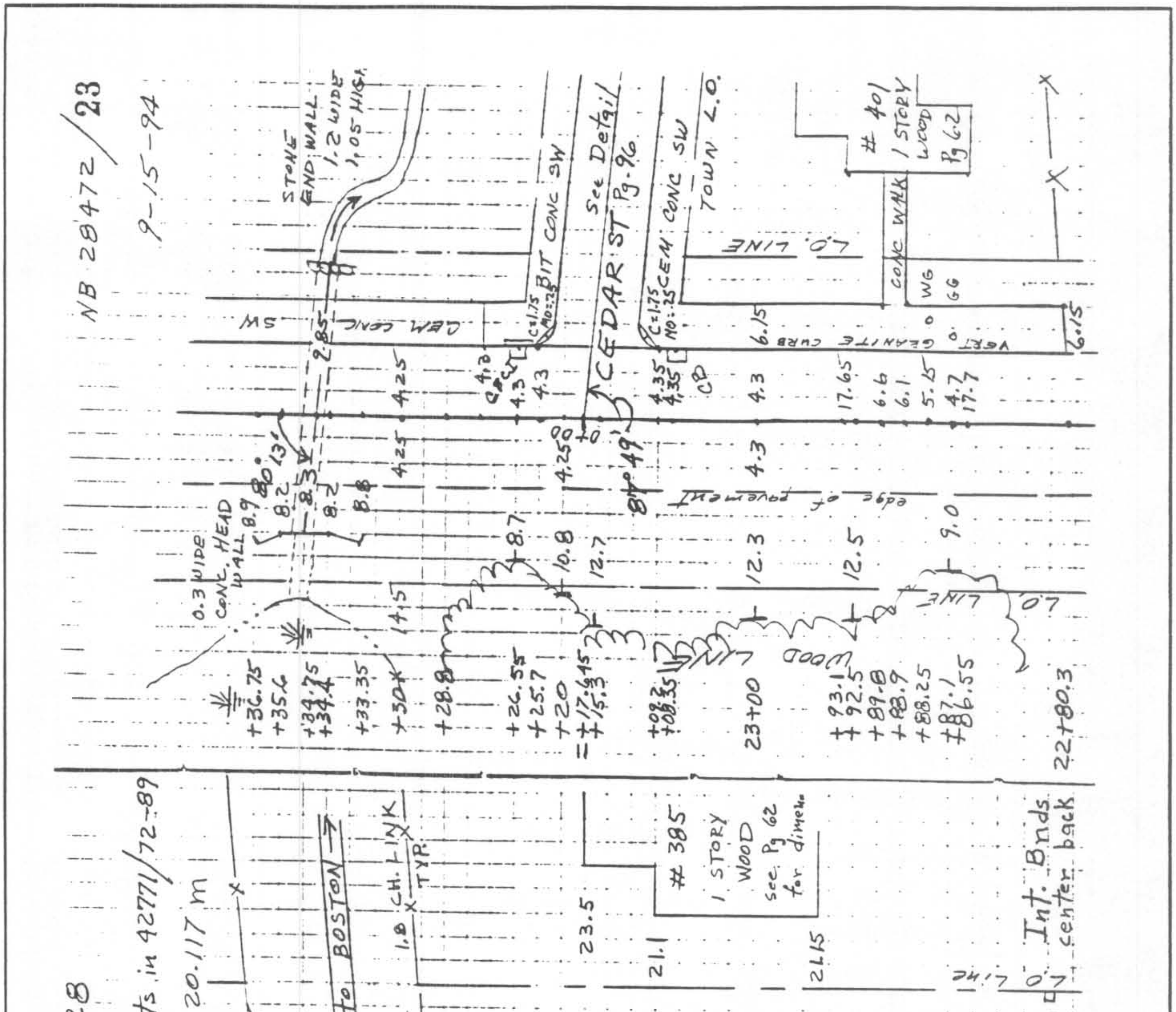
	LowScale 3 Wire	Mean	STD/INT	ΣS	Center High Scale
1A@SESD6					
	1303				
9	1292	12917	11	23	4302
	1280		12		
10	2591				
	2571	25707	20	41	5579
	2550	38624	21	64	9881
11	2858				
	2838	28380	20	40	5857
	2818	67004	20	104	15738
12	1668				
	1649	16490	19	38	4664
	1630	83494	19	142	20402
		-12144			-13268
BACK	ΔLOW = 7.135			ΔHIGH =	7.134

MEAN = 7.134^{5} m

ZEISS 33309
WILD 5129 A&B

CHIEF
TRANSITMAN
RODMAN

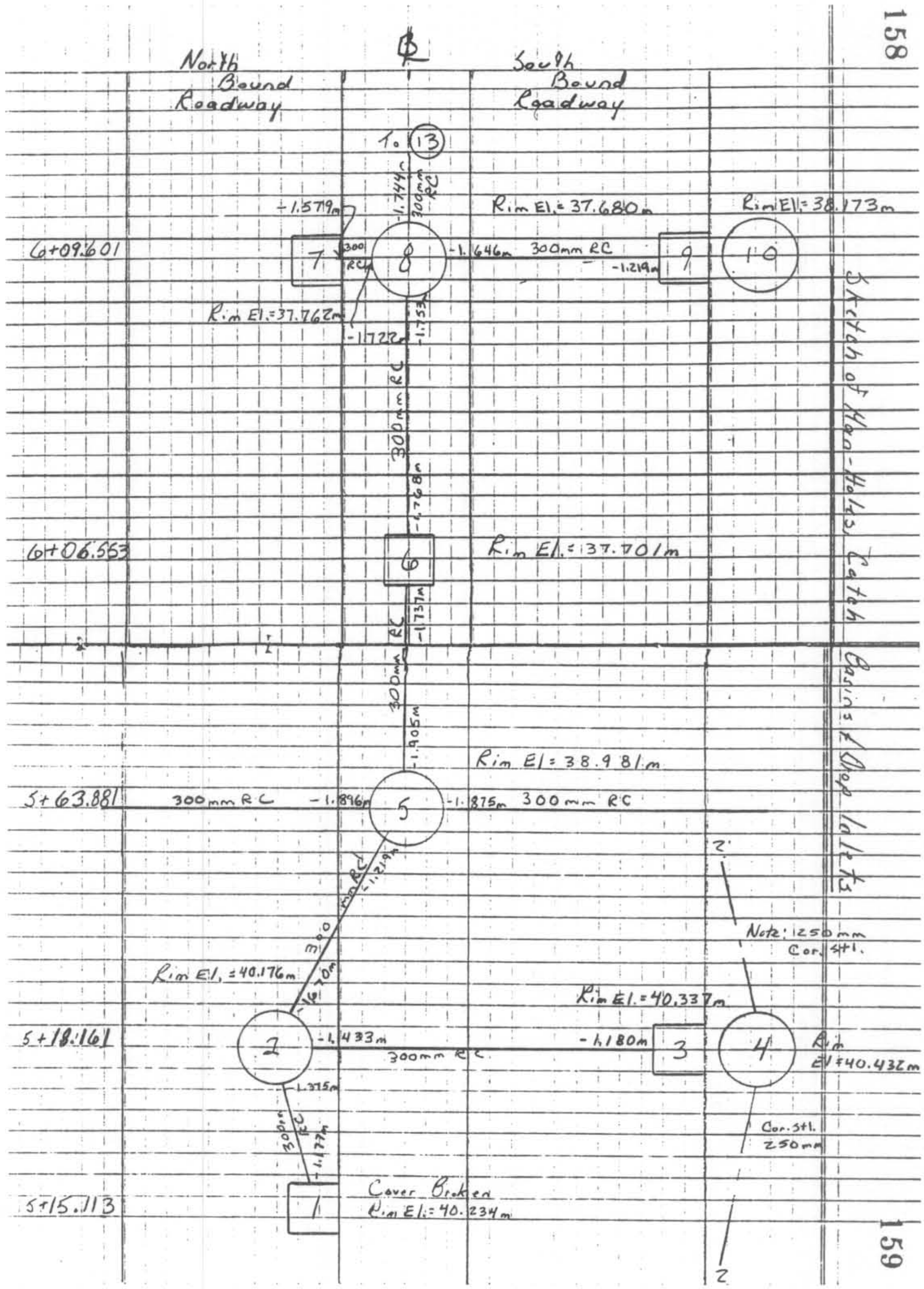
LowScale 3 Wire	Mean	STD/INT	ΣS	Center High Scale
0408				
0396	03957	12	25	3407
0383		13		
0158				
0141	01410	17	34	3149
0124	05367	17	59	6556
0422				
0400	03997	22	45	3419
0377	09364	23	104	9975
1A@7AC				
0298				
0278	02780	20	40	3293
0258	12144	20	144	13268



MASS HIGHWAY SURVEY MANUAL

DETAIL NOTES TAKEN MANUALLY
(TAPE AND RIGHT ANGLE PRISM)

DATE OF ISSUE
SEPTEMBER 1996



MASS HIGHWAY SURVEY MANUAL

EXAMPLE OF DRAINAGE NOTES FOR ELECTRONIC FIELD BOOKS

DATE OF ISSUE
SEPTEMBER 1996

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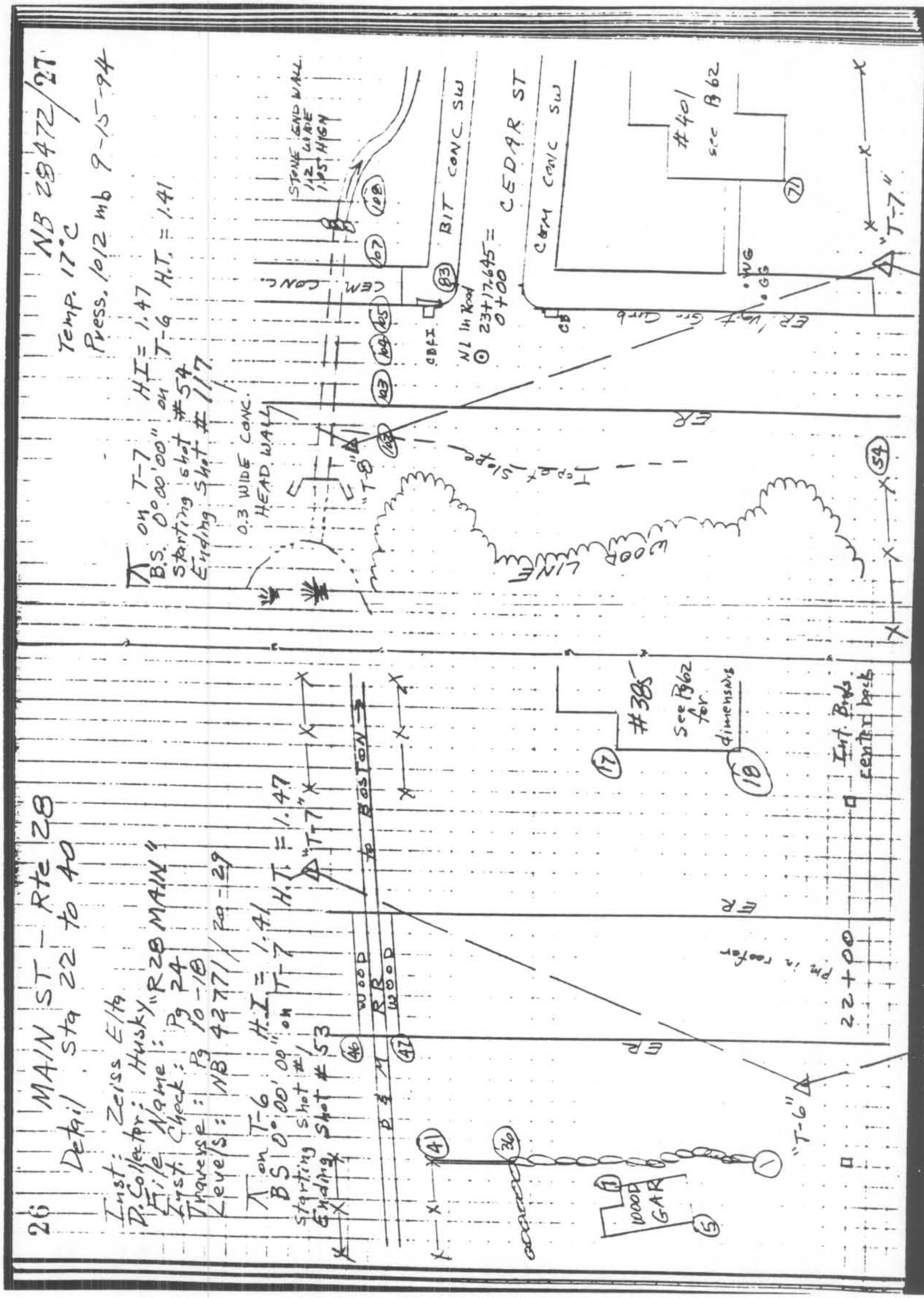

MASS HIGHWAY SURVEY MANUAL

DETAIL NOTES TAKEN ELECTRONICALLY (DATA COLLECTOR)

DATE OF ISSUE SEPTEMBER 1996

96

THORNDIKE RD. PRELIM X SECS.

DETAIL 3 ℄ NB 30102

BM 7 □ CUT LEDGE 20+21 16.m RT

BS 2.637 166.478

19+20	CONC SW 5.95 / 1.515	4.36 / 1.520	GRAN VERT CURB 4.35 / 1.670	3 / 1.650	TYPE 10 / 1.645	
19+20	SILL / 1.505		12.25 / 1.70	10 / 1.65	0/	
19+29.5 ASK 75° RT DRIVEWAY PROFILE	0 / 1.675	4.75 / 1.735	TYPE 6 / 1.525			
19+36.5 ASK 88°-13' RT CULVERT	TOP HEADWALL 17.90 / 2.070	℄ 4.65 / 1.735	TYPE 0 / 1.710			
19+36.5 ASK CONT.	15 / 2.135	10 / 2.680	INVERT 7.95 / 2.635	0/		

6/13/76

CHIEF OF PARTY
TRANSITMAN
RODMAN

97

NB 30101/11 163.841 m

I	3 / 1.655	4.45 / 1.675	GRAN VERT CURB 4.48 / 1.770	CONC SW 6.0 / 1.765	10 / 1.52	LAWN 15 / 1.22
	LAWN 17.05 / 1.22	TOP FIELDSTONE WALL 17.2 / 0.3				
I	10 / 1.480	12.20 / 1.400	CONC APRON 13.45 / 1.315	GARAGE FLOOR 13.5 / 1.305		
I	℄ 4.75 / 1.740	TOP HEADWALL 6.15 / 2.200	INVERT 15" RC 6.20 / 2.760	DITCH 10 / 2.895	15 / 3.050	

EXAMPLE OF PRELIMINARY CROSS-SECTION NOTES

BEDFORD ST - Rte's 18/28
GRADE STAKES

CHIEF
TRANSITMAN
RODMAN

NB 42771/91
11-18-94

℄ in NB 28472 / 20-35
Benches this book / 8-13

BM 19	0.835				17.400		16.565	☐-cut in ledge, pg 10
42		Lt	10.70	1.340			16.060	
		Rt	9.95	1.155			16.245	
42+20		Lt	10.15	1.715			15.685	
		Rt	11.05	1.630			15.770	
42+40		Lt	12.20	1.970			15.430	
		Rt	11.65	1.795			15.605	
42+60		Lt	9.90	2.010			15.390	
		Rt	10.25	1.960			15.440	
42+80		Lt	12.55	2.155			15.245	
		Rt	11.70	2.300			15.100	
TP 42+83 10.8 Rt.	1.120				15.825	2.690	14.710 (14.705)	STK up .2 m, pg 10
43		Lt	8.80	0.180			15.645	roofer in drive
		Rt	10.60	0.385			15.440	
43+20		Lt	10.55	0.630			15.195	
		Rt	11.40	0.725			15.100	
43+40		Lt	12.75	0.810			15.015	
		Rt	10.90	0.930			14.895	
BM 20						1.415	14.410 (14.410)	RR SPK in root, .3 m Pine, pg 10

APPENDIX A-7

* SURVEY COMPLETION REPORT

CITY/TOWN: ______________________ STREET: ______________

JOB NUMBER: ______________________ WORKDATE: ______________

CHIEF	WORKTYPE	STATIONS (FROM–TO)
1) ________	1) ________	1) ________
2) ________	2) ________	2) ________
3) ________	3) ________	3) ________

BOOKNUMBER	PAGE NUMBERS (FROM–TO)
1) ________	1) ________
2) ________	2) ________
3) ________	3) ________

ARE ALL BOOKS INDEXED, CROSS–INDEXED, AND DATED? ______________

ARE NAMES OF PARTY MEMBERS SHOWN? ______________

ARE TIE–INS TO OTHER SURVEYS, TOWNS OR COUNTIES, OR MASS.

GEODETIC SURVEY STATIONS SHOWN? ______________

NOTEBOOK NUMBERS USED FOR REFERENCE: ______________

PLANS USED FOR REFERENCE: ______________

REMARKS: ______________

DATE: ______________ SIGNED: ______________

APPENDIX A-8

* CALIBRATION BASE LINES STATE OF MASSACHUSETTS

CALIBRATION BASE LINES

STATE OF MASSACHUSETTS

Compiled and published by
National Geodetic Survey
Silver Spring, MD
December 01, 1994

U.S. DEPARTMENT OF COMMERCE
National Oceanic and Atmospheric Administration
National Ocean Service
Charting and Geodetic Services

Calibration Baselines

The following consists of data on five calibration sites that were set by the National Geodetic Survey.

These sites are used for checking/calibrating electronic distance measurement (EDM) equipment. There are four very solid monuments set on line at precisely known horizontal distances. This enables one to check an EDM at a variety of distances (150 m. to 1000+ m.). The presence of multiple monuments also allows one to ensure that they have remained stable.

A secondary use of the sites is for checking/calibrating measuring tapes. The sites also contain 100 and/or 200 ft. (note English dimension) bases which can be converted to metric equivalents (30.480 and 60.960 m. respectively). Note the limits of this as the sites are not protected from the environment and the monuments do not project above the ground.

Surveyors are expected to take advantage of the opportunity offered by the existence of these baselines, especially for EDM equipment. It is not uncommon for an uncalibrated EDM to provide distances which are off by a centimeter or more.

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US DEPARTMENT OF COMMERCE - NOAA
NOS - NATIONAL GEODETIC SURVEY
ROCKVILLE MD 20852 - OCTOBER 29, 1986

CALIBRATION BASE LINE DATA
BASE LINE DESIGNATION: CAPE COD
PROJECT ACCESSION NUMBER: G16364

QUAD: N410701
MASSACHUSETTS
BARNSTABLE COUNTY

LIST OF ADJUSTED DISTANCES (JANUARY 26,1980)

FROM STATION	ELEV. (M)	TO STATION	ELEV. (M)	ADJ. DIST. (M) HORIZONTAL	ADJ. DIST. (M) MARK - MARK	STD. ERROR (MM)
0	6.10	150	7.01	149.9867	149.9895	0.2
0	6.10	430	8.87	429.9773	429.9862	0.3
0	6.10	1400	16.88	1399.9655	1400.0070	0.5
150	7.01	430	8.87	279.9905	279.9967	0.3
150	7.01	1400	16.88	1249.9788	1250.0177	0.4
430	8.87	1400	16.88	969.9882	970.0213	0.3

DESCRIPTION OF CAPE COD BASE LINE
YEAR MEASURED: 1979
CHIEF OF PARTY: CWW

THE BASE LINE IS LOCATED ON CAPE COD ABOUT 5 MILES NORTHEAST OF HYANNIS AND 0.4 MILE SOUTHEAST OF YARMOUTH IN THE MEDIAN STRIP BETWEEN EXITS 8 AND 9 OF UNITED STATES HIGHWAY 6.

TO REACH THE 0 METER POINT FROM THE JUNCTION OF HIGHWAY 6 AND UNION STREET AT EXIT 8 GO EAST ON HIGHWAY 6 FOR 1.5 MILES TO THE 0 METER POINT ON THE LEFT.

THE 0 METER POINT IS 153.5 FEET WEST OF THE SOUTHWEST CORNER OF THE CENTER PIER OF AN OVERPASS, 31 FEET NORTH OF THE CENTER LINE OF THE EAST BOUND LANES OF HIGHWAY 6, 11.5 FEET NORTH-NORTHWEST OF MILE POST 75.9 AND 5 FEET SOUTH OF A METAL WITNESS POST.

ALL OF THE MARKS ARE STANDARD CALIBRATION BASE LINE DISKS, STAMPED WITH THEIR BASE LINE DESIGNATION AND THE YEAR 1979, SET IN THE TOP OF ROUND CONCRETE MONUMENTS THAT ARE ABOUT 2 INCHES BELOW THE SURFACE.

THE BASE LINE IS A EAST-WEST LINE WITH THE 0 METER POINT ON THE EAST END. IT IS MADE UP OF THE 0, 150, 430, AND 1400 METER POINTS WITH TWO MARKS FOR THE CALIBRATION OF 100 AND 200 FOOT TAPES SET ON LINE WEST OF THE 0 METER POINT.

THIS BASE LINE WAS ESTABLISHED IN CONJUNCTION WITH THE MASSACHUSETTS ASSOCIATION OF LAND SURVEYORS AND CIVIL ENGINEERS AND THE MASSACHUSETTS GEODETIC SURVEY. FOR FURTHER INFORMATION CONTACT THE MASSACHUSETTS GEODETIC SURVEY, ~~ROOM 717, 100 NASHUA STREET, BOSTON, MASSACHUSETTS 02114. TELEPHONE 617-727-8287 OR MR. GERALD S. SHELBY, MOORE SURVEY AND MAPPING CORPORATION, 29 GRAFTON CIRCLE, SHREWSBURY, MASSACHUSETTS 01545. TELEPHONE 617-845-4181~~.

US DEPARTMENT OF COMMERCE - NOAA
NOS - NATIONAL GEODETIC SURVEY
ROCKVILLE MD 20852 - OCTOBER 29, 1986

CALIBRATION BASE LINE DATA
BASE LINE DESIGNATION: GEORGETOWN
PROJECT ACCESSION NUMBER: G16364

QUAD: N420704
MASSACHUSETTS
ESSEX COUNTY

LIST OF ADJUSTED DISTANCES (JANUARY 26,1980)

FROM STATION	ELEV. (M)	TO STATION	ELEV. (M)	ADJ. DIST. (M) HORIZONTAL	ADJ. DIST. (M) MARK - MARK	STD. ERROR (MM)
0	23.65	150	23.37	150.0018	150.0021	0.2
0	23.65	430	24.39	430.0026	430.0032	0.4
0	23.65	1200	24.47	1200.0000	1200.0003	0.5
150	23.37	430	24.39	280.0007	280.0026	0.3
150	23.37	1200	24.47	1049.9981	1049.9987	0.4
430	24.39	1200	24.47	769.9974	769.9974	0.3

DESCRIPTION OF GEORGETOWN BASE LINE
YEAR MEASURED: 1979
CHIEF OF PARTY: CWW

THE BASE LINE IS LOCATED ABOUT 25 MILES NORTH OF BOSTON AND 1.5 MILES SOUTHEAST OF GEORGETOWN ALONG THE EASTERN EDGE OF THE NORTH BOUND LANES OF INTERSTATE 95.

TO REACH THE 1200 METER POINT FROM THE JUNCTION OF STATE ROUTE 133 AND INTERSTATE 95, GO NORTH ON INTERSTATE 95 FOR 0.9 MILE TO THE STATION ON THE RIGHT.

THE 1200 METER POINT IS ABOUT 0.2 MILE NORTH OF THE FIRST OVERPASS NORTH OF ROUTE 133, 80 FEET NORTHEAST OF A GUARD RAIL, 47 FEET EAST OF THE CENTER LINE OF THE NORTH BOUND LANES OF INTERSTATE 95 AND 5 FEET WEST OF A METAL WITNESS POST.

ALL OF THE MARKS ARE STANDARD CALIBRATION BASE LINE DISKS, STAMPED WITH THEIR BASE LINE DESIGNATION AND THE YEAR 1979, SET IN THE TOP OF ROUND CONCRETE MONUMENTS THAT ARE ABOUT 1 INCH BELOW THE SURFACE.

THE BASE LINE IS A NORTH-SOUTH LINE WITH THE 0 METER POINT ON THE NORTH END. IT IS MADE UP OF THE 0, 150, 430, AND 1200 METER POINTS WITH TWO MARKS FOR THE CALIBRATION OF 100 AND 200 FOOT TAPES SET ON LINE SOUTH OF THE 0 METER POINT. ALL OF THE MARKS ARE SET ON A LINE PARALLEL WITH THE EASTERN EDGE OF THE NORTH BOUND LANES OF INTERSTATE 95.

THIS BASE LINE WAS ESTABLISHED IN CONJUNCTION WITH THE MASSACHUSETTS ASSOCIATION OF LAND SURVEYORS AND CIVIL ENGINEERS AND THE MASSACHUSETTS GEODETIC SURVEY. FOR FURTHER INFORMATION CONTACT THE MASSACHUSETTS GEODETIC SURVEY, ~~ROOM 717, 100 NASHUA STREET, BOSTON, MASSACHUSETTS 02114. TELEPHONE 617-727-8387 OR MR. GERALD S. SHELBY, MOORE SURVEY AND MAPPING CORPORATION, 39 GRAFTON CIRCLE, SHREWSBURY, MASSACHUSETTS 01545. TELEPHONE 617-845-4181~~.

US DEPARTMENT OF COMMERCE - NOAA
NOS - NATIONAL GEODETIC SURVEY
ROCKVILLE MD 20852 - OCTOBER 29, 1986

CALIBRATION BASE LINE DATA
BASE LINE DESIGNATION: MARLBORO
PROJECT ACCESSION NUMBER: G16364

QUAD: N420713
MASSACHUSETTS
MIDDLESEX COUNTY

LIST OF ADJUSTED DISTANCES (JANUARY 26,1980)

FROM STATION	ELEV. (M)	TO STATION	ELEV. (M)	ADJ. DIST. (M) HORIZONTAL	ADJ. DIST. (M) MARK - MARK	STD. ERROR (MM)
0	76.20	150	75.37	149.9972	149.9995	0.2
0	76.20	430	74.11	430.0017	430.0068	0.4
0	76.20	1330	73.90	1329.9806	1329.9825	0.5
150	75.37	430	74.11	280.0046	280.0074	0.3
150	75.37	1330	73.90	1179.9833	1179.9842	0.4
430	74.11	1330	73.90	899.9787	899.9787	0.3

DESCRIPTION OF MARLBORO BASE LINE
YEAR MEASURED: 1979
CHIEF OF PARTY: CWW

THE BASE LINE IS LOCATED ABOUT 10 MILES NORTHEAST OF WORCESTER IN THE TOWN OF MARLBORO AND ALONG THE SOUTHERN EDGE OF THE MEDIAN STRIP OF INTERSTATE HIGHWAY 290.

TO REACH THE BASE LINE FROM THE JUNCTION OF INTERSTATE HIGHWAYS 290 AND 495 IN MARLBORO GO WEST ON INTERSTATE 290 FOR 2.1 MILE TO THE 0 METER POINT ON THE LEFT.

THE 0 METER POINT IS 39 FEET SOUTH OF A CONCRETE CATCH BASIN, 13 FEET NORTH-NORTHWEST OF THE NORTH EDGE OF THE EAST BOUND LANES OF INTERSTATE 290 AND 3 FEET SOUTH OF A METAL WITNESS POST.

ALL OF THE MARKS ARE STANDARD CALIBRATION BASE LINE DISKS SET IN THE TOP OF SQUARE CONCRETE MONUMENTS THAT ARE ABOUT 2 INCHES BELOW THE SURFACE. EACH METER POINT HAS TWO REFERENCE MARKS AND AN UNDERGROUND MARK. ALL OF THE MARKS ARE STAMPED WITH THEIR BASE LINE DESIGNATION AND THE YEAR 1979.

THE BASE LINE IS A EAST-WEST LINE WITH THE 0 METER POINT ON THE WEST END. IT IS MADE UP OF THE 0, 150, 430, AND 1330 METER POINTS WITH TWO MARKS FOR THE CALIBRATION OF 100 AND 200 FOOT TAPES SET ON LINE EAST OF THE 0 METER POINT.

THIS BASE LINE WAS ESTABLISHED IN CONJUNCTION WITH THE STATE OF MASSACHUSETTS. FOR FURTHER INFORMATION CONTACT THE MASSACHUSETTS GEODETIC SURVEY, ~~ROOM 717, 100 NASHUA STREET, BOSTON, MASSACHUSETTS 02114. TELEPHONE 617-727-8287~~.

US DEPARTMENT OF COMMERCE - NOAA
NOS - NATIONAL GEODETIC SURVEY
ROCKVILLE MD 20852 - OCTOBER 29, 1986

CALIBRATION BASE LINE DATA
BASE LINE DESIGNATION: PITTSFIELD
PROJECT ACCESSION NUMBER: G16364

QUAD: N420732
MASSACHUSETTS
BERKSHIRE COUNTY

LIST OF ADJUSTED DISTANCES (JANUARY 26,1980)

FROM STATION	ELEV. (M)	TO STATION	ELEV. (M)	ADJ. DIST. (M) HORIZONTAL	ADJ. DIST. (M) MARK - MARK	STD. ERROR (MM)
0	346.23	150	347.70	149.9981	150.0053	0.3
0	346.23	430	350.52	429.9837	430.0051	0.5
0	346.23	1400	360.22	1399.9646	1400.0345	0.7
150	347.70	430	350.52	279.9857	279.9999	0.4
150	347.70	1400	360.22	1249.9665	1250.0292	0.6
430	350.52	1400	360.22	969.9809	970.0294	0.4

DESCRIPTION OF PITTSFIELD BASE LINE
YEAR MEASURED: 1979
CHIEF OF PARTY: CWW

THE BASE LINE IS LOCATED ABOUT 2 MILES SOUTHWEST OF PITTSFIELD ALONG THE SOUTHERN EDGE OF THE EAST-WEST RUNWAY AT THE PITTSFIELD MUNICIPAL AIRPORT.

THE 0 METER POINT IS 112 FEET WEST OF THE SOUTHEAST CORNER OF THE EAST-WEST RUNWAY, 27 FEET NORTH OF A METAL WITNESS POST AND 26 FEET SOUTH OF THE SOUTHERN EDGE OF THE RUNWAY.

ALL OF THE MARKS ARE STANDARD CALIBRATION BASE LINE DISKS, STAMPED WITH THEIR BASE LINE DESIGNATION AND THE YEAR 1979, SET IN THE TOP OF ROUND CONCRETE MONUMENTS THAT ARE ABOUT 2-INCHES BELOW THE SURFACE.

THE BASE LINE IS A EAST-WEST LINE WITH THE 0 METER POINT ON THE EAST END. IT IS MADE UP OF THE 0, 150, 430, AND 1400 METER POINTS WITH TWO MARKS FOR THE CALIBRATION OF 100 AND 200 FOOT TAPES SET ON LINE WEST OF THE 430 METER POINT. ALL OF THE MARKS ARE SET ON A LINE PARALLEL WITH THE SOUTHERN EDGE OF THE RUNWAY.

THIS BASE LINE WAS ESTABLISHED IN CONJUNCTION WITH THE MASSACHUSETTS ASSOCIATION OF LAND SURVEYORS AND CIVIL ENGINEERS AND THE MASSACHUSETTS GEODETIC SURVEY. FOR FURTHER INFORMATION CONTACT THE MASSACHUSETTS GEODETIC SURVEY, ~~ROOM 717, 100 NASHUA STREET, BOSTON, MASSACHUSETTS 02114. TELEPHONE 617-727-8287 OR MR. GERALD S. SHELBY, MOORE SURVEY AND MAPPING CORPORATION, 29 GRAFTON CIRCLE, SHREWSBURY, MASSACHUSETTS 01545. TELEPHONE 617-845-4181.~~

US DEPARTMENT OF COMMERCE - NOAA
NOS - NATIONAL GEODETIC SURVEY
ROCKVILLE MD 20852 - OCTOBER 29, 1986

CALIBRATION BASE LINE DATA
BASE LINE DESIGNATION: TURNERS FALLS
PROJECT ACCESSION NUMBER: G16364

QUAD: N420724
MASSACHUSETTS
FRANKLIN COUNTY

LIST OF ADJUSTED DISTANCES (JANUARY 26,1980)

FROM STATION	ELEV. (M)	TO STATION	ELEV. (M)	ADJ. DIST. (M) HORIZONTAL	ADJ. DIST. (M) MARK - MARK	STD. ERROR (MM)
0	108.59	150	107.66	149.9981	150.0010	0.2
0	108.59	430	106.30	429.9969	430.0030	0.4
0	108.59	1030	105.15	1030.0005	1030.0063	0.5
150	107.66	430	106.30	279.9989	280.0022	0.3
150	107.66	1030	105.15	880.0024	880.0060	0.4
430	106.30	1030	105.15	600.0035	600.0046	0.3

DESCRIPTION OF TURNERS FALLS BASE LINE
YEAR MEASURED: 1979
CHIEF OF PARTY: CWW

THE BASE LINE IS LOCATED ABOUT 2.0 MILES SOUTHEAST OF TURNERS FALLS, 1.6 MILES NORTHEAST OF MILLERS FALLS AND 1.4 MILES SOUTHWEST OF THE FRENCH KING BRIDGE ALONG THE SOUTHERN EDGE OF THE MAIN RUNWAY AT THE TURNERS FALLS AIRPORT.

THE 0 METER POINT IS LOCATED NEAR THE SOUTHEAST END OF THE RUNWAY, 140 FEET NORTHWEST OF THE SOUTHWEST CORNER OF THE RUNWAY APRON, 76 FEET SOUTHWEST OF THE SOUTHERN EDGE OF THE RUNWAY, 44 FEET SOUTHWEST OF THE SOUTHWEST CORNER OF A RECTANGULAR LIGHT BOX AND 8 FEET EAST OF A METAL WITNESS POST.

ALL OF THE MARKS ARE STANDARD CALIBRATION BASE LINE DISKS, STAMPED WITH THEIR BASE LINE DESIGNATION AND THE YEAR 1979, SET IN THE TOP OF ROUND CONCRETE MONUMENTS THAT ARE ABOUT 2 INCHES BELOW THE SURFACE.

THE BASE LINE IS A NORTHWEST-SOUTHEAST LINE WITH THE 0 METER POINT ON THE SOUTHEAST END. IT IS MADE UP OF THE 0, 150, 430, AND 1030 METER POINTS WITH TWO MARKS FOR THE CALIBRATION OF 100 AND 200 FOOT TAPES SET ON LINE NORTHWEST OF THE 0 METER POINT.

THIS BASE LINE WAS ESTABLISHED IN CONJUNCTION WITH THE MASSACHUSETTS ASSOCIATION OF LAND SURVEYORS AND CIVIL ENGINEERS AND THE MASSACHUSETTS GEODETIC SURVEY. FOR FURTHER INFORMATION CONTACT THE MASSACHUSETTS GEODETIC SURVEY, ~~ROOM 717, 100 NASHUA STREET, BOSTON, MASSACHUSETTS 02114. TELEPHONE 617-727-8287 OR MR. GERALD S. SHELBY, MOORE SURVEY AND MAPPING CORPORATION, 29 GRAFTON CIRCLE, SHREWSBURY, MASSACHUSETTS 01545. TELEPHONE 617-845-4181~~.

APPENDIX A-9

* STATE PLANE COORDINATE SYSTEM OF 1983

NOAA Manual NOS NGS 5

State Plane Coordinate System of 1983

James E. Stem

Rockville, MD
January 1989

Reprinted with minor corrections
March 1990

U.S. DEPARTMENT OF COMMERCE
National Oceanic and Atmospheric Administration
National Ocean Service
Charting and Geodetic Services

NOAA Manual NOS NGS 5

State Plane Coordinate System of 1983

James E. Stem

National Geodetic Survey
Rockville, MD

January 1989

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March 1990

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U.S. DEPARTMENT OF COMMERCE
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William E. Evans, Under Secretary

National Ocean Service
Thomas J. Maginnis, Assistant Administrator

Charting and Geodetic Services
R. Adm. Wesley V. Hull

For sale by the National Geodetic Information Center, NOAA, Rockville, MD 20852

PREFACE

This manual explains how to perform computations on the State Plane Coordinate System of 1983 (SPCS 83). It supplements Coast and Geodetic Survey Special Publication No. 235, "The State coordinate systems," and replaces Coast and Geodetic Survey Publication 62-4, "State plane coordinates by automatic data processing." These two widely distributed publications provided the surveying and mapping profession with information on deriving 1927 State plane coordinates from geodetic coordinates based on the North American Datum of 1927 (NAD 27) plus information for traverse and other computations with these coordinates. This manual serves the similar purpose for users of SPCS 83 derived from the North American Datum of 1983 (NAD 83). Emphasis is placed on computations that have changed as a result of SPCS 83.

This publication is neither a textbook on the theory, development, or applications of general map projections nor a manual on the use of coordinates in survey computations. Instead it provides the practitioner with the necessary information to work with three conformal map projections: the Lambert conformal conic, the transverse Mercator, and the oblique Mercator. Derivatives of these three map projections produce the system which the National Geodetic Survey (NGS) has named the State Plane Coordinate System (SPCS). Referred to NAD 83 or NAD 27, this system of plane coordinates is identified as SPCS 83 or SPCS 27, respectively.

The equations in chapter 3, Conversion Methodology, form a significant portion of the manual. Chapter 3 is required reading for programmers writing software, but practitioners with software available may skip this chapter. Although a modification of terminology and notation was suggested by some reviewers, consistency with NGS software was deemed more important. Hence, chapter 3 documents the SPCS 83 software available from the National Geodetic Survey.

ACKNOWLEDGMENT

The mathematics given in this manual were compiled or developed by T. Vincenty prior to his retirement from the National Geodetic Survey (NGS). His consultation was invaluable to the author.

Principal reviewer was Joseph F. Dracup, NGS, retired. His many excellent suggestions were incorporated into the manual. Once again, Joe gave generously of his time to assist in the education of the surveying profession.

The author appreciates the review and contributions made by Earl F. Burkholder, Oregon Institute of Technology. Earl spent a summer at NGS researching the subject of map projections and maintains a continuing interest in the subject.

In addition, the manual was reviewed by Charles A. Whitten, B. K. Meade, and Charles N. Claire, all retired employees of the former Coast and Geodetic Survey (now NGS). The author was very fortunate to have such experts donate their services.

Finally, the author appreciates the helpful guidance of John G. Gergen and Edward J. McKay, present NGS employees.

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THE STATE PLANE COORDINATE SYSTEM OF 1983

James E. Stem
National Geodetic Survey
Charting and Geodetic Services
National Ocean Service, NOAA
Rockville, MD 20852

ABSTRACT. This manual provides information and equations necessary to perform survey computations on the State Plane Coordinate System of 1983 (SPCS 83), a map projection system based on the North American Datum of 1983 (NAD 83). Given the geodetic coordinates on NAD 83 (latitude and longitude), the manual provides the necessary equations to compute State plane coordinates (northing, easting) using the "forward" mapping equation ($\phi,\lambda \rightarrow N,E$). "Inverse" mapping equations are given to compute the geodetic position of a point defined by State plane coordinates ($N,E \rightarrow \phi,\lambda$). The manual addresses corrections to angles, azimuths, and distances that are required to relate these geodetic quantities between the ellipsoid and the grid. The following map projections are defined within SPCS 83: Lambert conformal conic, transverse Mercator, and oblique Mercator. A section on the Universal Transverse Mercator (UTM) projection is included. UTM is a derivative of the general transverse Mercator projection as well as another projection, in addition to SPCS 83, on which NAD 83 is published by NGS.

1. INTRODUCTION

1.1 Requirement for SPCS 83

The necessity for SPCS 83 arose from the establishment of NAD 83. When NAD 27 was readjusted and redefined by the National Geodetic Survey, a project which began in 1975 and finished in 1986, SPCS 27 became obsolete. NAD 83 produced new geodetic coordinates for all horizontal control points in the National Geodetic Reference System (NGRS). The project was undertaken because NAD 27 values could no longer provide the quality of horizontal control required by surveyors and engineers without regional recomputations (least squares adjustments) to repair the existing network. NAD 83 supplied the following improvements:

- o One hundred and fifty years of geodetic observations (approximately 1.8 million) were adjusted simultaneously, eliminating error propagation which occurs when projects must be mathematically assembled on a "piecemeal" basis.

- o The precise transcontinental traverse, satellite triangulation, Doppler positions, baselines established by electronic distance measurements (EDM), and baselines established by very long baseline interferometry (VLBI), improved the internal consistency of the network.

- A new figure of the Earth, the Geodetic Reference System of 1980 (GRS 80), which approximates the Earth's true size and shape, supplied a better fit than the Clarke 1866 spheroid, the reference surface used with NAD 27.

- The origin of the datum was moved from station MEADES RANCH in Kansas to the Earth's center of mass, for compatibility with satellite systems.

Not only will the published geodetic position of each control point change, but the State plane coordinates will change for the following reasons:

- The plane coordinates are mathematically derived (using "mapping equations") from geodetic coordinates.

- The new figure of the Earth, the GRS 80 ellipsoid, has different values for the semimajor axis "a" and flattening "f" (and eccentricity "e" and semiminor axis "b"). These ellipsoidal parameters are often embedded in the mapping equations and their change produces different plane coordinates.

- The mapping equations given in chapter 3 are accurate to the millimeter, whereas previous equations promulgated by NGS were derivatives of logarithmic calculations with generally accepted approximations.

- The defining constants of several zones have been redefined by the States.

- The numeric grid value of the origin of each zone has been significantly changed to make the coordinates appear clearly different.

- The State plane coordinates for all points published on NAD 83 by NGS will be in metric units.

- The SPCS 83 uses the Gauss-Kruger form of the transverse Mercator projection, whereas the SPCS 27 used the Gauss-Schreiber form of the equations.

1.2 SPCS 27 Background

The State Plane Coordinate System of 1927 was designed in the 1930s by the U.S. Coast and Geodetic Survey (predecessor of the National Ocean Service) to enable surveyors, mappers, and engineers to connect their land or engineering surveys to a common reference system, the North American Datum of 1927. The following criteria were applied in the design of the State Plane Coordinate System of 1927:

- Use of conformal mapping projections.

- Restricting the maximum scale distortion (sec. 2.6) to less than one part in 10,000.

- Covering an entire State with as few zones of a projection as possible.

- Defining boundaries of projection zones as an aggregation of counties.

It is impossible to map a curved Earth on a flat map using plane coordinates without distorting angles, azimuths, distances, or area. It is possible to design a map such that some of the four remain undistorted by selecting an appropriate "map projection." A map projection in which angles on the curved Earth are preserved after being projected to a plane is called a "conformal" projection. (See sec. 2.3.) Three conformal map projections were used in designing the original State plane coordinate systems, the Lambert conformal conic projection, the transverse Mercator projection, and the oblique Mercator projection. The Lambert projection was used for States that are long in the east-west direction (e.g., Kentucky, Tennessee, North Carolina), or for States that prefer to be divided into several zones of east-west extent. The transverse Mercator projection was used for States (or zones within States) that are long in the north-south direction (e.g., Vermont and Indiana), and the oblique Mercator was used in one zone of Alaska when neither of these two was appropriate. These same map projections are also often custom designed to provide a coordinate system for a local or regional project. For example, the equations of the oblique Mercator projection produced project coordinates for the Northeast Corridor Rail Improvement project when a narrow coordinate system from Washington, DC, to Boston, MA, was required.

Land survey distance measurements in the 1930s were typically made with a steel tape, or something less precise. Accuracy rarely exceeded one part in 10,000. Therefore, the designers of the SPCS 27 concluded that a maximum systematic distance scale distortion (see sec. 2.6, "Grid scale factor") attributed to the projection of 1:10,000 could be absorbed in the computations without adverse impact on the survey. If distances were more accurate than 1:10,000, or if the systematic scale distortion could not be tolerated, the effect of scale distortion could be eliminated by computing and applying an appropriate grid scale factor correction. Admittedly, the one in 10,000 limit was set at an arbitrary level, but it worked well for its intended purpose and was not restrictive on the quality of the survey when grid scale factor was computed and applied.

To keep the scale distortion at less than one part in 10,000 when designing the SPCS 27, some States required multiple projection "zones." Thus some States have only one State plane coordinate zone, some have two or three zones, and the State of Alaska has 10 zones that incorporate all three projections. With the exception of Alaska, the zone boundaries in each State followed county boundaries. There was usually sufficient overlap from one zone to another to accommodate projects or surveys that crossed zone boundaries and still limit the scale distortion to 1:10,000. In more recent years, survey accuracy usually exceeded 1:10,000. More surveyors became accustomed to correcting distance observations for projection scale distortion by applying the grid scale factor correction. When the correction is used, zone boundaries become less important, as projects may extend farther into adjacent zones.

1.3 SPCS 83 Design

In the mid-1970s NGS considered several alternatives to SPCS 83. Some geodesists advocated retaining the design of the existing State plane coordinate system (projection type, boundaries, and defining constants) and others believed that a system based on a single projection type should be adopted. The single projection proponents contended that the present SPCS was cumbersome, since three projections involving 127 zones were employed.

A study was instituted to decide whether a single system would meet the principal requirements better than SPCS 27. These requirements included ease of understanding, computation, and implementation. Initially, it appeared that adoption of the Universal Transverse Mercator (UTM) system (sec. 2.7) would be the best solution because the grid had long been established, to some extent was being used, and the basic formulas were identical in all situations. However, on further examination, it was found that the UTM 6-degree zone widths presented several problems that might impede its overall acceptance by the surveying profession. For example, to accommodate the wider zone width, a grid scale factor of 1:2,500 exists on the central meridian while a grid scale factor of 1:1,250 exists at zone boundaries. As already discussed, similar grid scale factors on the SPCS rarely exceeded 1:10,000. In addition, the "arc-to-chord" correction term (sec. 2.5) that converts observed geodetic angles to grid angles is larger, requiring application more frequently. And finally, the UTM zone definitions did not coincide with State or county boundaries. These problems were not viewed as critical, but most surveyors and engineers considered the existing SPCS 27 the simpler system and the UTM as unacceptable because of rapidly changing grid scale factors.

The study then turned to the transverse Mercator projection with zones of 2° in width. This grid met the primary conditions of a single national system. By reducing zone width, the scale factor and the arc-to-chord correction would be no worse than in the SPCS 27. The major disadvantage of the 2° transverse Mercator grid was that the zones, being defined by meridians, rarely fell along State and county boundaries. A more detailed review showed that while many States would require two or more zones, the 2° grid could be defined to accommodate those who wanted the zones to follow county lines. Furthermore, seldom did this cause larger scale factor or arc-to-chord corrections than in the existing SPCS 27, although several of the larger counties would require two zones. However, the average number of zones per State was increased by this approach.

Throughout this study, three dominant factors for retaining the SPCS 27 design were evident: SPCS had been accepted by legislative action in 37 States. The grids had been in use for more than 40 years and most surveyors and engineers were familiar with the definition and procedures involved in using them. Except for academic and puristic considerations the philosophy of SPCS 27 was fundamentally sound. With availability of electronic calculators and computers, little merit was found in reducing the number of zones or projection types. There was merit in minimizing the number of changes to SPCS legislation. For these reasons a decision was made to retain the basic design philosophy of SPCS 27 in SPCS 83.

The above decision was expanded to enable NGS to also publish UTM coordinates for those users who preferred that system. Both grids are now fully supported by NGS for surveying and mapping purposes. It is recognized that requirements will arise when additional projections may be required, and there is no reason to limit use to only the SPCS 83 and UTM systems.

1.4 SPCS 83 Local Selection

The policy decision that NGS would publish NAD 83 coordinates in SPCS 83, a system designed similar to SPCS 27, was first announced in the Federal Register on March 24, 1977. From April 1978 through January 1979, NGS solicited comments on this published policy by canvassing member boards of the National Council of Engineering Examiners, all individual land surveyor members of each board, the secretary of each section and affiliate of the American Congress on Surveying and Mapping (ACSM), and State and local public agencies familiar to NGS. As of August 1988, the 1978-79 solicitations and responses to subsequently published articles had produced committees or liaison contacts in 43 States. Through these people NGS presented the options to be considered in delineation of SPCS 83 zones and options in the adoption of the defining mathematical constants for each zone.

Although most States left unchanged the list of counties that comprised a zone, three States (South Carolina, Montana, and Nebraska) elected to have a single zone cover the entire State, replacing what was several zones on the SPCS 27. In these States the grid scale factor correction to distances now exceeds 1:10,000, and the arc-to-chord correction to azimuths and angles may become significant. (See secs. 2.5 and 4.3.) A zone definition change also occurred in New Mexico due to creation of a new county, and in California where zone 7 of the SPCS 27 was incorporated into zone 5 of the SPCS 83. Figure 1.4 depicts the zone identification numbers and boundaries of the SPCS 83.

In 1982, NGS printed a map titled "Index of State Plane Coordinate (SPC) Zone Codes," which depicts the boundaries and identification numbers of the SPCS 27 zones. Figure 1.4 differs from that map in the following States and Possessions.

CALIFORNIA: CA 7 No. 0407 was eliminated, and its area, the County of Los Angeles, included in CA 5, No. 0405.

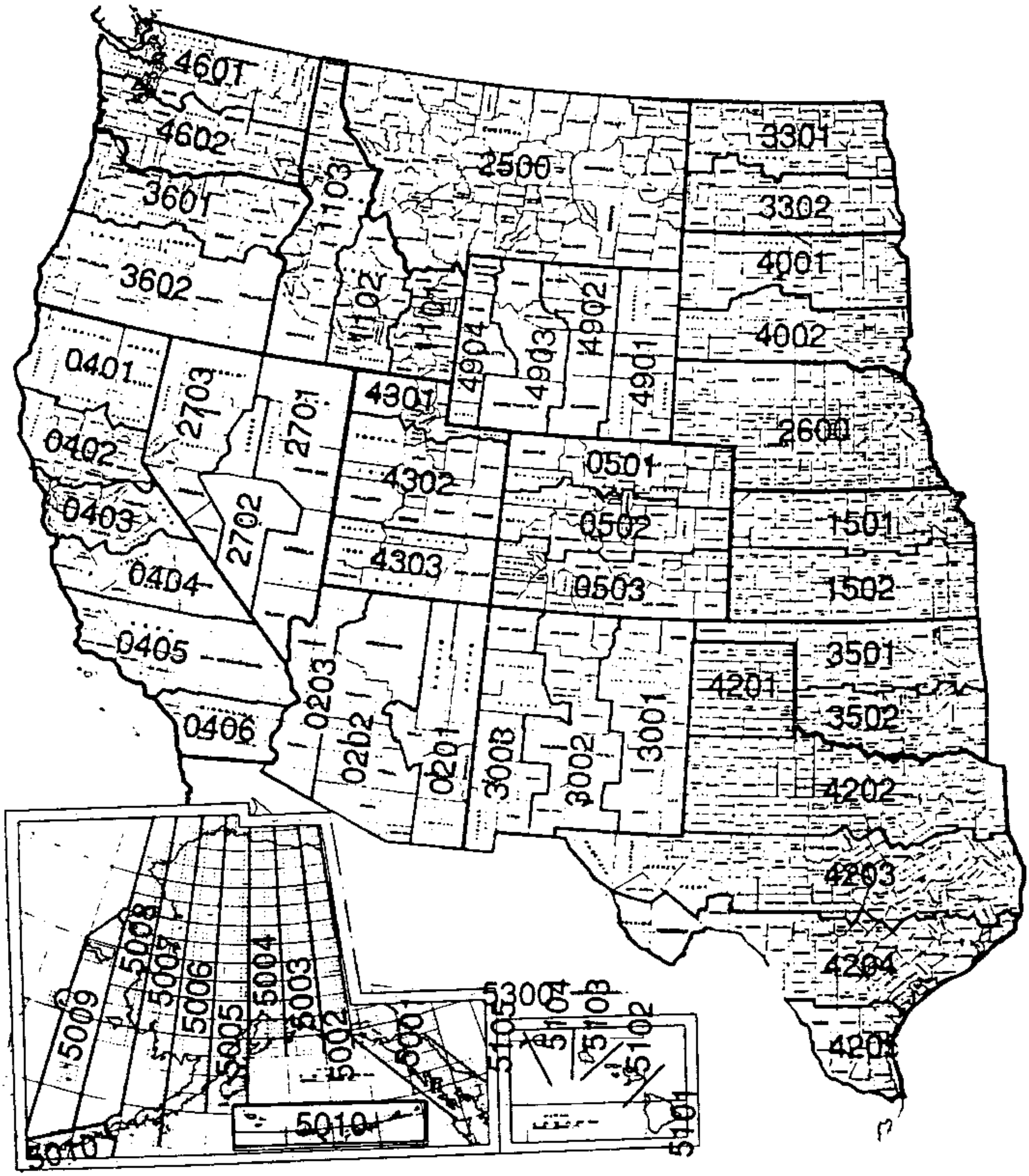
MICHIGAN: MI E No. 2101, MI C No. 2102, and MI W No. 2103, were eliminated in favor of the Lambert zones.

MONTANA: MT N No. 2501, MT C No. 2502, and MT S No. 2503 were eliminated in favor of a single State zone MT No. 2500.

NEBRASKA: NE N No. 2601 and NE S No. 2602 were eliminated in favor of a single State zone NE No. 2600.

SOUTH CAROLINA: SC N No. 3901 and SC S 3902 were eliminated in favor of a single State zone SC No. 3900.

PUERTO RICO AND VIRGIN ISLANDS: PR 5201, VI 5201, and VI SX 5202 were eliminated in favor of a single zone PR 5200.



4601
4602
3601
3602
1103
2500
3301
3302
4001
4002
1102
1101
4904
4903
4902
4901
0401
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3001
4202
4203
4204
5008
5007
5006
5004
5003
5009
5005
5002
5300
5104
5103
5105
5102
5101
5010

Figure 1.4.--State Plane Coordinate System of 1983 zones.

Several States chose to modify one or more of the defining constants of their zones. Appendix A contains the defining constants for all zones of the SPCS 83. Where the constant differs from the SPCS 27 definition, it is flagged with an asterisk. Some of these changes increase the magnitude of the grid scale factor and arc-to-chord correction terms. In addition to the flagged changes, all "grid origins" are different because, within the SPCS 83, origins that were redefined are defined in meters. This new grid origin was selected by liaison with the States based on the following criteria:

- o Keeping the number of digits in the coordinate to the minimum.
- o Creating a new range for easting and/or northing in meters on the 1983 datum that would not overlap the range of x and/or y in feet on the existing 1927 datum. If an overlap could not be avoided, the location of the band of overlap (i.e., where the range of x and/or y on the 1927 datum intersects with the range on the 1983 datum) could be positioned anywhere through selection of appropriate grid origin.
- o Selecting different grid origins (either in northing or easting) for each zone so the coordinate user could determine the zone from the magnitude of the coordinate. This usually requires the "false-easting" to be the smallest in the easternmost zone to avoid easting values close in magnitude for points near boundaries of adjacent transverse Mercator zones. It requires the "false-northing" of the northernmost zone to be the smallest for adjacent Lambert zones for the same reason.
- o Creating different orders of magnitude for northing and easting to reduce the possibility of transition errors.

The grid origin selection influenced only the appearance of the coordinate system, not its accuracy or usefulness.

1.5 SPCS 83 State Legislation

Before the NAD 83 project began, 37 States had passed a State Plane Coordinate System Act, the first in 1935. As of August 1988, 42 States had legislated a "1927 State Plane Coordinate System." The most recent additions during the NAD 83 project included Illinois, New Hampshire, North Dakota, South Carolina, and West Virginia. Of these five, only Illinois did not simultaneously include the definition of the SPCS 83 within its initial SPCS 27 legislation.

As of August 1988, 26 States had enacted 1983 State Plane Coordinate System legislation (table 1.5). For SPCS 83, as for SPCS 27, NGS prepared a model act to implement SPCS legislation by the States. The act was generally followed by the States except for minor changes, some of which are discussed below. The model SPCS 83 act may be found in appendix B. In addition to providing mathematical definitions of SPCS 83, enacted and proposed legislation contains other sections that warrant discussion.

In the old model a section stated that no coordinates "purporting to define the position of a point on a land boundary, shall be presented to be recorded in any public land records or deed records unless such point is within one-half mile" of

Table 1.5.--Status of SPCS 27 and SPCS 83 legislation
(as of August 1, 1988)

NO SPCS LEGISLATION (8 States)		EXISTING NAD 27 SPCS LEGISLATION (16 States)			ENACTED NAD 83 SPCS LEGISLATION (26 States)
No correspondence with NGS (4 States)	NAD 83 legislation drafted (4 States)	No correspondence with NGS (3 States)	Correspondence with recommendations (7 States)	NAD 83 legislation drafted (6 States)	
Hawaii Kansas Kentucky Oklahoma	Iowa Mississippi *Nebraska**(S) Wyoming**(S)	Arkansas Pennsylvania Tennessee	Alabama Delaware Florida Idaho Illinois North Dakota Wisconsin	Colorado(S) Massachusetts New Jersey New Mexico New York Washington	Alaska Arizona(I) *California(S) Connecticut(S) Georgia Indiana(S) Louisiana Maine Maryland(S) Michigan(I) Minnesota Missouri *Montana(I) Nevada New Hampshire North Carolina(S) Ohio Oregon(I) Rhode Island *South Carolina(I) South Dakota Texas(S) Utah(I) Vermont Virginia West Virginia

*NOTE: The only States that authorized changes in zone boundaries are: California, Montana, Nebraska, and South Carolina.

**NOTE: These States have not written legislation, but have corresponded definite new SPCS parameters to NGS.

UNITS: S = U.S. Survey feet and meters
I = International feet and meters
All others only meters

a first- or second-order control point. The new model changes only the "one-half mile" to "1 kilometer," and references the Federal Geodetic Control Committee (FGCC) as the source of the classifications of first- and second-order geodetic control points. The intent of this section has not been well understood.

To determine a boundary coordinate, the act explicitly states that at least a second-order monumented point must exist not more than 1 km away. It does not say that the second-order point must already exist. Adding that an "existing or newly established" control point needs to be within 1 km may clarify this confusion. The intent was that a property surveyor would either recover an existing point or use any survey methodology to establish a permanently monumented point of at least second-order, class II accuracy in an accessible but protected location within 1 km of the property to be surveyed. Then, using this point, coordinates of the "temporarily" monumented (essentially unmonumented) property corners would be determined. These corners, if determined from a second-order, class II point, are of third-order accuracy (1:10,000), following the usual practice of establishing the point to the next lower accuracy standard. Another approach would have been to legislate that property coordinates would be determined using FGCC third-order (1:10,000) positional standards but eliminate the monumentation standards. This approach may serve well with Global Positioning System methods, but it eliminates the nearby control point needed for retracement surveys by conventional means. The 1-km limit from monumented control is perhaps appropriate only for urban or suburban conditions. Of importance is not the distance, but the existence of monumented control. Land values may also affect the specifications for a State or county.

The following examples illustrate how some States have addressed the above requirement in their 1983 SPCS laws. South Carolina's law states that no point can be recorded unless "...such point is established in accordance with the Federal Geodetic Control Committee specification for second-order, class II ...." Virginia's law reads that no point can be recorded unless "...such point is within 2 km of a public or private monumented horizontal control station..." established in conformity with first- or second-order FGCC specifications. Minnesota is to be applauded for writing the most unambiguous section. The law reads that no point would be recorded unless "... coordinates have been established in conformity with the national prescribed standards for third-order, class II horizontal control surveys, and provided that these surveys have been tied to or originated off monumented first or second-order horizontal control stations, which are adjusted to and published in the national network of geodetic control and are within 3 km of the said boundary points or land corners." The statement continues by defining the national standards to be those of the FGCC.

Another debated portion of the model SPCS legislation involves the role of coordinates within legal land descriptions. The section that states, "It shall be considered a complete, legal, and satisfactory description of such location to give the position of the survey station or land boundary corner on the system of plane coordinates..." has been passed by many States. This section, in conjunction with the previously discussed section dealing with the accuracy and recording of such points, should be sufficient to permit, but not to require, the use of the SPCS 83 to describe real property or supplement parcel descriptions.

Often found in the 1927 SPCS laws, especially in Public Land Survey System (PLSS) States, and carried over to the SPCS 83 legislation, is a section that specifies coordinates are supplemental to any other means of land description, and in case of conflict the conventional description shall prevail over the description by coordinates. This legislates an unconditional priority to the order of evidence and could prevent the best surveyor from submitting sound boundary evidence based on coordinates. There are many who believe that the intent should be to evaluate each situation on its own merit and not to impose an invariable rule. Language such as, "In case of conflict between elements of a description, cast out doubtful data and adhere to the most certain" may suffice. Perhaps this is currently being accomplished by common law.

Several States have addressed the above situation with the statement, "...the conventional description shall prevail over the description by coordinates unless said coordinates are upheld by adjudication, at which time the coordinate description will prevail." This, at least, provides the opportunity for the competent land/property line surveyor to defend the use of coordinates in retracement surveys.

Pertaining to the use of coordinates on plats, States have inserted additional individual sections. Georgia defines, "Grid North" and requires the convergence term on "...maps of survey that are purported oriented to a Georgia Coordinate System Zone." Illinois states that plats of survey referencing the SPCS must indicate the zone and "...geodetic stations, azimuth, angles, and distances used for establishing the survey connection." Virginia added that "Nothing contained in this chapter shall be interpreted as preventing the use of the Virginia Coordinate System in any unrecorded deeds, maps, or computations."

The last section of most SPCS acts assigns responsibility for the act to a specific State agency. Our model law stated that sections of the law "could be modified by a State agency to meet local conditions." Many States specifically assign responsibility to a particular department. For example, the South Carolina Geodetic Survey "... shall maintain the South Carolina coordinate system, files, and such other maps and files as deemed necessary to make station information readily available...." Similarly, the Virginia Polytechnic Institute and State University is "...the authorized State agency to collect and distribute information," and it authorizes such modifications as are referred to elsewhere in the law.

NGS encourages the development of State level surveying and mapping offices. Responsibility for the States' geodetic networks would be one function of such an office. The need for SPCS 83 legislation provides an opportunity to designate this lead agency.

1.6 SPCS 83 Unit of Length

A Federal Register notice published jointly on July 1, 1959 (24 Fed. Reg. 5348) by the directors of the National Bureau of Standards (NBS) (now National Institute of Standards and Technology) and the U.S. Coast and Geodetic Survey refined the definition of the yard in metric terms. The notice also pointed out the very slight difference between the new definition of the yard (0.9144 m) and the 1893 definition (3600/3937 m), from which the U.S. survey foot is derived. The "international foot" of 0.3048 meter is shorter than the U.S. survey foot by 2 parts per million. The 1959 notice stated that the U.S. survey foot would continue to be used "until such time as it becomes desirable and expedient to readjust the basic geodetic survey networks in the United States, after which the ratio of a yard, equal to 0.9144 m, shall apply."

Because the profession desired to retain the U.S. survey foot, and because it is incorporated in legal definitions in many States as well as in practical usage, a tentative decision was made by NBS not to adopt the international foot of 0.3048 m for surveying and mapping activities in the United States. However, before reaching a final decision in this matter, it was deemed appropriate and necessary to solicit the comments of land surveyors; Federal, State, and local officials; and any others from among the public at large who are engaged in surveying and mapping or are interested in or affected by surveying and mapping operations.

NBS and NGS published their preliminary decision (Federal Register Doc. 88-16174) and as of this writing are awaiting comments. A final decision will be reached after careful consideration of all the comments received. The final decision will be published in the Federal Register and will be publicly announced in the communications media as deemed appropriate. Even if the final decision affirms the preliminary decision not to adopt the international definition of the foot in surveying and mapping services, it should be noted that the Office of Charting and Geodetic Services, National Ocean Service, NOAA, in a 1977 Federal Register notice (42 Fed. Reg. 15943), uses the meter exclusively and is providing the new SPCS 83 coordinates in meters.

In the 1927 SPCS legislation, the "foot" was the defining unit of measure, the conversion factor defined by the "U.S. survey foot" being implicit. In the States that have prepared 1983 SPCS legislation, the "U.S. survey foot" was explicitly stated as the unit of measure when using SPCS 27. If a foot unit has been selected for SPCS 83, it is explicitly written into the SPCS 83 legislation.

Most States define a metric SPCS 83. When the NAD 83-SPCS 83 publication policy was developed, and published in the Federal Register on March 24, 1977, the Department of Commerce had established a policy that the agency would use metric units exclusively. NGS concurs with the advantages of the metric system, and except for SPCS 27 applications, has always worked totally in metric units. Accordingly, a metric SPCS 83 was recommended to the States. Except for Arizona, States that have enacted legislation defined a metric system. In addition, 10 States defined which "foot" unit to use when converting from meters. (See table 1.5.)

When the metric grid origin of an SPCS 83 zone is other than a rounded number, it was derived from a rounded foot value using one of the definitions:

0.3048 m exactly = international foot

1200/3937 m = U.S. survey foot

1.7 The New GRS 80 Ellipsoid

The mathematics of map projection systems convert point and line data from the ellipsoid of a datum to a plane. Accordingly, the dimensions of the ellipsoid are an inherent part of the conversion process. As discussed above, NAD 83 adopted a new ellipsoid, the Geodetic Reference System of 1980 (GRS 80). Therefore, the dimensions of this ellipsoid must be incorporated within any map projection equations, SPCS 83 or otherwise, when the requirement is to project NAD 83 geodetic data into a plane system. Whereas the ellipsoid constants were imbedded within the map projection equations promulgated by NGS (and its predecessors) for the SPCS 27, and hence they were invisible to the user, the formulation given in chapter 3 requires entry of ellipsoid constants.

An ellipsoid is formed by rotating an ellipse about its minor axis. For geodetic purposes this regular mathematical surface is designed to approximate the irregular surface of the Earth or portion thereof. The SPCS 27 incorporated the defining parameters of an ellipsoid identified as the Clarke spheroid of 1866, the ellipsoid of NAD 27. The parameters defining the Clarke spheroid of 1866 were the semimajor axis "a" of 6,378,206.4 m and the semiminor axis "b" of 6,356,583.8 m.

The ellipsoid that forms the basis of NAD 83, and consequently the SPCS 83, is identified as the Geodetic Reference System of 1980 (GRS 80). GRS 80 provides an excellent global approximation of the Earth's surface. The Clarke spheroid of 1866, as used for NAD 27 approximated only the conterminous United States. Because the geoid separation at point MEADES RANCH was assumed equal to zero, a translation exists between ellipsoids. The ellipsoid change is the major contributor of the coordinate shift from NAD 27 to NAD 83.

The parameters of GRS 80 were adopted by the XVII General Assembly of the International Union of Geodesy and Geophysics meeting in 1979 at Canberra, Australia. Since only one of the four GRS 80 defining parameters (semimajor axis "a") is an element of the geometric ellipsoid, a second geometric constant ("b", "1/f," or "e^2") must be derived from the three GRS 80 parameters of physical geodesy. Accordingly, the geometric definition of the GRS 80 is:

$$a = 6{,}378{,}137.\ \text{m (exact by definition)}$$

$$1/f = 298.25722210088 \text{ (to 14 significant digits by computation)}$$

From these two numbers, any other desired constants of geometric geodesy may be derived. For example, to 14 significant digits:

$$b = 6{,}356{,}752.3141403$$

$$e^2 = 0.0066943800229034.$$

2. MAP PROJECTIONS

A "projection" is a function relating points on one surface to points on another surface so that for every point on the first surface there corresponds exactly one point on the second surface. A "map projection" is a function relating coordinates of points on a curved surface to coordinates of points on a surface of different curvature. The mathematical function defines a relationship of coordinates between the ellipsoid and a sphere, between that sphere and a plane, or directly between the ellipsoid and a plane.

In horizontal surveying operations, field observations are collected on the surface of the irregular nonmathematical Earth. For most applications, it is more convenient to represent the spatial relationship between surveyed points by a set of coordinates, the basis of which is a regular mathematical surface. Part of the process of reducing field survey observations consists of computing equivalent values for the survey observations from the measured value on the Earth's surface to their reduced value on the regular surface on which one wishes to compute coordinates. Because a selected regular surface can only approximate the physical surface on which the survey points are actually located, the degree of approximation, and hence selection of the regular surface, is usually a function of the ultimate accuracy requirements of the points. It would not show good judgment to perform difficult reductions of survey observations and place them on a complex mathematical surface if the final required accuracy of the points was such as could be achieved with more simple reducing procedures.

One mathematical surface traditionally used by surveyors is the local tangent plane with few, if any, reductions made to the field observations. Resulting plane coordinates from computations of this nature are sufficient for independent projects of a small extent. On the other end of the spectrum, survey observations are often reduced to an ellipsoid of revolution and its associated datum, with subsequent computations performed using geodetic coordinates and ellipsoidal geometry. To the majority of surveyors and engineers, the use of "map projections" provides a compromise solution to either of these two extremes.

2.1 Fundamentals

In the study of map projections the ultimate surface on which survey observations are reduced and on which coordinates are computed is a plane. Usually it is a plane that has been "developed" from another regular mathematical surface, as a cone in Lambert's conic projections or the cylinder in Mercator's projections. Survey observations are "projected" or reduced to a predefined cylinder or cone, as is the "graticule" of latitude and longitude. The regular mathematical surface of the cylinder or cone is then cut open, or "developed," and laid flat into a plane. The "grid" of northings and eastings is then overlaid. Map projection systems provide a compromise solution between a limited-in-extent and approximate local plane system, and performing ellipsoidal computations on the geodetic datum. Theoretically, field observations are first reduced to the ellipsoid and then to the map projection surface. But in practice this is often accomplished as one step. The conversion of angles, azimuths, distances, and coordinates between an ellipsoid (GRS 80 for NAD 83) and developable surfaces is one role of the science of map projections. Figure 2.1a illustrates the three basic projection surfaces.

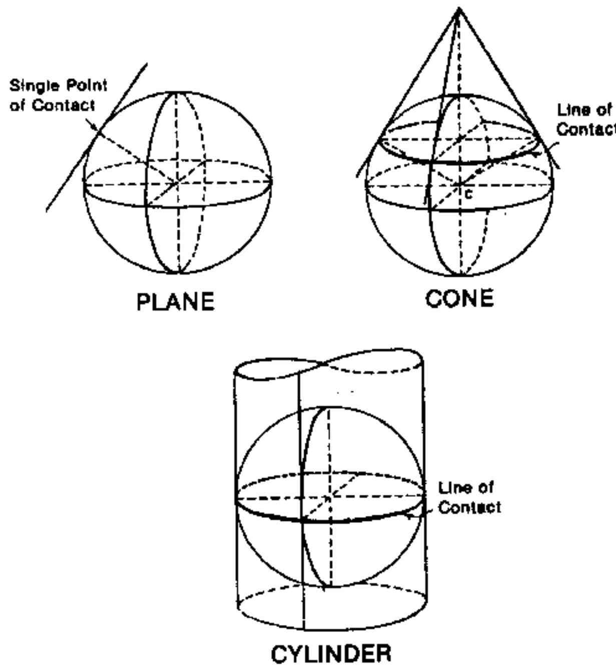


Figure 2.1a.--The three basic projection surfaces.

The plane, cone, or cylinder can be defined such that instead of being tangent to the datum surface, as illustrated in figure 2.1a, they intersect the datum surface as in figure 2.1b. This "secant" type of projection, a secant cone in Lambert's projections and secant cylinder in Mercator's projections, has been used for SPCS 27 and SPCS 83. In the Mercator projection, the secant cylinder has been rotated 90° so the axis of the cylinder is perpendicular to the axis of rotation of the datum surface, hence becoming a "transverse" Mercator projection. Occasionally the cylinder is rotated into a predefined azimuth, creating an "oblique" Mercator projection. Conceptually this is how one SPCS zone in Alaska was designed.

The secant cone intersects the surface of the ellipsoid along two parallels of latitude called "standard parallels" or "standard lines." Specifying these two parallels defines the cone. Specifying a "central meridian" orients the cone with respect to the ellipsoid. The transverse secant cylinder intersects the surface of the ellipsoid along two small ellipses equidistant from the meridian through the center of the zone. The secant cylinder is defined by specifying this central meridian, plus the desired grid scale factor on the central meridian. The ellipses of intersection are standard lines. Their location is a function of the selected central meridian grid scale factor. The specification of the latitude-longitude of the grid origin and the linear grid values assigned to that origin are all that remain to uniquely define a zone of either the Lambert or transverse Mercator projection. The above minimum specified values are the "defining constants" for a single zone of a projection.

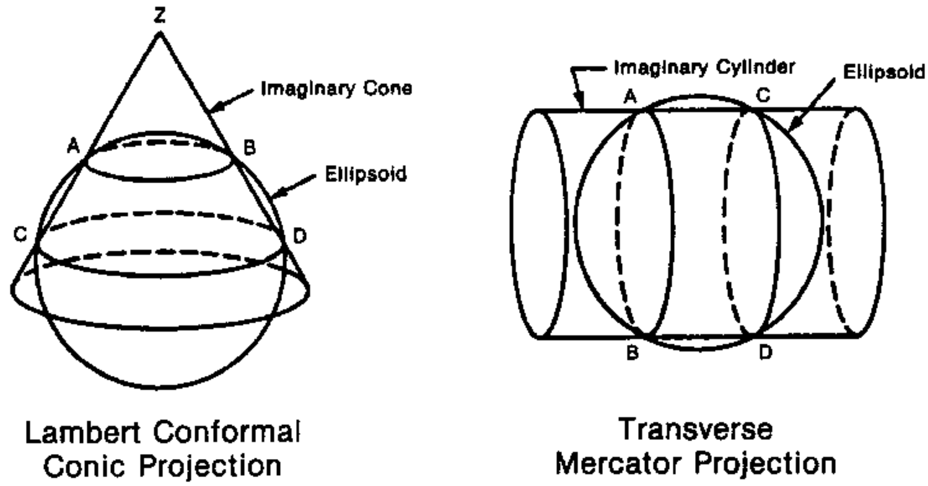


Figure 2.1b.--Surfaces used in State Plane Coordinate Systems.

Figure 2.1b illustrates the Lambert projection provides the closest approximation to the datum surface for a rectangular zone greatest in east-west extent, whereas the transverse Mercator projection provides the closest fit for an area north-south in extent. The narrower the strip of Earth's surface which it is desired to portray on a plane, the smaller will be the scale distortion designed in the projection. At a maximum width of 254 km, a maximum grid scale factor of 1:10,000 will exist at the zone boundaries. This maximum grid scale factor was designed into the SPCS 27 system, but in the SPCS 83 this maximum has been exceeded in the States of Montana, Nebraska, and South Carolina.

2.2 The SPCS 83 Grid

As with SPCS 27 or any plane-rectangular coordinate system, the SPCS 83 is represented on a map by two sets of uniformally spaced straight lines intersecting at right angles. The network thus formed is termed a "grid." One set of these lines is parallel to the plane of a meridian passing through the center of the grid, and the grid line corresponding to that meridian is the "northing axis" of the grid. It is also termed the "central meridian" of the grid. Forming right angles with the northing axis and to the south of the area covered by the grid is the easting axis. The point of intersection of these axes is the "grid origin" of the plane coordinate system. The grid origin differs from the "projection origin" by a constant. Knowledge of the origin is not required to use SPCS 83. The latitude and longitude of the grid origin are required defining-constants of a zone. The position of any point represented on the grid can be defined by stating two distances, termed "coordinates." One of these distances, known as the "northing coordinate," gives the position in a northern direction from the easting axis. The other distance, known as the "easting coordinate," gives the position in an east or west direction relative to the northing axis.

The northing coordinates increase numerically from south to north, the easting coordinates increase from west to east. Within the area covered by the grid, all northing coordinates are assured to be positive by placement of the grid origin south of the intended grid coverage. Easting coordinates are made positive by assigning the grid origin of the easting coordinates a large constant. For any point, the easting equals this value adopted for the grid origin, often identified as the "false easting," plus or minus the distance (E') of the point east or west from the central meridian (northing axis). Some zones have also assigned a "false northing" value at the grid origin. Accordingly, the northing equals this adopted value plus the distance of the point north of the easting axis.

The linear distance between two points on the SPCS 83, as obtained by computation or scaled from the grid, is termed the grid length of the line connecting those points. The angle between a line on the grid and the northing axis, reckoned clockwise from north through 360°, is the grid azimuth of the line. The computations involved in obtaining a grid length and a grid azimuth from grid coordinates are by means of the formulas of plane trigonometry.

2.3 Conformality

The commonly used examples of a developed cone for the Lambert grid and a developed cylinder for the transverse Mercator grid serve as excellent illustrations of the principles of map projections. Although some projections are truly "perspective," for SPCS 83 the mathematical equations of the map projection define the orderly system whereby the meridians and parallels of the ellipsoid are represented on the grid. Through the equations, controlled and computable distortion is placed into the map, the unavoidable result of representing a spherical surface on a flat plane. If correct relative depiction of an area is important, then "equal-area" mapping equations are selected. If correct depiction of select distances or azimuths is important, then other sets of mapping equations are selected.

In surveying and engineering, correct depiction of shapes is important. This is accomplished by mathematically constraining the grid scale factor (sec. 2.6) at a point, whatever it may be, such that it is the same in all directions from that point. This characteristic of a projection preserves angles between infinitesimal lines. That is, all lines on the grid cut each other at the same angles as do the corresponding lines on the ellipsoid for very short lines. Hence, for a small area, there is no local distortion of shape. But since the scale must change from point to point, distortion of shape can exist over large areas. Furthermore, for long lines the angle on the ellipsoid may not exactly equal the angle on the grid. This angular relationship is the property of conformality that has been mathematically imposed on SPCS 27, SPCS 83, the universal transverse Mercator projection, and most projections used in surveying and engineering. Although angles converted from the datum surface to the grid are preserved unchanged only for angles between lines of infinitesimal length, the angular difference of a single direction between the infinitesimal length and a finite length is a computable quantity identified as the "arc-to-chord" or (t-T) correction. (See secs. 2.5 and 4.3.)

By numerical example, the reader may verify that the distortion injected into the map projection is an exactly defined and computable quantity. All too often the concept of distortion in a map projection system is interpreted as an error of the system. However, map projection systems provide for a rigorous mathematical conversion of quantities between surfaces, and, as such, any inexactness that enters the conversion is caused by computational approximations.

2.4 Convergence Angle

The construction of all SPCS grids is such that geodetic north and grid north do not coincide at any point in a zone except along the central meridian. This condition is caused by the fact that the meridians converge toward the poles while the north-south grid lines are parallel to the central meridian. since geodetic azimuths are referred to meridians and grid azimuths are referred to north-south grid lines, it is evident that geodetic azimuths and grid azimuths must differ by a certain amount that depends on the position of the point of origin of the azimuth in relation to the central meridian of the SPCS zone. The "convergence angle," often also identified as the "mapping angle," is this angular difference between grid north and geodetic north. Defined another way, the convergence angle is the difference between a geodetic azimuth and the projection of that azimuth on the grid. Convergence is not the difference between geodetic and grid azimuths. (See sec. 2.5.) The "projected geodetic" azimuth is not the grid azimuth. Geodetic azimuths are symbolized as "α" and the convergence angle is symbolized as "γ". Note the change from SPCS 27, where the symbol "θ" was used for convergence within Lambert projections and "$\Delta\alpha$" for convergence within transverse Mercator systems, to SPCS 83 where "γ" represents convergence regardless of the projection type.

2.5 Grid Azimuth "t" and Projected Geodetic Azimuth "T"

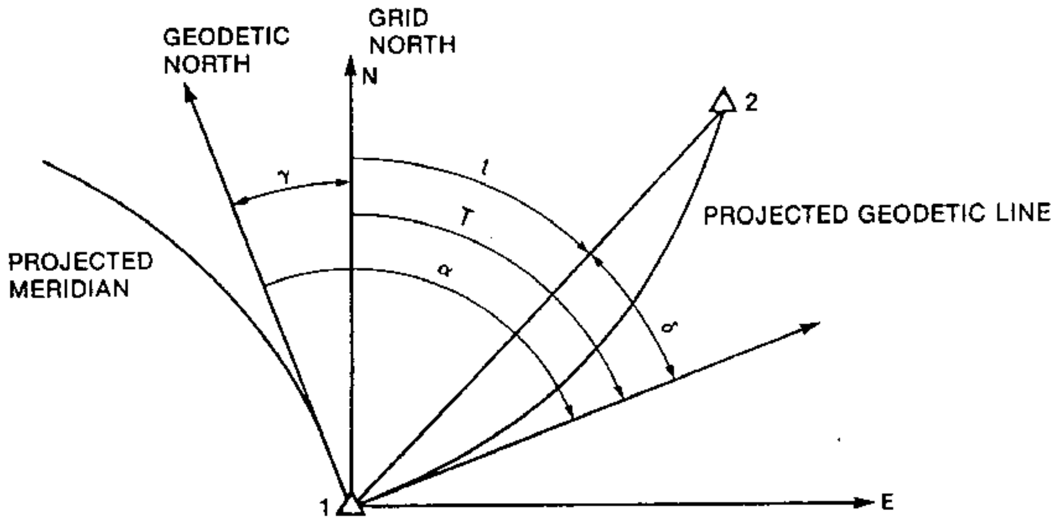
The projection of the geodetic azimuth "α" onto the grid is not the grid azimuth, but the "projected geodetic azimuth" symbolized as "T". Convergence "γ" is defined as the difference between geodetic and projected geodetic azimuths. Hence by definition, $\alpha = T+\gamma$, and the sign of "γ" should be applied accordingly. The angle obtained from two projected geodetic azimuths is a true representation of an observed angle.

When an azimuth is computed from two plane coordinate pairs, the resulting quantity is the grid azimuth symbolized as "t", or sometimes "α_g". The relationship between projected geodetic azimuth "T" and grid azimuth "t" is subtle and may be more clearly understood in figure 2.5. The difference between these azimuths is a computable quantity symbolized as "δ", or more often as (t-T). For the purpose of sign convention it is defined as $\delta = t-T$. For reasons apparent in figure 2.5, this term is also identified as the "arc-to-chord" correction. Given the above definition of α and δ, we obtain $t = \alpha-\gamma+\delta$.

Sometimes the convergence/mapping angle is incorrectly defined as the difference between the geodetic azimuth and the grid azimuth. This incorrect definition assumes the magnitude of (t-T) to be insignificant. While for many applications that assumption may be correct, (t-T) is often considered a "second-term" correction to the convergence term. Whether identified as δ, (t-T), arc-to-chord or second-term, the correction should be understood and always considered.

2.6 Grid Scale Factor at a Point

The grid scale factor is the measure of the linear distortion that has been mathematically imposed on ellipsoid distances so they may be projected onto a plane. At a given point, the ratio of the length of a linear increment on the grid to the length of the corresponding increment on the ellipsoid is identified



$$t = \alpha - \gamma + \delta$$

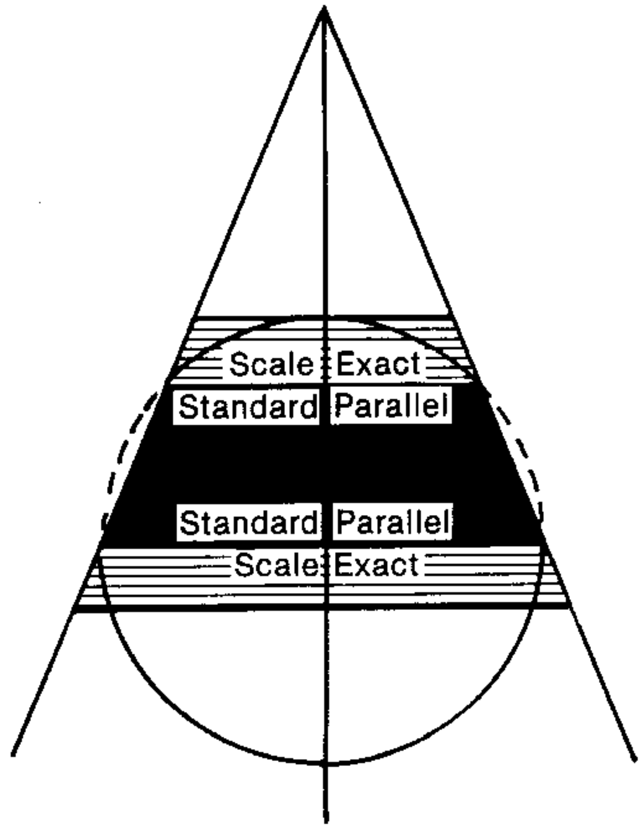
α = GEODETIC AZIMUTH RECKONED FROM NORTH
T = PROJECTED GEODETIC AZIMUTH
t = GRID AZIMUTH RECKONED FROM NORTH
γ = MAPPING ANGLE = CONVERGENCE ANGLE
δ = t-T = SECOND-TERM CORRECTION = ARC-TO-CHORD CORRECTION

Figure 2.5.--Azimuths.

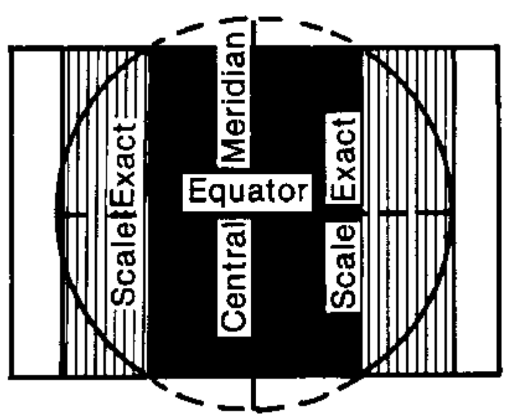
as the grid scale factor at that point, and symbolized by the letter "k". The grid scale factor is constant at a point, regardless of the azimuth, when conformal map projections are used as in the SPCS 83 and UTM systems. (See sec. 2.3, Conformality.) The grid scale factor is variable from point to point; mathematics refers to it as applying only to infinitesimal distances at a point.

The grid scale factor is equal to 1.0 along the "standard lines" (sec. 2.1) of the projection. Since the SPCS and UTM grids are secant type projections grid scale factors are less than 1.0 for the portion of the grid within the standard lines and greater than 1.0 for the remainder of the grid. In Lambert zones, the grid scale factor is less than 1.0 between the two standard parallels that define the zone. In transverse Mercator zones the scale factor is less than 1.0 between two north-south lines--the projection of the "ellipse of intersection" (sec. 2.1), their distance from the central meridian being a function of the scale factor assigned to that central meridian as part of the zone definition. Figure 2.6, although exaggerated, illustrates this concept.

Sometimes grid scale factor is defined as "scale distortion." On a map, map scale is correct only along the standard lines of the projection on which the map was cast. Everywhere else on the map, scale distortion exists and is defined as the ratio of the map scale at a given point to the map scale along a standard line. The scale distortion is identical to the grid scale factor. In small scale mapping, scale distortion is often expressed as "scale error" in percent, where scale error (%) = (scale distortion minus 1.0)*100.



Lambert Projection — Cone Secant to Sphere
Defined by Two Standard Parallels and the Origin



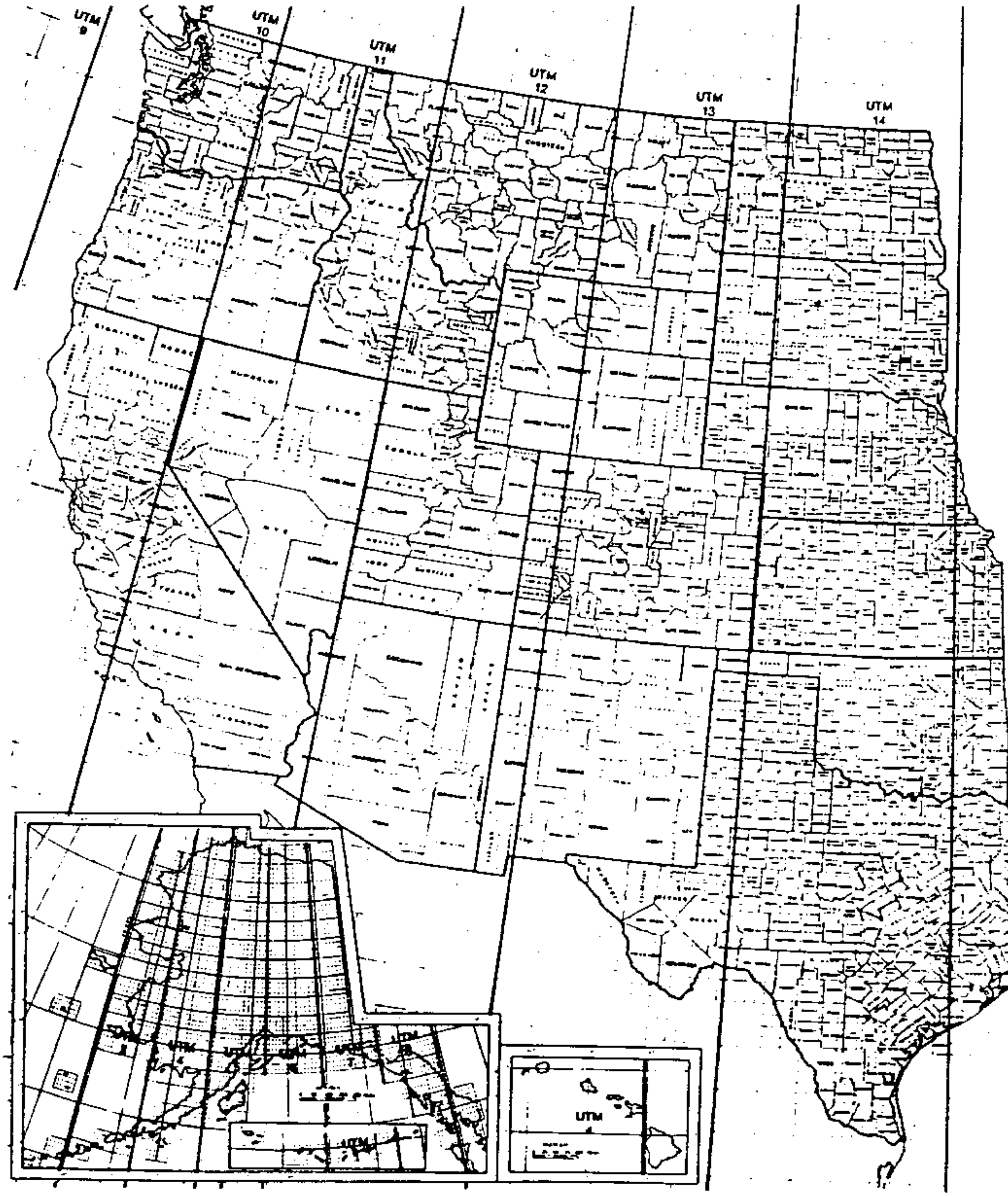
Transverse Mercator Projection —
Cylinder Secant to Sphere
Defined by Central Meridian and Its Scale
Factor, and the Origin

Figure 2.6.--Scale factor.

2.7 Universal Transverse Mercator Projection

The zones of the UTM projection system differ from the zones of other transverse Mercator projections by only the zone-defining constants. The basic mapping equations given in this manual for the transverse Mercator zones of the SPCS 83 may be used to obtain NAD 83 UTM coordinates upon substitution of UTM zone constants. The UTM zone constants have not changed, but when the constants are referenced to the GRS 80 ellipsoid of NAD 83, then 1983 UTM coordinates will be obtained. The UTM specifications, i.e., defining constants, on NAD 27 appear in many manuals of the Department of Army, originator of the system (e.g., Department of the Army 1958). To update the Department of Army specifications for NAD 83, only the ellipsoid ("spheroid" in the Army specifications) requires changing. The 1983 UTM specifications for the northern hemisphere are as follows.

Projection: Transverse Mercator (Gauss-Kruger type) in 6° wide zones
Ellipsoid: GRS 80 in North America
Longitude of origin: Central meridian of each zone
Latitude of origin: 0° (the equator)
Unit: Meter
False northing: 0
False easting: 500,000
Scale factor at central meridian: 0.9996 (exactly)
Zone numbering: Starting with No. 1 on the zone from 180° west to 174° west, and increasing eastward to No. 60 on the zone from 174° east to 180° east. (See fig. 2.7.)
Latitude limits of system: 0° to 80° north
Limits of zones: The zones are bounded by meridians whose longitudes are multiples of 6° west or east of Greenwich.


UTM 9
UTM 10
UTM 11
UTM 12
UTM 13
UTM 14
UTM 4

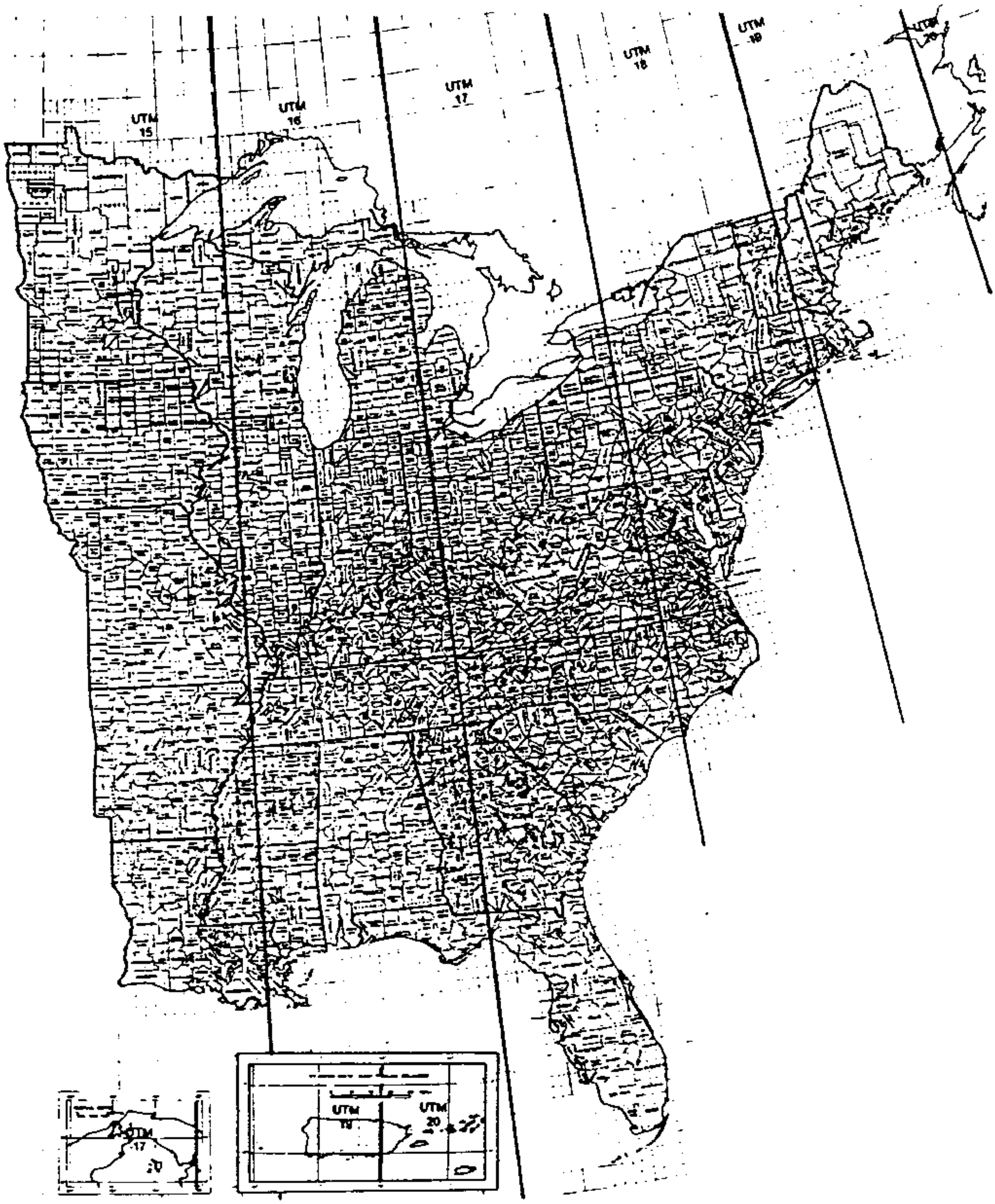


Figure 2.7.--Universal Transverse Mercator zones.

3. CONVERSION METHODOLOGY

This chapter addresses both "manual" and "automated" methods for performing "conversions" on any Lambert conformal conic, transverse Mercator, or oblique Mercator projections. Included is conversion from NAD 83 latitude/longitude to SPCS 83 northing/easting, plus the reverse process. For these processes this manual uses the term "conversion," leaving the term "transformation" for the process of computing coordinate values between datums, for example, transforming from NAD 27 to NAD 83 or transforming from SPCS 27 to SPCS 83. In addition to converting point coordinates, methods for conversion of distances, azimuths, and angles are also given.

The "automated" methods for conversions given in sections 3.1 through 3.3 are equations that have been sequenced and structured to facilitate programming. "Manual" methods are generally a combination of simple equations, tables, and intermediate numerical input, requiring only a calculator capable of basic arithmetic operations. Section 3.4 provides such a manual method for the Lambert projection where the intermediate numerical input is polynomial coefficients. Table 3.0 summarizes the conversion computational methods that were used for SPCS 27 and the methods discussed in this manual for SPCS 83.

Table 3.0.--Summary of conversion methods

Datum	Mode	Projection	Method
SPCS 27	Manual	Lambert and Transverse Mercator	Projection tables
		Oblique Mercator	Intersection tables
	Automated	Lambert, transverse Mercator, and oblique Mercator	Equations and constants described in C&GS <u>Publication</u> 62-4 (Claire 1968)
SPCS 83	Manual	Lambert	Polynomial coefficients (sec. 3.4)
		Transverse Mercator	New projection tables (future)
		Oblique Mercator	Automated only
	Automated	Lambert	Polynomial coefficients or new mapping equations (sec. 3.1)
		Transverse Mercator	New mapping equations (sec. 3.2)
		Oblique Mercator	New mapping equations (sec. 3.3)

The mapping equations given in sections 3.1 through 3.3 are not really "new" and may differ little from equations found in geodetic literature. However, they are new in the sense that they are not in the same form as the equations published or programmed by NGS or its predecessors in connection with SPCS 27. Whereas the SPCS 27 equations given in C&GS Publication 62-4 were designed to reproduce exactly the numerical results of an earlier manual method using logarithmic computations and projection tables, the equations here were designed for accuracy and computational efficiency. The Gauss-Kruger form of the transverse Mercator equations was used in SPCS 83 and the Gauss-Schreiber form in SPCS 27 equations.

Because the mapping equations of the automated approach apply equally to main-frame computers and programmable hand-held calculators, the availability of sufficient significant digits warrants consideration. The equations of transverse and oblique projections as given here will produce millimeter accuracy on any machine handling 10 significant digits. For the Lambert projection, the method of polynomial coefficients (sec 3.4) was developed for machines with only 10 significant digits. With less than 12 digits, the general mapping equations could not guarantee millimeter accuracy in all Lambert zones, particularly in Florida, Louisiana, Texas, South Carolina, Nebraska, and Montana. However, the polynomial coefficient method may also prove to be the most efficient for any machine. The general mapping equations will produce submillimeter accuracy when adequate significant digits are available for the computation.

Since the equations are not difficult, the polynomial coefficient method also fills the requirement for a manual method for the Lambert projection. A manual method for the SPCS 83 transverse Mercator projections has not been fully developed by NGS pending the demonstrated requirement for such a method.

While it is easy to visualize map projections by considering them a perspective projection of the meridians and parallels of the datum surface onto a surface that develops into a plane, in this age of coordinate plotters a graticule is generally not constructed by these means. Although a set of mechanical procedures can sometimes be defined by which meridians and parallels can be geometrically constructed on the grid using a ruler, compass, and scale, a pair of functions, $N = f_1(\phi,\lambda)$ and $E = f_2(\phi,\lambda)$, always exist. That is, for a point of given latitude (ϕ) and longitude (λ), there exist equations to yield the northing coordinate and equations to yield the easting coordinate when ϕ and λ are substituted into the equations. Likewise, equations must exist to compute the convergence angle, $\gamma = f_3(\phi,\lambda)$, and grid scale factor, $k = f_4(\phi,\lambda)$. These four functions, or equations, comprise the direct conversion process.

Furthermore, it must be possible to perform the inverse computation, requiring another pair of formulas, latitude $(\phi) = f_5(N,E)$ and longitude $(\lambda) = f_6(N,E)$. Similarly needed are convergence and grid scale factor as a function of the plane coordinates, $\gamma = f_7(N,E)$ and $k = f_8(N,E)$. Because these are one-to-one mappings, the inverse computation must reproduce the original values.

This chapter provides these eight "mapping equations" for each of these projections: Lambert conformal conic (sec. 3.1), transverse Mercator (sec. 3.2), and oblique Mercator (sec. 3.3). For each projection, the definition of the adopted symbols will be given first. Two sets of symbols are listed, the conventional set which incorporates the Greek alphabet and a set available on standard keyboards. The equations in this chapter will use the conventional notation. The entries in the notation section flagged with an asterisk are the constants required to uniquely define one specific zone of that general type of map projection. The values of those zone-defining constants as adopted and legislated by the States are listed in appendix A.

Included within the notation section are the symbols and definition of ellipsoid constants. Although several geometric ellipsoid constants are used within the mapping equations, only two geometric constants are required to define an ellipsoid. The SPCS 83 uses the GRS 80 ellipsoid. Those constants are discussed in section 1.7. All other geometric ellipsoid constants are then derived from the two defining constants, usually for the purpose of eliminating repeated computations.

A section on computation of zone constants follows each section on notation and definitions. Within this section are equations to compute intermediate quantities derived from the zone-defining constants of appendix A. These need only to be derived once. The derived "intermediate computing constants" of this section that need to be saved for future computations are flagged with an asterisk. The advantage of segmenting the general mapping equations is to eliminate repeated computations.

Subsequent sections under each projection type list the equations of the direct and inverse coordinate conversion process. The equations for the (t-T) line correction term (see also sec. 4.3) are provided in a final section. The solution of the ultimate mapping equations will require the values of the asterisked terms of the first two sections (defining constants plus intermediate constants).

3.1 Lambert Conformal Conic Mapping Equations

3.11 Notation and Definitions

For some terms an optional symbol appears in parentheses. This optional symbol available on all keyboards is used exclusively in section 3.4 and appendix C. Asterisked terms define the projection. Their values are listed in appendix A. These terms are the "zone defining constants" included within State SPCS legislation where enacted.

	ϕ (B)	Parallel of geodetic latitude, positive north
*	ϕ_s (B_s)	Southern standard parallel
*	ϕ_n (B_n)	Northern standard parallel
	ϕ_0 (B_0)	Central parallel, the latitude of the true projection origin
*	ϕ_b (B_b)	Latitude of the grid origin
	λ (L)	Meridian of geodetic longitude, positive west
*	λ_0 (L_0)	Central meridian, longitude of the true and grid origin
	k	Grid scale factor at a general point
	k_{12}	Grid scale factor of a line (between point 1 and point 2)
	k_0	Grid scale factor at the central parallel ϕ_0
	γ (C)	Convergence angle
	δ (t-T)	Arc-to-chord or second-term correction
	N	Northing coordinate (formerly y)
*	N_b	The northing value for ϕ_b at the central meridian (the grid origin). Sometimes identified as the false northing.
	N_0	Northing value at the intersection of the central meridian with the central parallel (the true projection origin)

	E	Easting coordinate (formerly x)
*	E_0	The easting value at the central meridian λ_0. Sometimes identified as the false easting
	R	Mapping radius at latitude ϕ
	R_b	Mapping radius at latitude ϕ_b
	R_0	Mapping radius at latitude ϕ_0
	K	Mapping radius at the equator
	Q	Isometric latitude
*	a	Semimajor axis of the geodetic ellipsoid
	b	Semiminor axis of the geodetic ellipsoid
*	f	Flattening of the geodetic ellipsoid = $(a-b)/a$
	e	First eccentricity of the ellipsoid = $(2f-f^2)^{1/2}$

3.12 Computation of Zone Constants

In this section the zone defining constants, ellipsoid constants, and parts of the Lambert mapping equations are combined to form several intermediate computing constants that are zone specific. These intermediate constants, flagged with an asterisk, will be required within the working equations of sections 3.13 through 3.15. All angles are in radian measure where 1 radian equals $180/\pi$ degrees. Linear units are identical to the units of the ellipsoid (a and b) and grid origin (N_b and E_0).

$$Q_s = \frac{1}{2}\left[\ell n\, \frac{1 + \sin\ \phi_s}{1 - \ \sin\ \phi_s} \quad - \quad e\ \ell n\, \frac{1 + e \sin\ \phi_s}{1 - e \sin\ \phi_s}\right]$$

$$W_s \ = \ (1 - e^2 \sin^2\phi_s)^{1/2}.$$

Similarly for Q_n, W_n, Q_b, Q_0, and W_0 upon substitution of the appropriate latitude

$$* \quad \sin\ \phi_0 = \frac{\ell n\ [W_n \cos\ \phi_s/(W_s \cos\ \phi_n)]}{Q_n - Q_s}$$

$$* \ K = \frac{a \cos\ \phi_s \exp(Q_s \sin\phi_0)}{W_s \sin\ \phi_0} = \frac{a \cos\ \phi_n \exp(Q_n \sin\ \phi_0)}{W_n \sin\ \phi_0}$$

NOTE: $\exp(x) = \varepsilon^x$

where ε = 2.718281828... (the base of natural logarithms)

$$* \qquad R_b = K/\exp(Q_b \sin\ \phi_0)$$

* $R_0 = K/\exp(Q_0 \sin\phi_0)$ (R_0 used in δ computation)

* $k_0 = (W_0 \tan\phi_0 R_0)/a$

* $N_0 = R_b + N_b - R_0$.

3.13 Direct Conversion Computation

This computation starts with the geodetic coordinates of a point (ϕ,λ) from which the Lambert grid coordinates (N,E) are to be computed, with convergence angle (γ), and grid scale factor (k).

$$Q = \frac{1}{2}\left[\ln\frac{1+\sin\phi}{1-\sin\phi} - e\ln\frac{1+e\sin\phi}{1-e\sin\phi}\right]$$

$$R = K/\exp(Q\sin\phi_0)$$

$$\gamma = (\lambda_0 - \lambda)\sin\phi_0$$

$$N = R_b + N_b - R\cos\gamma$$

$$E = E_0 + R\sin\gamma$$

$$k = (1 - e^2\sin^2\phi)^{1/2}(R\sin\phi_0)/(a\cos\phi).$$

3.14 Inverse Conversion Computation

In this computation the Lambert grid coordinates of a point (N,E) are given and the geodetic coordinates (ϕ,λ), convergence (γ), and grid scale factor (k) are to be computed.

$$R' = R_b - N + N_b$$

$$E' = E - E_0$$

$$\gamma = \tan^{-1}(E'/R')$$

$$\lambda = \lambda_0 - \gamma/\sin\phi_0$$

$$R = (R'^2 + E'^2)^{1/2}$$

$$Q = [\ln(K/R)]/\sin\phi_0.$$

Computation of latitude is iterative. Starting with the approximation

$$\sin\phi = \frac{\exp(2Q) - 1}{\exp(2Q) + 1}$$

solve for sin ϕ three times, as follows:

$$f_1 = \frac{1}{2}\left[\ell n\,\frac{1 + \sin\phi}{1 - \sin\phi} - e\,\ell n\,\frac{1 + e\sin\phi}{1 - e\sin\phi}\right] - Q$$

$$f_2 = \frac{1}{1 - \sin^2\phi} - \frac{e^2}{1 - e^2\sin^2\phi}\;.$$

Add a correction of $(-f_1/f_2)$ to sin ϕ and iterate two times before obtaining ϕ.

$$k = (1 - e^2\sin^2\phi)^{1/2}\,(R\sin\phi_0)/(a\cos\phi).$$

Latitude can also be obtained without iteration as shown in computations for the oblique Mercator projection. Four additional ellipsoid constants required for this alternative, F_0, F_2, F_4, F_6, are computed in section 3.32.

If only k is desired from the grid coordinates, an approximate ϕ will suffice and its computation shortened. After computing Q, compute

$$\sin\theta = \frac{\exp(2Q) - 1}{\exp(2Q) + 1}$$

$$\phi = \theta + (A\sin\theta\cos\theta)(1 + B\cos^2\theta)$$

in which $A = e^2(1 - e^2/6)$ and $B = 7e^2/6$. For the GRS 80 ellipsoid A = 0.0066869 and B = 0.0078.

The grid scale factor may be approximated by the equation

$$k = k_0 + (N-N_0)^2/2r_0^2 + (N-N_0)^3(\tan\phi_0)/6r_0^3.$$

The quantity r_0 is defined in section 3.15. Values of r_0, k_0 and N_0 for each zone are given in appendix C.

A further approximation is given by the equation:

$$k = k_0 + (N-N_0)^2(1.231 \times 10^{-14}) + (N-N_0)^3(\tan\phi_0)(6.94 \times 10^{-22}).$$

These approximations may not be sufficiently accurate in the States with a single Lambert zone.

To derive the grid scale factor at a point directly from the grid coordinates, the method given in section 3.4, the method of polynomial coefficients, also warrants consideration.

3.15 Arc-to-Chord Correction "δ" (alias "t-T") (see also sec. 4.3.)

The relationship among grid azimuth (t), geodetic azimuth (α), convergence angle (γ), and arc-to-chord correction (δ) at any given point is

$$t = \alpha - \gamma + \delta.$$

To compute δ requires knowledge of the coordinates of both ends of the line to which δ is to be applied. If geodetic coordinates of the endpoints are available (ϕ_1, λ_1 and ϕ_2, λ_2), the δ from point 1 to point 2 can be computed from

$$\delta_{12} = (\sin \phi_3 - \sin \phi_0)\ (\lambda_1 - \lambda_2)/2$$

where $\phi_3 = (2\phi_1 + \phi_2)/3$ and ϕ_0 is the computed constant for the zone. In normal practice, however, δ is desired as a function of the grid coordinates. To that end the following sequence of equations will produce the best possible determination of δ_{12}, given points N_1, E_1 and N_2, E_2:

$$p_1 = N_1 - N_0 \qquad p_2 = N_2 - N_0$$
$$q_1 = E_1 - E_0 \qquad q_2 = E_2 - E_0$$
$$R'_1 = R_0 - p_1 \qquad R'_2 = R_0 - p_2$$
$$\Delta N = N_2 - N_1$$
$$M_0 = k_0 a\ (1 - e^2)/(1 - e^2\sin^2\phi_0)^{3/2}$$

NOTE: M_0 is the scaled radius of curvature in the meridian at ϕ_0 scaled to the grid. The value of M_0 for each zone appears in appendix 3 as a "computed constant."

$$u_1 = p_1 - q_1{}^2/2R'_1$$

$$\phi_3 = \phi_0 + (u_1 + \Delta N/3)/M_0$$

$$\delta_{12} = (\sin \phi_3/\sin \phi_0 - 1)(q_2/R'_2 - q_1/R'_1)/2. \tag{1}$$

For most applications a less accurate determination of ϕ will suffice. For example, the original Coast and Geodetic Survey formula (Adams and Claire 1948: p. 13) should be adequate for all applications except the most precise surveys in the largest Lambert zones.

$$\delta_{12} = (p_1 + \Delta N/3)\Delta E/2r_0{}^2 \tag{2}$$

where

$$p_1 = N_1 - N_0$$

$$\Delta N = N_2 - N_1 \qquad \Delta E = E_2 - E_1$$

$$r_0 = k_0 a(1 - e^2)^{1/2}/(1 - e^2\sin^2\phi_0).$$

The quantity r_0 is the geometric mean radius of curvature at ϕ_0 scaled to the grid and is constant for any one zone. The value of r_0 for each zone has been included with the computed constants in appendix C. A single value of $1/(2\ r_0{}^2)$ is often used and combined with the constant to convert radians to seconds (1 radian = $648000/\pi$ seconds).

Hence, $\delta_{12} = 25.4\ (p_1 + \Delta N/3)(\Delta E)10^{-10}$ seconds, where the coordinates are in meters. Sometimes the notation (ΔN) replaces $(p_1 + \Delta N/3)$. Then the above equation is analogous to the expression often used in connection with NAD 27:

$$\delta_{12} = 2.36\ \Delta x\ \Delta y\ 10^{-10} \text{ seconds}$$

where the coordinates are in feet. This expression also serves for NAD 83 coordinates that have been converted to feet.

For the SPCS 27 Lambert systems NGS suggested two other appropriate methods that provided more accurate (t-T) corrections at the zone extremities. One was similar to equation 3.15 (1) and gave essentially the same results. Since the computing effort was somewhat greater than for 3.15 (1) it is not given here. The second, while not as accurate as 3.15(1), may be simpler for manual calculations because it uses the SPCS 83 zone constants and readily understood rotation and translation formulas.

$$\delta_{12} = (e_2 - e_1)(2n_1 + n_2)/6r_0^2 \quad \text{(in radian measure)}$$

where

$n = D + E' \sin\gamma + N' \cos\gamma$
$e = E' \cos\gamma - N' \sin\gamma$
"γ" is the average convergence angle for the survey area and is considered positive. γ to minutes is sufficient.
$D = 2R_0 \sin^2\gamma/2$
$N' = N - N_0$
$E' = E - E_0$

The size of δ varies linearly with the length of the ΔE ($\Delta\lambda$) component of the line and with the distance of the standpoint from the central parallel. It does not vary with distance of standpoint from the central meridian. Hence the size of δ depends on the direction of the line, varying from a zero value between points on the same meridian to maximum values over east-west lines.

Table 3.1 gives an overview of the true numeric value of the arc-to-chord correction (δ) and of the computational errors expected from equations (1) and (2). The examples were computed for a hypothetical zone with central parallel of approximately 42° (standard parallels 41° and 43°), on the GRS 80 ellipsoid. Two cases, 1° and 2°, are illustrated for the distance of the standpoint from the central parallel ϕ_0. Two cases, 5° and 10°, are given for the distance of the standpoint from the central meridian. Although the magnitude of δ is not a function of the distance of the standpoint from the central meridian, equation (2) becomes less accurate as this distance increases. Table 3.1 also gives three cases for the orientation of the line, in azimuths of 90°, 135°, and 180°. Again note that although the true δ equals zero in an azimuth of 180°, the equations only approximate zero.

The final assumption in table 3.1 is that the length of the line for which δ is being computed is 20 km. Dividing the line into several traverse legs results in a proportional decrease in the required correction to a direction. It does nothing to diminish the closure error in azimuth because errors due to omission of δ are cumulative.

From data given in table 3.1 the persons performing the computing must decide which reduction formula is appropriate for their needs, remembering that the accuracy of the formula should exceed the expected accuracy of the field work by one order of magnitude and that an error of 1" in direction corresponds to a linear error of about 1:200,000, or 5 ppm.

Table 3.1.--True values of (t-T) and computational errors in their determination (in seconds of arc)

$\phi_1 - \phi_0$	1°	2°	1°	2°
$\lambda_1 - \lambda_0$	5°	5°	10°	10°
Azimuth	90°	90°	90°	90°
True δ	5.67	11.44	5.67	11.44
Error by (1)	0.00	0.02	0.03	0.11
Error by (2)	0.53	0.38	2.28	2.07
Azimuth	135°	135°	135°	135°
True δ	3.83	7.91	3.83	7.91
Error by (1)	0.00	0.01	0.02	0.08
Error by (2)	0.14	-0.20	0.99	0.38
Azimuth	180°	180°	180°	180°
True δ	0.00	0.00	0.00	0.00
Error by (1)	0.00	0.00	0.00	0.00
Error by (2)	-0.34	-0.67	-0.90	-1.55

3.2 Transverse Mercator Mapping Equations

3.21 Notation and Definitions

Asterisked terms define the projection for which values are given in appendix A. These zone specific "defining constants" are included within State SPCS legislation where enacted.

	ϕ	Parallel of geodetic latitude, positive north
	λ	Meridian of geodetic longitude, positive west
	ω	Rectifying latitude
	N	Northing coordinate on the projection (formerly y)
	E	Easting coordinate on the projection (formerly x)
*	λ_0	Central meridian
*	E_0	False easting (value assigned to the central meridian)
	S	Meridional distance
*	ϕ_0	Latitude of grid origin
*	N_0	False northing (value assigned to the latitude of grid origin)
	S_0	Meridional distance from the equator to ϕ_0, multiplied by the central meridian scale factor
*	k_0	Grid scale factor assigned to the central meridian
	k	Grid scale factor at a point
	k_{12}	Grid scale factor for a line (between points 1 and 2)
	ΔN	$N_2 - N_1$
	ΔE	$E_2 - E_1$
	E'	$E - E_0$
	γ	Meridian convergence
	δ_{12}	Arc-to-chord correction (t-T) (from point 1 to point 2)
	a	Semimajor axis of the ellipsoid
	b	Semiminor axis of the ellipsoid
	f	Flattening of the ellipsoid = (a - b)/a
	e^2	First eccentricity squared = $(a^2 - b^2)/a^2 = 2f - f^2$

e'^2	Second eccentricity squared = $(a^2 - b^2)/b^2 = e^2/(1 - e^2)$
n	$(a - b)/(a + b) = f/(2 - f)$
R	Radius of curvature in the prime vertical = $a/(1 - e^2\sin^2\phi)^{1/2}$
r_0	Geometric mean radius of curvature scaled to the grid
r	Radius of the rectifying sphere
t	$\tan\phi$ (secs. 3.23 and 3.24)
t	grid azimuth (sec. 3.25 and others)
η^2	$e'^2\cos^2\phi$

3.22 Constants for Meridional Distance

In this section nine ellipsoid specific constants and one zone specific intermediate computing constant are derived. These intermediate constants, flagged with an asterisk, will be required within the working equations of sections 3.23 through 3.26.

The following ellipsoid specific constants may be directly entered into software. The equations are given for those with requirements for other ellipsoids.

$$* \; r = a(1 - n)(1 - n^2)(1 + 9n^2/4 + 225n^4/64) = 6367449.14577 \text{ m (GRS 80)}$$
$$u_2 = -3n/2 + 9n^3/16$$
$$u_4 = 15n^2/16 - 15n^4/32$$
$$u_6 = -35n^3/48$$
$$u_8 = 315n^4/512$$

$$* \; U_0 = 2(u_2 - 2u_4 + 3u_6 - 4u_8) = -0.00504\;82507\;76 \text{ (GRS 80)}$$
$$* \; U_2 = 8(u_4 - 4u_6 + 10u_8) = 0.00002\;12592\;04 \text{ (GRS 80)}$$
$$* \; U_4 = 32(u_6 - 6u_8) = -0.00000\;01114\;23 \text{ (GRS 80)}$$
$$* \; U_6 = 128\,u_8 = 0.00000\;00006\;26 \text{ (GRS 80)}$$
$$v_2 = 3n/2 - 27n^3/32$$
$$v_4 = 21n^2/16 - 55n^4/32$$
$$v_6 = 151n^3/96$$
$$v_8 = 1097n^4/512$$

$$* \; V_0 = 2(v_2 - 2v_4 + 3v_6 - 4v_8) = 0.00502\;28939\;48 \text{ (GRS 80)}$$
$$* \; V_2 = 8(v_4 - 4v_6 + 10\,v_8) = 0.00002\;93706\;25 \text{ (GRS 80)}$$
$$* \; V_4 = 32(v_6 - 6v_8) = 0.00000\;02350\;59 \text{ (GRS 80)}$$
$$* \; V_6 = 128v_8 = 0.00000\;00021\;81 \text{ (GRS 80)}$$

The following meridional constant is a zone specific constant computed once for a zone. Table 3.22 contains S_0 for each SPCS 83 transverse Mercator zone.

$$\omega_0 = \phi_0 + \sin\phi_0 \cos\phi_0(U_0 + U_2\cos^2\phi_0 + U_4\cos^4\phi_0 + U_6\cos^6\phi_0)$$
$$* \; S_0 = k_0\omega_0 r$$

3.23 Direct Conversion Computation

The following computation starts with the geodetic coordinates of a point (ϕ, λ) from which the transverse Mercator grid coordinates (N,E), convergence angle (γ), and the grid scale factor (k) are computed. All angles are in radian measure where one radian equals $180/\pi$ degrees. Linear units match the units of the ellipsoid and false origin.

Table 3.22.--Intermediate constants for transverse Mercator projections

State-zone-code	S_0 (m)	$1/(2\ r_0^2)$* (10^{14})	State-zone-code	S_0 (m)	$1/(2\ r_0^2)$* (10^{14})
AL-E-0101	3,375,406.7112	1.23256	IL-W-1202	4,056,280.6721	1.23068
AL-W-0102	3,319,892.0570	1.23262	IN-E-1301	4,151,863.7425	1.23062
AK-2-5002	5,985,317.4367	1.22473	IN-W-1302	4,151,863.7425	1.23062
AK-3-5003	5,985,317.4367	1.22473	ME-E-1801	4,836,302.3615	1.22906
AK-4-5004	5,985,317.4367	1.22473	ME-W-1802	4,744,046.5583	1.22918
AK-5-5005	5,985,317.4367	1.22473	MS-E-2301	3,264,526.0416	1.23258
AK-6-5006	5,985,317.4367	1.22473	MS-W-2302	3,264,526.0416	1.23258
AK-7-5007	5,985,317.4367	1.22473	MO-E-2401	3,966,785.2908	1.23126
AK-8-5008	5,985,317.4367	1.22473	MO-C-2402	3,966,785.2908	1.23126
AK-9-5009	5,985,317.4367	1.22473	MO-W-2403	4,003,800.5632	1.23124
AZ-E-0201	3,430,631.2260	1.23244	NV-E-2701	3,846,473.6437	1.23106
AZ-C-0202	3,430,631.2260	1.23244	NV-C-2702	3,846,473.6437	1.23106
AZ-W-0203	3,430,745.5918	1.23236	NV-W-2703	3,846,473.6437	1.23106
DE---0700	4,207,476.9816	1.23083	NH---2800	4,707,019.0442	1.22976
FL-E-0901	2,692,050.5001	1.23387	NJ---2900	4,299,571.6693	1.23078
FL-W-0902	2,692,050.5001	1.23387	NM-E-3001	3,430,662.4167	1.23242
GA-E-1001	3,319,781.3865	1.23271	NM-C-3002	3,430,631.2260	1.23244
GA-W-1002	3,319,781.3865	1.23271	NM-W-3003	3,430,688.4089	1.23240
HI-1-5101	2,083,150.1655	1.23570	NY-E-3101	4,299,571.6693	1.22992
HI-2-5102	2,249,193.4045	1.23532	NY-C-3102	4,429,252.1847	1.22983
HI-3-5103	2,341,506.4725	1.23527	NY-W-3103	4,429,252.1847	1.22983
HI-4-5104	2,415,321.4658	1.23507	RI---3800	4,549,799.4141	1.22998
HI-5-5105	2,396,891.1333	1.23505	VT---4400	4,707,007.8366	1.22948
ID-E-1101	4,614,370.6555	1.22980	WY-E-4901	4,484,768.4357	1.22983
ID-C-1102	4,614,370.6555	1.22952	WY-EC-4902	4,484,768.4357	1.22983
ID-W-1103	4,614,305.8890	1.22926	WY-WC-4903	4,484,768.4357	1.22983
IL-E-1201	4,059,417.9793	1.23060	WY-W-4904	4,484,768.4357	1.22983

$L = (\lambda - \lambda_0) \cos \phi$

Note: The sign convention used in SPCS 27 was $(\lambda_0 - \lambda)$.

$\omega = \phi + \sin \phi \cos \phi(U_0 + U_2\cos^2\phi + U_4\cos^4\phi + U_6\cos^6\phi)$.

Suggestion: Use nested form.

$\omega = \phi + (\sin \phi \cos \phi)[U_0 + \cos^2\phi\{U_2 + \cos^2\phi(U_4 + U_6\cos^2\phi)\}]$

$S = k_0\omega\ r$

$R = k_0a/(1 - e^2\sin^2\phi)^{1/2}$

$A_2 = \frac{1}{2} R\ t$

$$A_4 = \frac{1}{12}[5 - t^2 + \eta^2(9 + 4\eta^2)]$$

$$A_6 = \frac{1}{360}[61 - 58t^2 + t^4 + \eta^2(270 - 330t^2)]$$

$$N = S - S_0 + N_0 + A_2L^2[1 + L^2(A_4 + A_6L^2)]$$

$$A_1 = -R$$

$$A_3 = \frac{1}{6}(1 - t^2 + \eta^2)$$

$$A_5 = \frac{1}{120}[5 - 18t^2 + t^4 + \eta^2(14 - 58t^2)]$$

$$A_7 = \frac{1}{5040}(61 - 479t^2 + 179t^4 - t^6)$$

$$E = E_0 + A_1L[1 + L^2(A_3 + L^2(A_5 + A_7L^2))]$$

$$L = (\lambda - \lambda_0)\cos\phi$$

$$C_1 = -t$$

$$C_3 = \frac{1}{3}(1 + 3\eta^2 + 2\eta^4)$$

$$C_5 = \frac{1}{15}(2 - t^2)$$

$$\gamma = C_1L[1 + L^2(C_3 + C_5L^2)]$$

$$F_2 = \frac{1}{2}(1 + \eta^2)$$

$$F_4 = \frac{1}{12}[5 - 4t^2 + \eta^2(9 - 24t^2)]$$

$$k = k_0[1 + F_2L^2(1 + F_4L^2)].$$

The A_6, A_7, C_5, and F_4 terms are negligible when computing within the approximate boundaries of the SPCS 83 zones. To use the SPCS 83 beyond the defined SPCS 83 boundaries and to compute UTM coordinates, the significance of these terms should be evaluated.

3.24 Inverse Conversion Computation

This computation starts with the transverse Mercator grid coordinates of a point (N,E) from which the geodetic coordinates (ϕ,λ), convergence angle (γ), and grid scale factor (k) are computed.

$$\omega = (N - N_0 + S_0)/(k_0r)$$
$$\phi_f = \omega + (\sin\omega\cos\omega)(V_0 + V_2\cos^2\omega + V_4\cos^4\omega + V_6\cos^6\omega).$$

(This is sometimes referred to as the "footpoint latitude.")

Suggestion: Use nested form.

$\phi_f = \omega + (\sin\omega\cos\omega)[V_0 + \cos^2\omega\,\{V_2 + \cos^2\omega\,(V_4 + V_6\cos^2\omega)\}]$

$R_f = k_0 a/(1 - e^2\sin^2\phi_f)^{1/2}$

$Q = E'/R_f$

$B_2 = -\frac{1}{2}\,t_f(1 + \eta_f^2)$

$B_4 = -\frac{1}{12}\,[5 + 3t_f^2 + \eta_f^2(1 - 9t_f^2) - 4\eta_f^4]$

$B_6 = \frac{1}{360}\,[61 + 90t_f^2 + 45t_f^4 + \eta_f^2(46 - 252t_f^2 - 90t_f^4)]$

$\phi = \phi_f + B_2Q^2[1 + Q^2(B_4 + B_6Q^2)]$

$B_3 = -\frac{1}{6}\,(1 + 2t_f^2 + \eta_f^2)$

$B_5 = \frac{1}{120}\,[5 + 28t_f^2 + 24t_f^4 + \eta_f^2(6 + 8t_f^2)]$

$B_7 = -\frac{1}{5040}\,(61 + 662t_f^2 + 1320t_f^4 + 720t_f^6)$

$L = Q[1 + Q^2\{B_3 + Q^2(B_5 + B_7Q^2)\}]$

$\lambda = \lambda_0 - L/\cos\phi_f$

$D_1 = t_f$

$D_3 = -\frac{1}{3}\,(1 + t_f^2 - \eta_f^2 - 2\eta_f^4)$

$D_5 = \frac{1}{15}\,(2 + 5t_f^2 + 3t_f^4)$

$\gamma = D_1Q[1 + Q^2(D_3 + D_5Q^2)]$

$G_2 = \frac{1}{2}\,(1 + \eta_f^2)$

$G_4 = \frac{1}{12}\,(1 + 5\eta_f^2)$

$k = k_0[1 + G_2Q^2(1 + G_4Q^2)]$

The B_6, B_7, D_5, and G_4 terms are negligible when using the SPCS 83 within the approximate boundaries of the SPCS 83 zones. To compute beyond the defined SPCS 83 boundaries and to compute UTM coordinates, the use of these terms should be evaluated.

For most requirements the point grid scale factor k may be determined from the approximation:

$$k = k_0 + (E')^2/2r_0^2$$

where r_0 is the geometric mean radius of curvature scaled to the grid defined in section 3.15, and evaluated at the mean latitude of the zone. Table 3.22 contains $(1/2r_0^2)$ for each of the transverse Mercator zones.

3.25 Arc-to-Chord Correction (t-T)

The relationship among grid azimuth (t), geodetic azimuth (α), convergence angle (γ), and arc-to-chord correction (δ) at any given point is

$$t = \alpha - \gamma + \delta. \quad \text{(Remember that } \delta \text{ is defined as t-T).}$$

To compute δ requires knowledge of the coordinates of both ends of the line to which δ is to be applied. The following equations will compute δ_{12}, the δ from point (N_1, E_1) to (N_2, E_2):

$$N_m = \frac{1}{2}(N_1 + N_2)$$

$$\omega = (N_m - N_0 + S_0)/(k_0 r)$$

$$\phi_f = \omega + V_0 \sin\omega \cos\omega$$

$$F = (1 - e^2\sin^2\phi_f)(1 + \eta_f^2)/(k_0 a)^2$$

$$E_3 = 2E'_1 + E'_2$$

$$\delta_{12} = -\frac{1}{6}\,\Delta N\, E_3 F\left(1 - \frac{1}{27} E_3^2 F\right).$$

When computing within the approximate boundaries of the SPCS 83 zones, the term "$\frac{1}{27}\ E_3^2F$" is negligible and a single value of F can be precomputed for a mean latitude.

Often a single value of (F/2) is combined with the constant to convert radians to seconds (1 radian = $648000/\pi$ seconds) yielding the expression:

$$\delta_{12} = -25.4\ \Delta N(E_3/3)\ 10^{-10} \text{ seconds}$$

where the coordinates are in meters. Sometimes the notation ΔE replaces $(E_3/3)$. Then this equation is analogous to the expression often used in connection with the SPCS 27:

$$\delta_{12} = 2.36\ \Delta x\ \Delta y\ 10^{-10} \text{ seconds}$$

where the coordinates are in feet. This expression would serve for SPCS 83 coordinates that have been converted to feet upon insertion of the negative sign to conform to the sign convention. This expression with SPCS 27 coordinates derived the correct sign graphically or from a table.

3.26 Grid Scale Factor of a Line

As covered in section 4.2, the grid scale factor is different at each end of a line, but a single value is required to reduce a measured line. Given the grid scale factor of endpoints of a line (k_1 and k_2), a grid scale factor of the line (k_{12}) is required. Below is an alternative to the methods stated in section 4.2. This equation computes k_{12} using the function "F" derived in section 3.25 for the δ_{12} correction.

$$G = F(E'^2_1 + E'_1E'_2 + E'^2_2)/6$$

$$k_{12} = k_0[1 + G(1 + G/6)].$$

As above, the term "G/6" is often negligible within the bounds of the SPCS 83 and a single value of F will usually give results within $\pm(3)(10^{-7})$ at zone extremes.

3.3 Oblique Mercator Mapping Equations

3.31 Notation and Definition

Asterisked terms define the projection. Their value can be found in appendix A for the one zone in SPCS 83 that uses this projection.

	ϕ	Parallel of geodetic latitude, positive north
	λ	Meridian of geodetic longitude, positive west
	Q	Isometric latitude
	χ	Conformal latitude
	N	Northing coordinate
	E	Easting coordinate
*	N_0	False northing
*	E_0	False easting
	k	Point grid scale factor
	γ	Convergence
*	ϕ_c	Latitude of local origin
*	λ_c	Longitude of local origin
*	α_c	Azimuth of positive skew axis (u-axis) at local origin
*	k_c	Grid scale factor at the local origin
	α_0	Azimuth of positive skew axis at equator
	λ_0	Longitude of the true origin
	k_{12}	Line scale factor (between points 1 and 2)
	δ_{12}	Arc-to-chord correction (t-T) (from point 1 to point 2)
	a	Equatorial radius of the ellipsoid
	f	Flattening of the ellipsoid

3.32 Computation of GRS 80 Ellipsoid Constants

This section lists the equations for the ellipsoid-specific constants and the constants derived for the GRS 80 ellipsoid. The asterisked terms are required in section 3.34 through 3.36.

$$e^2 = 2f - f^2$$

$$e'^2 = e^2/(1 - e^2)$$

$$c_2 = e^2/2 + 5e^4/24 + e^6/12 + 13e^8/360$$

$$c_4 = 7e^4/48 + 29e^6/240 + 811e^8/11520$$

$$c_6 = 7e^6/120 + 81e^8/1120$$

$$c_8 = 4279e^8/161280$$

* $F_0 = 2(c_2 - 2c_4 + 3c_6 - 4c_8)$ $= 0.00668\ 69209\ 27$ (GRS 80)

* $F_2 = 8(c_4 - 4c_6 + 10c_8)$ $= 0.00005\ 20145\ 84$ (GRS 80)

* $F_4 = 32(c_6 - 6c_8)$ $= 0.00000\ 05544\ 30$ (GRS 80)

* $F_6 = 128\ c_8$ $= 0.00000\ 00068\ 20$ (GRS 80)

3.33 Computation of Zone Constants

In this section the zone defining constants, ellipsoid constants, and expressions within the oblique Mercator mapping equations are combined to form several intermediate computing constants that are zone and ellipsoid specific. These intermediate constants, flagged with an asterisk, will be required within the working equations of section 3.34 through 3.36.

All angles are in radian measure, where 1 radian equals $180/\pi$ degrees. Linear units are in meters.

* $$B = (1 + e'^2\cos^4\phi_c)^{1/2}$$

$$W_c = (1 - e^2\sin^2\phi_c)^{1/2}$$

$$A = aB(1 - e^2)^{1/2}/W_c^{\,2}$$

$$Q_c = (1/2)\left[\ln\frac{1 + \sin\phi_c}{1 - \sin\phi_c} - e\,\ln\frac{1 + e\sin\phi_c}{1 - e\sin\phi_c}\right]$$

* $$C = \cosh^{-1}\frac{B\,(1 - e^2)^{1/2}}{W_c\,\cos\phi_c} - BQ_c$$

Note: $\cosh^{-1}x = \ln[x+(x^2-1)^{1/2}]$

* $D = k_c A/B$

$\sin \alpha_0 = (a \sin \alpha_c \cos \phi_c)/(AW_c)$

For zone 5001:

$\tan \alpha_c = -0.75$
$\sin \alpha_c = -0.6$
$\cos \alpha_c = +0.8$

* $\lambda_0 = \lambda_c + \{\sin^{-1} [\sin \alpha_0 \sinh(BQ_c + C)/\cos \alpha_0]\}/B$

Note: $\sinh x = (e^x - e^{-x})/2$ (e = base of natural logarithms)

* $F = \sin \alpha_0$

* $G = \cos \alpha_0$

* $I = k_c A/a$

For Alaska zone 1, these constants are:

$B = 1.00029\ 64614\ 04$
$C = 0.00442\ 68339\ 26$
$D = 6\ 386\ 186.73253$
$F = -0.32701\ 29554\ 38$
$G = 0.94501\ 98553\ 34$
$I = 1.00155\ 89176\ 62$
$\lambda_0 = 101.51383\ 9560$ degrees

3.34 Direct Conversion Computation

This computation starts with the geodetic coordinates of a point (ϕ, λ), and computes the oblique Mercator grid coordinates (N,E), convergence angle (γ), and the grid scale factor (k).

$$L = (\lambda - \lambda_0)B$$

$$Q = (1/2)\left[\ln \frac{1 + \sin \phi}{1 - \sin \phi} - e \ln \frac{1 + e \sin \phi}{1 - e \sin \phi}\right]$$

$$J = \sinh(BQ + C)$$

$$K = \cosh(BQ + C)$$

Note: $\cosh x = (e^{x} + e^{-x})/2$ (e = base of natural logarithms)

$$u = D \tan^{-1} [(JG - F \sin L)/\cos L]$$

$$v = \frac{D}{2} \ln \frac{K - FJ - G \sin L}{K + FJ + G \sin L}$$

$$N = u \cos \alpha_c - v \sin \alpha_c + N_0$$

$$E = u \sin \alpha_c + v \cos \alpha_c + E_0$$

For zone 5001:

$$N = 0.8\ u + 0.6\ v - 5{,}000{,}000.$$

$$E = -0.6\ u + 0.8\ v + 5{,}000{,}000.$$

$$\gamma = \tan^{-1} \frac{F - JG \sin L}{KG \cos L} - \alpha_c$$

$$k = \frac{I(1 - e^2 \sin^2 \phi)^{1/2} \cos(u/D)}{\cos \phi \cos L} .$$

3.35 Inverse Conversion Computation

This computation starts with the oblique Mercator grid coordinates (N,E) and computes the geodetic coordinates (ϕ, λ). To compute the convergence angle (γ) and the grid scale factor (k), the computed (ϕ, λ) is then used in the equations of the direct conversion computation.

$$u = (E - E_0) \sin \alpha_c + (N - N_0) \cos \alpha_c$$

$$v = (E - E_0) \cos \alpha_c - (N - N_0) \sin \alpha_c$$

For zone 5001:

$$u = -0.6E + 0.8N + 7{,}000{,}000.$$

$$v = 0.8E + 0.6N - 1{,}000{,}000.$$

$$R = \sinh(v/D)$$

$$S = \cosh(v/D)$$

Note: $\cosh x = (e^{x} + e^{-x})/2$

$$T = \sin(u/D)$$

$$Q = \left[(1/2) \ln \frac{S - RF + GT}{S + RF - GT} - C \right] / B$$

$$\chi = 2\tan^{-1}\frac{\exp(Q) - 1}{\exp(Q) + 1}$$

where $\exp(Q) = e^Q$ and $e = 2.718281828...$(base of natural logarithms)

$$\phi = \chi + (\sin\chi\cos\chi)(F_0 + F_2\cos^2\chi + F_4\cos^4\chi + F_6\cos^6\chi)$$

$$\lambda = \lambda_0 - \frac{1}{B}\tan^{-1}\frac{RG + TF}{\cos(u/D)} .$$

3.36 Arc-to-chord Correction (t-T) and Grid Scale Factor of a Line

Having first obtained coordinates (u_1,v_1) and (u_2,v_2) from either the direct or inverse conversion computation, the (t-T) correction for the line from point 1 to point 2 (δ_{12}) and line correction k_{12} may be computed.

$$\phi = (\phi_1 + \phi_2)/2$$

$$Q = \frac{1}{2}\left[\ln\frac{1 + \sin\phi}{1 - \sin\phi} - e\ln\frac{1 + e\sin\phi}{1 - e\sin\phi}\right]$$

$$\delta_{12} = (u_1 - u_2)\,(2v_1 + v_2)/(6D^2)$$

$$k_{12} = \frac{k_c\,[1 + (v_1^2 + v_1v_2 + v_2^2)/(6D^2)](1 - e^2\sin^2\phi)^{1/2}}{\cos\phi\cosh(BQ + C)}$$

3.4 Polynomial Coefficients for the Lambert projection

Conversion of coordinates from NAD 83 geodetic positions to SPCS 83 plane coordinate positions, and vice versa, can be greatly simplified for the Lambert projection using precomputed zone constants obtained by polynomial curve fitting. NGS developed the Lambert "polynomial coefficient" approach as an alternative to the rigorous mapping equations given section 3.1. For many zones the solution of the textbook mapping equations for the Lambert projection requires the use of more than 10 significant digits to obtain millimeter accuracy, and in light of the programmable calculators generally in use by surveyors/engineers, an alternative approach was warranted. The mapping equations of the transverse Mercator projection do not present the same numerical problem as does the Lambert projection. Therefore, 10 significant digits are adequate. For the polynomial coefficient method of the Lambert projection, 10 significant digits will produce millimeter accuracy in all zones.

Given the precomputed polynomial coefficients, the conversion process by this method reduces to the solution of simple algebraic equations, requiring no exponential or logarithmic functions. It is therefore very efficient for hand calculators and small computers. In addition, the conversion is not too difficult to apply manually without the aid of programming. For this reason, the polynomial coefficient approach has also been listed as a manual approach in table 3.0. When programmed, this approach may be more efficient than the mapping equations of section 3.1.

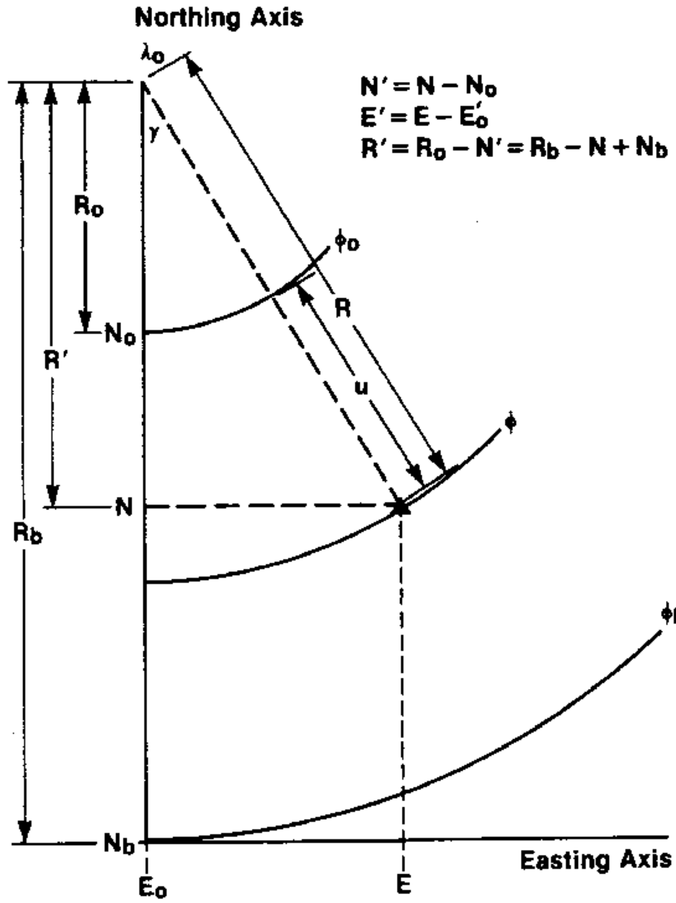


Figure 3.4.--The Lambert grid.

The equations in this section are similar to those in section 3.1, with the symbols representing the same quantities. Four new symbols are introduced, three of which are for polynomial coefficients--L's, G's, and F's--and the fourth is the symbol "u". From the equations and figure 3.4, it will be discovered that "u" is a distance on the mapping radius "R" between the central parallel and a given point. The "L" coefficients (L_1, L_2, L_3, etc.) are used in the forward conversion process (sec. 3.41), the "G" coefficients (G_1, G_2, G_3, etc.) are used in the inverse conversion process (sec. 3.42), and the "F" coefficients are used in the computation of grid scale factor. For the computation of (t-T), the methods in section 3.15 are applicable.

The fundamental polynomial equations of this method are

$$u = L_1\Delta\phi + L_2\Delta\phi^2 + L_3\Delta\phi^3 + L_4\Delta\phi^4 + L_5\Delta\phi^5 \text{ (forward conversion)}$$

$$\Delta\phi = \phi - \phi_0 = G_1u + G_2u^2 + G_3u^3 + G_4u^4 + G_5u^5 \text{ (inverse conversion).}$$

The determination of "u" in meters on a plane by a polynomial, given point (ϕ,λ) in the forward conversion, and the determination by a polynomial of $\Delta\phi$ in radians on the ellipsoid given point (N,E) in the inverse conversion, is the unique aspect of this method. The L-coefficients perform the functions: (1) computing the length of the meridian arc between ϕ and ϕ_0, and (2) converting that length to (R_0-R) which is its equivalent on the mapping radius. The G-coefficients serve the same two stage process, but in reverse. The polynomial coefficients of these equations, L's and G's, were separately determined by a least squares curve fitting program that also provided information as to the accuracy of the fit. Ten data points were used for each Lambert zone and the model solved for the fewest number of coefficients possible that provided 0.5 mm coordinate accuracy in the conversion. Consequently, some small zones required only three coefficients, three L's and three G's, whereas a few large zones required five coefficients for each the forward and inverse conversion.

Appendix C discusses the computed constants and coefficients required for this method, which are defined as follows:

The Defining Constants of a Zone:

ϕ_s or B_s	Southern standard parallel
ϕ_n or B_n	Northern standard parallel
ϕ_b or B_b	Latitude of grid origin
λ_0 or L_0	Central meridian - longitude of true and grid origin
N_b	Northing value at grid origin (B_b)
E_0	Easting value at grid and projection origin (L_0)

The Derived Constants:

ϕ_0 or B_0	Central parallel - Latitude of the projection origin
N_0	Northing value at projection origin (B_0)
k_0	Grid scale factor at the central parallel
R_0	Mapping radius at (B_0)
R_b	Mapping radius at (B_b)
M_0	Scaled radius of curvature in the meridian at B_0 used in section 3.15.

The Polynomial Coefficients:

L_1 through L_5 used in the forward conversion
G_1 through G_5 used in the inverse conversion
F_1 through F_3 used in the grid scale-factor computation.

3.41 Direct Conversion Computation

The computation starts with the geodetic position of a point (ϕ,λ), and computes the Lambert grid coordinates (N,E), convergence angle (γ), and grid scale factor (k).

$$\Delta\phi = \phi - B_0 \quad (\Delta\phi \text{ in decimal degrees})$$

$$u = L_1\Delta\phi + L_2\Delta\phi^2 + L_3\Delta\phi^3 + L_4\Delta\phi^4 + L_5\Delta\phi^5.$$

Note: The only required terms are those for which polynomial coefficients are provided in appendix C. Either three, four, or five L's are required depending on the size of the zone.

Suggestion: Use nested form.

$$u = \Delta\phi[L_1 + \Delta\phi\{L_2 + \Delta\phi(L_3 + \Delta\phi(L_4 + L_5\Delta\phi))\}]$$

$$R = R_0 - u$$

$\gamma = (L_0 - \lambda)\sin(B_0)$ convergence angle

$$E' = R \sin\gamma$$
$$N' = u + E' \tan(\gamma/2)$$

$E = E' + E_0$ easting

$N = N' + N_0$ northing

$k = F_1 + F_2u^2 + F_3u^3$ grid scale factor

3.42 Inverse Conversion Computation

This computation starts with the Lambert grid coordinates (N,E) from which are computed the geodetic coordinates (ϕ,λ), convergence angle (γ), and grid scale factor (k):

$$N' = N - N_0$$

$$E' = E - E_0$$

$$R' = R_0 - N'$$

$\gamma = \tan^{-1}(E'/R')$ convergence angle

$\lambda = L_0 - \gamma/\sin(B_0)$ longitude

$$u = N' - E'\tan(\gamma/2)$$

$\Delta\phi = \phi - B_0 = G_1u + G_2u^2 + G_3u^3 + G_4u^4 + G_5u^5$ ($\Delta\phi$ in decimal degrees)

Note: The only required terms are those for which polynomial coefficients are provided in appendix C. Either three, four, or five G's are required depending on the size of the zone.

Suggestion: Use factored form.

$$\Delta\phi = u[G_1 + u\{G_2 + u(G_3 + u(G_4 + G^5u))\}]$$

$\phi = B_0 + \Delta\phi$ latitude

$k = F_1 + F_2u^2 + F_3u^3$ grid scale factor

4. LINE CONVERSION METHODS REQUIRED TO PLACE A SURVEY ON SPCS 83

State plane coordinates are derived from latitudes and longitudes. Latitudes and longitudes are based on an ellipsoid of reference and a horizontal datum that approximates the surface of the Earth. Accordingly, field observations measured on the ground must first be reduced to the surface of the horizontal datum before they are further reduced to the map projection surface--the grid. The mathematical process of reducing field observations does not necessarily imply that the numbers are reduced in magnitude although often that is the case.

Section 4.1 addresses the reduction of measured distances to the datum surface, not a subject of map projections, but included here for convenience. Only the geometric aspect of reduction is discussed. Reductions relating to the influence of the atmosphere are not included. Section 4.2 contains the further reduction of measured distances to the grid, expanding on section 2.6 and applying the concept of point grid scale factors to an entire measured line. Section 4.3 discusses the reduction of azimuths and angles from the ellipsoid to the grid, applying the concepts stated in section 2.5. Reduction of angles and azimuths to the ellipsoid is beyond the scope of this manual. The reader is referred to texts on higher geodesy.

4.1 Reduction of Observed Distances to the Ellipsoid

Before a measured distance can be reduced to a grid distance in a zone of the SPCS 83, it must first be reduced to a geodetic distance. Classically, observed distances have been reduced to one of two surfaces, either the geoid (sea level) or the ellipsoid. (See fig. 4.1a.) To which surface distances were reduced depended on available information. Generally, in conjunction with NAD 27, distances were reduced only to sea level, although subsequent computations using those distances were performed on the ellipsoid. This incomplete reduction was adequate for NAD 27, as the ellipsoid of NAD 27 (Clarke Spheroid of 1866) closely approximated sea level. For NAD 83, due to availability of information on geoid-ellipsoid separation, distances may be reduced to the ellipsoid. Furthermore,

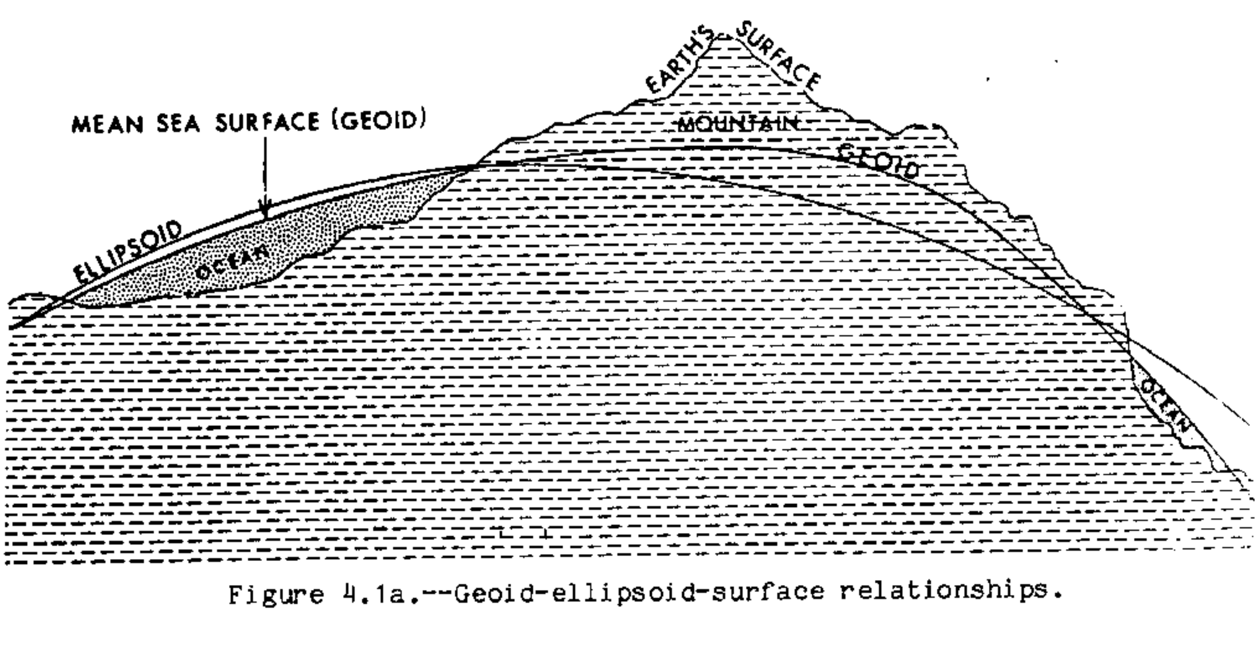


Figure 4.1a.--Geoid-ellipsoid-surface relationships.

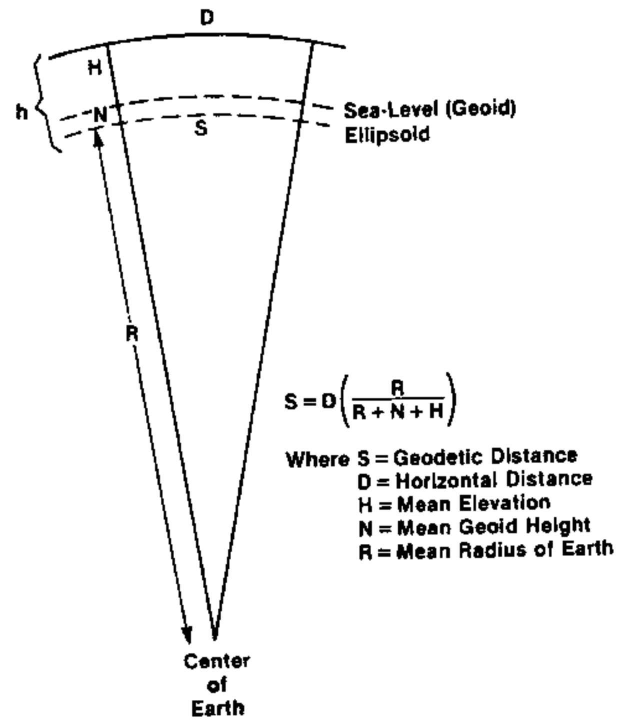


Figure 4.1b.--Reduction to the ellipsoid.

the worldwide datum of NAD 83 does not fit the North American continent as well as the previous NAD 27. The impact of this on surveyors may be the requirement to use this geoidal separation information in connection with the reduction of observed distances to the ellipsoid. The approximation of using sea level may not always be adequate, but those occurrences should be few and affect only surveys of highest order.

To reduce measurements to the ellipsoid instead of sea level requires the addition of the geoid height (sea level/ellipsoid separation) to the station elevations prior to reduction. In the conterminous United States the ellipsoid is above the geoid. In Alaska the ellipsoid is below the geoid. Since the geoid height of a station is defined as the height above the ellipsoid minus the height above the geoid, except in Alaska it is a negative value. The geoid height is published by NGS together with NAD 83 coordinate information. The geodetic height of a control station (height above ellipsoid) "h" is the sum of elevation above mean sea level "H" and geoid height "N". The failure to use geoid height will introduce an error in reduced distance of 0.16 ppm for each meter of geoid height. A geoid height of -30 m systematically affects all reduced distances by -4.8 ppm (1:208,000). Clearly a single geoid height may be applied for a region or project, and even ignored for many types of surveys.

The application of geoid height and the precise determination of a radius of curvature on the ellipsoid (below) are the only occurrences where NAD 83 may affect changes to distance reduction procedures.

Knowledge of radii of curvature on ellipsoids is paramount to the reduction of distances measured on the surface of the Earth to either sea level or the ellipsoid surface. Ellipsoid radii are a function of both latitude and azimuth. Fortunately, for many uses any mean radius of curvature is often a satisfactory approximation. Distance reductions are often performed on a sphere having a radius equal to the mean radius of curvature of the ellipsoid at an average latitude of the conterminous United States. Figure 4.1b illustrates the quantities involved in the reduction assuming this sphere. From the proportion:

$$S/D = R/(R+h),$$

we solve: $S = D*R/(R+h)$

$h = N + H$ by definition.

Therefore,

$$S = D*R/(R+N+H).$$

The ratio R/(R+N+H) is similar to the familiar sea level factor except that the average elevation of line "D" above the ellipsoid, usually denoted as "h" and called sea level in most NAD 27 literature, is replaced by "H" as the height above sea level (the geoid) and "N" the ellipsoidal separation. To emphasize the difference, the ratio has been designated as an elevation factor. On NAD 83, "h" remains the height above the ellipsoid, but is obtained by adding together the geoid-ellipsoid separation "N" and the height of the station above the geoid "H". In line reductions by this method, a mean geoid height "N" and mean elevation "H" are used to obtain a mean height of the line "h". Figure 4.1c depicts the situation of a negative N, as is the case in the conterminous United States.

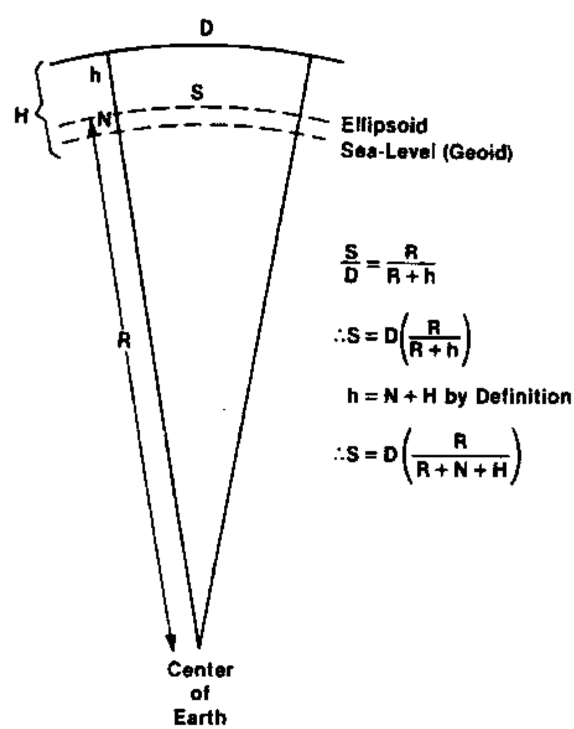


Figure 4.1c.--Reduction to the ellipsoid (shown with a negative geoid height).

The mean radius "R" used in connection with NAD 27 was 20,906,000 ft, or 6,372,000 m. This approximate radius serves equally well for NAD 83.

The elevation factor is often combined with the grid scale factor of a line (see 4.2) to form a single multiplier that reduces an observed horizontal distance at an average elevation directly to the SPCS grid. These two factors are quickly combined by obtaining the product of the factors. This product is approximated by subtracting "1" from the sum of the two factors. Identified as the "combined factor," when multiplied by the horizontal distance it has the same effect as each factor multiplied separately, yielding the grid distance. If the area of a parcel of land at ground elevation is desired, the area obtained from using SPCS 83 coordinates should be divided by the square of the combined factor (the same factor that was used to reduce the measured distances) to obtain the area at ground elevation.

Although the above approximate method serves most surveyors and engineers well, sometimes a more rigorous reduction procedure will be required. Such a procedure is found in NOAA Technical Memorandum NOS NGS-10, Use of calibration base lines, appendix I: "The geometrical transformation of electronically measured distances" (1977).

4.2 Grid scale factor k_{12} of a Line

As discussed in section 2.6 on grid scale factor, an incremental length on the ellipsoid must be multiplied by a grid scale factor to obtain the length of that increment on the grid. However, measured survey lines are not infinitesimal increments, and grid scale factor ratios change from point to point. Therefore, we are faced with the problem of deriving a single grid scale factor that can be applied to an entire measured length (that has first been reduced to the ellipsoid), when in fact the value of the grid scale factor is changing from point to point. Required is a grid scale factor ratio that when multiplied by the measured ellipsoid-reduced distance will yield the grid distance. (See fig. 4.2.) This grid scale factor which applies to a line between points 1 and 2 is symbolized as k_{12}.

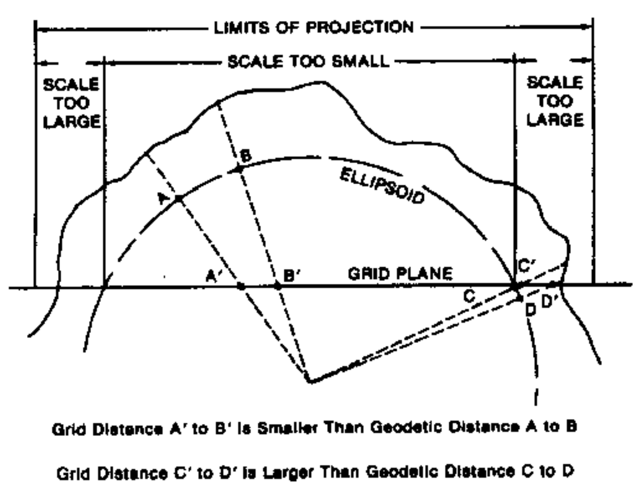


Figure 4.2.--Geodetic vs. grid distances.

Recalling we are computing on a conformal projection, the grid scale factor is the same in any direction, but increases in magnitude with distance of the point from the central meridian in a transverse Mercator projection, or central parallel in a Lambert projection. This means that the grid scale factor is different at each end of a measured line, and that this difference is greatest for an east-west line in the transverse Mercator projection, or north-south line in the Lambert projection. There are several solutions to the problem of deriving a grid scale factor for a line. The solution depends on the required accuracy of the reduction, the lengths of lines involved, and, to a lesser degree, the areal extent of the zone and location of the line within the zone.

The application of the grid scale factor to measured lengths for each project should begin with an analysis of the magnitude of the correction within that project--a function of the average length of measured lines and location of the project within the zone--compared with the desired project accuracy. For surveys of third-order accuracy or less (as classified by the Federal Geodetic Control Committee) a single scale factor for all lines in the project may suffice. This grid scale factor would be computed at the center of the project. It may be determined that in the reduction of measured geodetic lengths to the grid, the grid scale factor could be ignored.

To determine an appropriate method for computing the line grid scale factor for any project, it is suggested that one first determine point grid scale factors for the worst case situation in the project--ends of the longest line that run in a direction perpendicular to the central axis of the projection, and at the greatest distance from the central axis. The central axis is the central meridian in the transverse Mercator or central parallel in the Lambert projection. The appropriateness of approximations for each line or for a project, i.e., a single project grid scale factor, is dependent on the computing error that can be tolerated.

When a single grid scale factor for a project is acceptable, it may also be an acceptable approximation to use a single elevation reduction factor, a similar looking multiplier that reduces a measured horizontal line on the Earth's surface to its equivalent ellipsoid length. (See sec. 4.1.) Sometimes a combined project factor is used to reduce all measured horizontal distances from the average elevation of the project directly to the grid. The appropriateness of a single project elevation reduction factor requires the similar analysis as a project grid scale factor.

When a line grid scale factor must be determined for each measured line of the survey, there are several approaches for handling the fact that point grid scale factors are different at each end of a line. Each approach requires computing one or more point grid scale factors. Approximate equal results are obtained from either using the point scale factor of the midpoint of the line or a mean scale factor computed from the point scale factor for each end of the line. The most accurate determination of the line grid scale factor (k_{12}) requires computing point scale factors at each end of the line (k_1 and k_2) plus the midpoint (k_m) and combining according to:

$$k_{12} = (k_1 + 4\ k_m + k_2)/6.$$

4.3 Arc-to-Chord Correction (t-T)

As given in section 2.5, the representation (projection) of a geodetic azimuth "α" on any plane grid does not produce the grid azimuth "t" but the projected geodetic azimuth "T". Figure 2.5 illustrates the small difference, "t-T". "t-T" (alias "arc-to-chord" and "second difference") is an angular correction to the line of sight between two points, whether that "direction" is an azimuth or part of an angle measure. The "t-T" is the difference between the "pointing" observed on the ellipsoid (generally the same as on the ground) and the pointing on the grid. The difference is often insignificant.

From a purely theoretical perspective, grid azimuths should be used with grid angles and directions, while geodetic azimuths should be used with observed angles and directions. To perform survey computations on a plane, observed directions should be corrected for the arc-to-chord (t-T) correction to derive an equivalent value for the observed direction on the grid; otherwise observed angles should be used with geodetic azimuths with survey computations performed on the ellipsoid. For example, in a traverse computation on the ellipsoid, the azimuth would need to be carried forward by geodetic methods where forward and backward azimuths differ by approximately $\Delta\lambda''\sin\phi_m \pm 180°$.

From a practical perspective in many survey operations the (t-T) correction is negligible, observed angles are used with grid azimuths, and survey computations are done on a plane. In a precise survey it is necessary to evaluate the magnitude of (t-T). Table 4.3a provides an approximation.

Table 4.3a.--Approximate size of (t-T) in seconds of arc for Lambert or transverse Mercator projection (see note 1)

ΔE or ΔN (See note 2) (km)	Perpendicular distance from central axis to midpoint of the line (see note 3) (km)				
	50	100	150	200	250
2	0.3	0.5	0.8	1.0	1.3
5	0.6	1.3	1.9	2.5	3.2
10	1.3	2.5	3.8	5.1	6.4
20	2.5	5.1	7.6	10.2	12.7

(1) (t-T) is also a function of latitude, but often is estimated by $(t-T) = 25.4\,(\Delta N)(\Delta E)10^{-10}$ seconds, where ΔN and ΔE are in meters.

(2) The length of the line to which the correction is to be applied is in a direction parallel to central axis.

(3) A better approximation is obtained by taking the distance from the central axis to a point one-third of the distance from point 1 to point 2 when estimating (t-T) at point 1.

Note on the use of Table 4.3a

* IN A STRAIGHT TRAVERSE OF EQUAL LINE LENGTHS, THE *
* CORRECTION TO AN ANGLE WILL BE DOUBLE THE ABOVE *
* CORRECTION TO EACH DIRECTION, AND THESE ANGULAR *
* CORRECTIONS SYSTEMATICALLY ACCUMULATE ALONG THE *
* TRAVERSE. BECAUSE OF THE DOUBLING ACTION, IF *
* THIS TABLE IS USED TO ESTIMATE THE PORTION OF AZIMUTH *
* MISCLOSURE OF AN ENTIRE TRAVERSE SURVEY ATTRIBUTED *
* TO IGNORING THIS CORRECTION, THE CONTRIBUTION WOULD *
* BE TWICE THE TABLE VALUE. *

Use of the table requires knowledge of two items. First, the approximate perpendicular distance from the central axis (central parallel in Lambert projection or the central meridian in Mercator projection) to the midpoint of your line is required. The midpoint is derived from differencing mean coordinates of the line (northings for Lambert and eastings for Mercator) from the northing of the central parallel or the easting of the central meridian.

Also needed to use table 4.3a is the length of the line in a direction parallel to the central parallel (Lambert) or parallel to the central meridian (Mercator). Again ΔE or ΔN is derived from the point coordinates. From studying table 4.3a it is apparent that the (t-T) correction will be its largest on lines parallel, and the greatest distance, from the central axis of the projection zone.

Table 4.3b.--Sign of (t-T) correction

Map projection		Azimuth of the line from north	
Lambert:	Sign of $N-N_0$	0 to 180°	180 to 360°
	Positive	+	-
	Negative	-	+
		270-90°	90-270°
Transverse Mercator:	Sign of $E-E_0$ (or E_3)		
	Positive	-	+
	Negative	+	-

Figure 4.3 illustrates the relative orientation of projected geodetic lines (T) and grid lines (t) for traverses located on either side of the central axis. It should be observed that the projected geodetic line is always concave towards the central meridian or parallel. This fact provides a visual check on the correct sign of the (t-T) correction. For a conventional nearly straight traverse, the signs of the (t-T) correction on each direction of an observed angle are opposite, thus the corrections accumulate. Therefore, for a straight traverse of

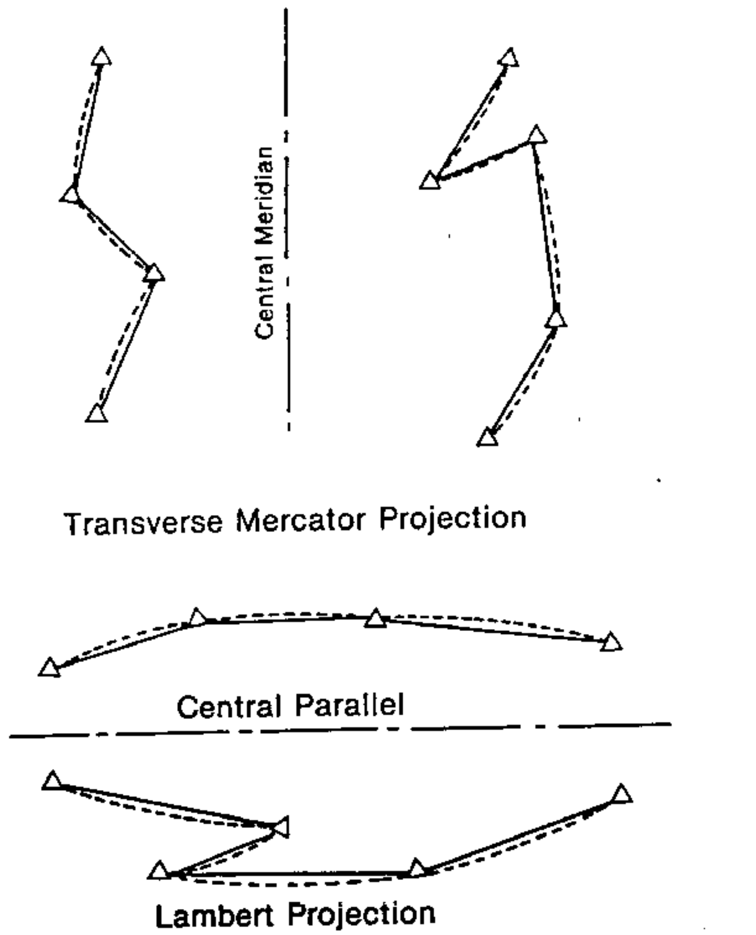


Figure 4.3.--Projected geodetic vs. grid angles.

approximately equal line lengths, the (t-T) correction for the observed angle will be twice the computed correction to a single direction.

Although the formulas in chapter 3 will provide the proper sign of (t-T), table 4.3b may also be used as a guide.

4.4 Traverse Example

This section illustrates the solution to a traverse computation. Given the terrestrial survey observations for a closed connecting traverse between points of known position, final adjusted coordinates are computed for new points. The emphasis in the example is the procedure to reduce angles and distances measured on the surface of the Earth to an equivalent value on the grid of the SPCS 83. These reduction procedures are applicable regardless of the positioning method or network geometry. After the field data are reduced to the SPCS 83 grid, the data are made consistent with the SPCS 83 plane coordinates of the fixed control station using the method of the compass rule adjustment. Mathematical rigor plus data encoding and programming consideration often make adjustment by least squares the preferred method, but the compass rule adjustment is widely used and serves well for this example where computations on a plane are being illustrated. Furthermore, the example in this section was a sample NAD 27 problem used at more than 30 workshops instructed by NGS, so adjusting identical field data to NAD 83 control illustrates datum differences.

Figure 4.4a is a sketch of the sample traverse. For instructional consideration, it purposely violates Federal Geodetic Control Committee specifications for network design. Four new points are to be positioned between two points of known position. The starting azimuth (azimuth at station number 1) is derived from published angles while the closing azimuth (azimuth at station number 6) is derived from published coordinates. The solid lines depict grid ines; dotted lines show projected geodetic lines. The difference is (t-T). The illustrated northing of the central axis together with the approximate point coordinates are used in the computation of (t-T). The computation of (t-T) requires the distance of the traverse line from the central axis. Similarly, a traverse line in a transverse Mercator zone requires E_0, the easting of the central axis.

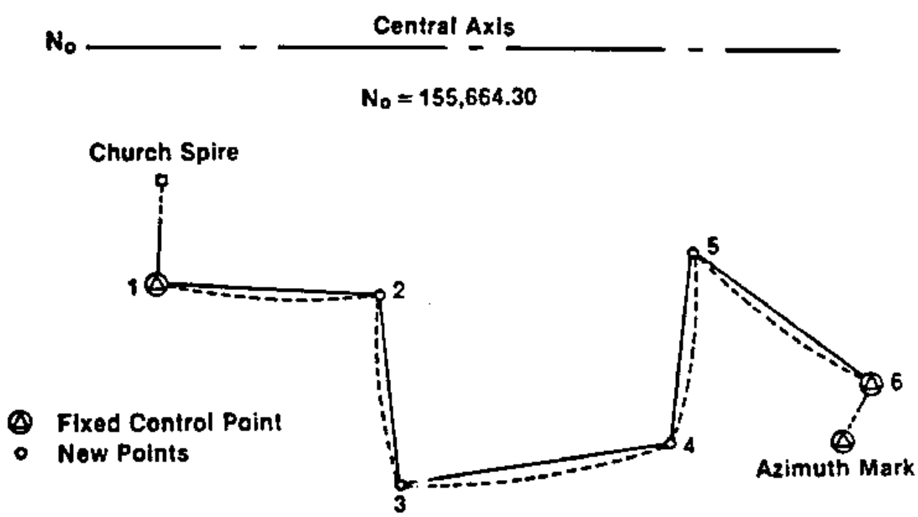
The following steps are required to compute any traverse:

1. Obtain starting and closing azimuth.

2. Analyze the grid scale factor for the project. A mean of the published point grid scale factors of the control points may be adequate for all lines in the project, or a grid scale factor for each line may be required.

3. Analyze the elevation factor for the project. A mean of the published elevations of the control points corrected for the geoid height (N) may be adequate to compute the elevation factor. Otherwise each line may need to be reduced individually.

4. If a project grid scale factor and project elevation factor are applicable, compute a project combined factor.



Approximate Coordinates

Point	N	E
1	61,400 meters	660,300 meters
2	61,300	665,100
3	57,300	665,400
4	58,200	670,300
5	61,800	670,500
6	58,900	674,000

Figure 4.4a.--Sample traverse. (N specifies northing component and E specifies easting component.)

5. Reduce the horizontal distances to the grid.

6. Using preliminary azimuths derived from unreduced angles and grid distances, compute approximate coordinates.

7. Analyze magnitude of (t-T) corrections, and if their application is required, compute the (t-T) corrections for each line using approximate coordinates for each point.

8. Apply (t-T) corrections to the measured angles to obtain grid angles.

9. Adjust the traverse.

10. Compute the final adjusted 1983 State Plane Coordinates for the new points, adjusted azimuths and distances between the points, and if required ground level distances.

Step 1

To obtain the starting azimuth, use the published azimuth and angle information (Fig. 4.4b) and compute the grid azimuth from point 1 to the church spire.

Plane azimuth point 1 to azimuth mark	30°30'12.6"
Plus observed clockwise angle from azimuth mark to church spire	329 50 18.6
Starting azimuth	0 °20'31.2"

Recalling that plane angles are used with plane azimuths and observed spherical angles are used with geodetic azimuths, in theory the above observed angle should have been corrected for the (t-T) correction. However, because each direction of the angle was short, the (t-T) correction is zero.

To obtain the closing azimuth, use the published coordinate information in figure 4.4b and compute the grid azimuth from Point 6 to Point 6 Azimuth Mark using a plane coordinate inverse:

$$\tan \text{azimuth}_{12} = (E_1 - E_2)/(N_1 - N_2)$$

$$\text{azimuth}_{12} = \text{arc tan } (121.457/485.047) = 194°03'28.5''$$

Steps 2 through 4

Grid scale factor point 1	=	1.0000420
Grid scale factor point 6	=	1.0000480
Mean grid scale factor	=	1.0000450

Elevation point 1	=	830.0 ft
Elevation point 6	=	900.0 ft
Mean H	=	865.0 ft
Mean N	=	-30.5 m = -100 ft

$$\begin{aligned}\text{Elevation factor} &= (20{,}906{,}000)/(20{,}906{,}000 + H + N) \\ &= (20{,}906{,}000)/(20{,}906{,}765) \\ &= 0.9999634\end{aligned}$$

NORTH AMERICAN DATUM 1983
ADJUSTED HORIZONTAL CONTROL DATA

NAME OF STATION: Point 1

STATE: Wisconsin YEAR: 1980 Second ORDER

SOURCE: G-17289 Geoid Height = -30.5 meters

	° ′ ″	ELEVATION:
GEODETIC LATITUDE:	42 33 00.01150	METERS
GEODETIC LONGITUDE:	89 15 56.24590	830.5 FEET

STATE COORDINATES				
STATE & ZONE	CODE	Northing	Easting	Mapping angle ° ′ ″
Wisconsin	4803	61,367.006	660,318.626	+ 0 30 16.5

Scale Factor = 1.0000420

TO STATION OR OBJECT	GEODETIC AZIMUTH ° ′ ″	PLANE AZIMUTH* ° ′ ″	CODE
Point 1 Azimuth Mark	31 00 29.1	30 30 12.6	4803

DESCRIPTION OF TRAVERSE STATION

NAME OF STATION: Point 1 STATE: Wisconsin COUNTY: LEO

CHIEF OF PARTY: E.J. McKay YEAR: 1980 DESCRIBED BY: JES

NOTE.* HEIGHT OF TELESCOPE ABOVE STATION MARK METERS.† HEIGHT OF LIGHT ABOVE STATION MARK METERS.

SURFACE-STATION MARK, UNDERGROUND-STATION MARK — DISTANCES AND DIRECTIONS TO AZIMUTH MARK, REFERENCE MARKS AND PROMINENT OBJECTS WHICH CAN BE SEEN FROM THE GROUND AT THE STATION

OBJECT	BEARING	DISTANCE FEET	DISTANCE METERS	DIRECTION† ° ′ ″
Point 6	E		13.886.795	0 00 00.0
RM1	ESE		30.287	33 46 28.1
RM2	WSW		22.984	123 33 29.2
Church Spire (unidentified)	N		(2.3 miles)	260 19 01.5
Azimuth Mark	NNE		(0.2 miles)	290 28 42.9

Figure 4.4b.--Fixed station control information.

NORTH AMERICAN DATUM 1983
ADJUSTED HORIZONTAL CONTROL DATA

NAME OF STATION: Point 6

STATE: Wisconsin YEAR: 1980 Second -ORDER

SOURCE: G-17289

Geoid Height = -30.5 meters

	°	'	"		
GEODETIC LATITUDE:	42	31	37.32888	ELEVATION:	METERS
GEODETIC LONGITUDE:	89	05	58.04271	900.0	FEET

1983 STATE COORDINATES (meters)				
STATE & ZONE	CODE	Northing	Easting	Mapping Angle ° ' "
Wisconsin, S	4803	58,949.532	673,994.015	+ 0 37 07.5

Scale Factor = 1.0000480

ADJUSTED HORIZONTAL CONTROL DATA

NAME OF STATION: Point 6 Azimuth Mark

STATE: Wisconsin YEAR: 1980 Second -ORDER

SOURCE: G-14402

	°	'	"		
GEODETIC LATITUDE:	42	31	21.65360	ELEVATION:	METERS
GEODETIC LONGITUDE:	89	06	03.59289	750.0	FEET

STATE COORDINATES				
STATE & ZONE	CODE	Northing	Easting	Mapping angle ° ' "
Wisconsin, S	4803	58,464.485	673,872.558	+ 0 37 03.7

Scale Factor = 1.0000491

Figure 4.4b.--Fixed station control information (continued).

Combined factor = (1.0000450)(0.9999634)
= 1.0000084

Step 5

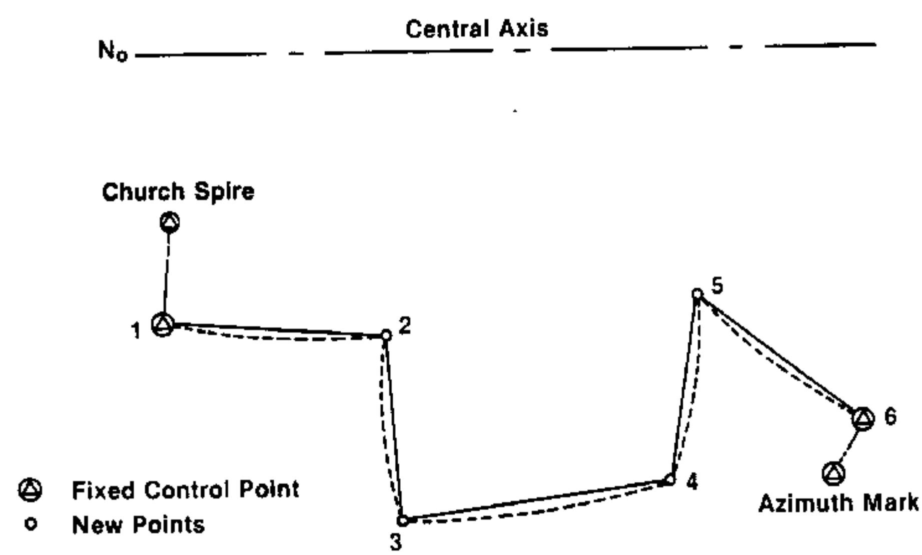
From	To	Measured horizontal lengths	Grid lengths
1	2	4,805.468	4,805.508
2	3	3,963.694	3,963.727
3	4	4,966.083	4,966.125
4	5	3,501.223	3,501.252
5	6	4,466.935	4,466.973

Step 6

The computation of preliminary coordinates is not illustrated. The procedure used to obtain preliminary coordinates for the adjustment is generally used.

Step 7

Figure 4.4c illustrates computation of (t-T) using the abbreviated formula $(t-T) = (25.4)(\Delta N)(\Delta E)(10^{-10})$ seconds. The (ΔN) is the distance of the midpoint of the line from the central axis. For example:



From	To	ΔN	ΔE	25.4 (ΔN) (ΔE)
1	2	−0.943	0.048	−1.1
2	3	−0.964	0.003	−0.1
3	4	−0.979	0.049	−1.2
4	5	−0.957	0.002	−0.1
5	6	−0.953	0.035	−0.8

Figure 4.4c.--(t-T) correction (Δ northing and Δ easting expressed in meters).

$$\Delta N_{12} = [(61,400 + 61,300)/2 - 155,664]10^{-5} = -0.943$$

(ΔE) is the difference of eastings of the endpoints of the line. For example:

$$\Delta E_{12} = (E_2 - E_1)10^{-5} = 0.048$$

Note that (ΔN) and (ΔE) are each scaled by (10^{-5}) to account for the (10^{-10}) in the equation for (t-T).

Step 8

The corrections computed in step 7 are applied to an observed pointing in one direction. Using this approximate equation for (t-T), the correction from the other end of the line is identical but with opposite sign. Figure 4.4d lists the observed traverse angles, (t-T) corrections to each direction, angle correction, and the grid angle.

Point	Observed Angle			Backsight Correction	Foresight Correction	Angle Correction	Grid Angle
	°	'	"	"	"	"	"
1	90	44	18.3	0	-1.1	-1.1	17.2
2	265	15	55.2	+1.1	-0.1	-1.2	54.0
3	82	48	26.9	+0.1	-1.2	-1.3	25.6
4	105	03	08.6	+1.2	-0.1	-1.3	07.3
5	304	33	46.2	+0.1	-0.8	-0.9	45.3
6	245	17	39.5	+0.8	0	-0.8	38.7

Station		Preliminary Azimuth			Correction for Closure	Corrected Azimuth		
From	To	°	'	"	"	°	'	"
1	Azimuth Mk.	00	20	31.2*		00	20	31.2*
∠		90	44	17.2	-1.8	90	44	15.4
1	2	91	04	48.4		91	04	46.6
2	1	271	04	48.4		271	04	46.6
∠		265	15	54.0	-1.8	265	15	52.2
2	3	176	20	42.4		176	20	38.8
3	2	356	20	42.4		356	20	38.8
∠		82	48	25.6	-1.8	82	48	23.8
3	4	79	09	08.0		79	09	02.6
4	3	259	09	08.0		259	09	02.6
∠		105	03	07.3	-1.8	105	03	05.5
4	5	4	12	15.3		4	12	08.1
5	4	184	12	15.3		184	12	08.1
∠		304	33	45.3	-1.8	304	33	43.5
5	6	128	46	00.6		128	45	51.6
6	5	308	46	00.6		308	45	51.6
∠		245	17	38.7	-1.8	245	17	36.9
6	Azimuth Mark	194	03	39.3		194	03	28.5*
				Closure =	-10.8			
∠ *Fixed Grid Azimuths								

Figure 4.4d.--Azimuth adjustment.

STATION	PLANE AZIMUTH	GRID DISTANCE	SIN AZIMUTH / COS AZIMUTH	LATITUDE	DEPARTURE	GRID COORDINATES	
	° ' "	Meters		Meters	Meters	Northing	Easting
1					Fixed	61,367.006	660,318.626
			+0.99982248		+4804.655		
	91 04 46.6	4805.508				61,276.462	665,123.281 (1)
			-0.01884165	-90.544		- 0.223	+ 0.232 (2)
2						61,276.239	665,123.513 (3)
			+0.06376401		+252.743		
	176 20 38.8	3963.727				57,320.801	665,376.024
			-0.99796501	-3955.661		- 0.407	+ 0.423
3						57,320.394	665,376.447
			+0.98212573		+4877.359		
	79 09 02.6	4966.125				58,255.555	670,253.383
			+0.18822607	+934.754		- 0.637	+ 0.662
4						58,254.918	670,254.045
			+0.07327736		+256.563		
	4 12 08.1	3501.252				61,747.394	670,509.946
			+0.99731160	+3491.839		- 0.799	+ 0.831
5						61,746.595	670,510.777
			+0.77972788		+3483.023		
	128 45 51.6	4466.973				58,950.539	673,992.969
			-0.62611855	-2796.855		- 1.007	+ 1.046
6					Fixed	58,949.532	673,994.015

(1) Preliminary Coordinate
(2) Accumulative Correction
(3) Adjusted Coordinate

Figure 4.4e.--Traverse computation by latitudes and departures.

FINAL ADJUSTED RESULTS

FROM	TO	LATITUDE	DEPARTURE	GRID LENGTH	TAN (OR COT*)	GROUND LENGTH	AZIMUTH
1	2	- 90.767	+4804.887	4805.744	-0.01889056*	4805.704	91 04 56.0
2	3	-3955.845	+ 252.934	3963.923	-0.06393931	3963.890	176 20 29.5
3	4	+ 934.524	+4877.598	4966.316	+0.19159513*	4966.274	79 09 13.9
4	5	+3491.677	+ 256.732	3501.103	+0.07352685	3501.074	4 12 18.8
5	6	-2797.063	+3483.238	4467.271	-0.80300657*	4467.233	128 45 52.9

Figure 4.4f.--Adjusted traverse data.

Step 9

Using the starting grid azimuth and grid angles, the closing azimuth is computed. (See fig. 4.4d.) The misclosure of (-10.8) seconds is prorated among the grid angles and final corrected azimuths computed. The adjusted azimuths and grid distances are transferred to figure 4.4e where the coordinate misclosures are determined and misclosures prorated according to the compass rule adjustment method.

Step 10

Plane coordinate inverses between adjusted coordinates provide adjusted grid azimuths and distances. If ground level distances are required, the adjusted grid distance is divided by the combined factor that was previously used to reduce the observed distances. Figure 4.4f shows the adjusted data.

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APPENDIX A.--DEFINING CONSTANTS FOR THE 1983 STATE PLANE COORDINATE SYSTEM

Transverse Mercator (T.M.), Oblique Mercator (O.M.), and Lambert (L.) Projections

State/Zone/Code			Projection	Central Meridian and Scale Factor (T.M.) or Standard Parallels (L.)	Grid Origin Longitude Latitude	Grid Origin Easting Northing (meters)
Alabama	AL					
East	E	0101	T.M.	85 50 1:25,000	85 50 30 30	200,000. 0
West	W	0102	T.M.	87 30 1:15,000	87 30 30 00	600,000. 0
Alaska	AK					
Zone 1		5001	O.M.	Axis Azimuth = arc tan -3/4 1:10,000	133 40 57 00	5,000,000. -5,000,000.
Zone 2		5002	T.M.	142 00 1:10,000	142 00 54 00	500,000. 0
Zone 3		5003	T.M.	146 00 1:10,000	146 00 54 00	500,000. 0
Zone 4		5004	T.M.	150 00 1:10,000	150 00 54 00	500,000. 0
Zone 5		5005	T.M.	154 00 1:10,000	154 00 54 00	500,000. 0
Zone 6		5006	T.M.	158 00 1:10,000	158 00 54 00	500,000. 0
Zone 7		5007	T.M.	162 00 1:10,000	162 00 54 00	500,000. 0
Zone 8		5008	T.M.	166 00 1:10,000	166 00 54 00	500,000. 0
Zone 9		5009	T.M.	170 00 1:10,000	170 00 54 00	500,000. 0
Zone 10		5010	L	51 50 53 50	176 00 51 00	1,000,000. 0

State/Zone/Code			Projection	Central Meridian and Scale Factor (T.M.) or Standard Parallels (L.)	Grid Origin Longitude Latitude	Grid Origin Easting Northing
Arizona	AZ					
East	E	0201	T.M.	110 10	110 10	213,360.
				1:10,000	31 00	0
Central	C	0202	T.M.	111 55	111 55	213,360.
				1:10,000	31 00	0
West	W	0203	T.M.	113 45	113 45	213,360.
				1:15,000	31 00	0

(State law defines the origin in International Feet)
(213,360M. = 700,000 International Feet)

Arkansas	AR					
North	N	0301	L	34 56	92 00	400,000.
				36 14	34 20	0
South	S	0302	L	33 18	92 00	400,000.
				34 46	32 40	400,000.
California	CA					
Zone 1		0401	L	40 00	122 00	2,000,000.
				41 40	39 20	500,000.
Zone 2		0402	L	38 20	122 00	2,000,000.
				39 50	37 40	500,000.
Zone 3		0403	L	37 04	120 30	2,000,000.
				38 26	36 30	500,000.
Zone 4		0404	L	36 00	119 00	2,000,000.
				37 15	35 20	500,000.
Zone 5		0405	L	34 02	118 00	2,000,000.
				35 28	33 30	500,000.
Zone 6		0406	L	32 47	116 15	2,000,000.
				33 53	32 10	500,000.
Colorado	CO					
North	N	0501	L	39 43	105 30	914,401.8289
				40 47	39 20	304,800.6096
Central	C	0502	L	38 27	105 30	914,401.8289
				39 45	37 50	304 800.6096
South	S	0503	L	37 14	105 30	914 401.8289
				38 26	36 40	304 800.6096

State/Zone/Code			Projection	Central Meridian and Scale Factor (T.M.) or Standard Parallels (L.)	Grid Origin Longitude Latitude	Grid Origin Easting Northing
Connecticut	CT					
		0600	L	41 12 41 52	72 45 40 50	304,800.6096 152,400.3048
Delaware	DE					
		0700	T.M.	75 25 1:200,000	75 25 38 00	200,000. 0
Florida	FL					
East	E	0901	T.M.	81 00 1:17,000	81 00 24 20	200,000. 0
West	W	0902	T.M.	82 00 1:17,000	82 00 24 20	200,000. 0
North	N	0903	L	29 35 30 45	84 30 29 00	600,000. 0
Georgia	GA					
East	E	1001	T.M.	82 10 1:10,000	82 10 30 00	200,000. 0
West	W	1002	T.M.	84 10 1:10,000	84 10 30 00	700,000. 0
Hawaii	HI					
Zone 1		5101	T.M.	155 30 1:30,000	155 30 18 50	500,000. 0
Zone 2		5102	T.M.	156 40 1:30,000	156 40 20 20	500,000. 0
Zone 3		5103	T.M.	158 00 1:100,000	158 00 21 10	500,000. 0
Zone 4		5104	T.M.	159 30 1:100,000	159 30 21 50	500,000. 0
Zone 5		5105	T.M.	160 10 0	160 10 21 40	500,000. 0

State/Zone/Code			Projection	Central Meridian and Scale Factor (T.M.) or Standard Parallels (L.)	Grid Origin Longitude Latitude	Easting Northing
Idaho	ID					
East	E	1101	T.M.	112 10 1:19,000	112 10 41 40	200,000. 0
Central	C	1102	T.M.	114 00 1:19,000	114 00 41 40	500,000. 0
West	W	1103	T.M.	115 45 1:15,000	115 45 41 40	800,000. 0
Illinois	IL					
East	E	1201	T.M.	88 20 1:40,000	88 20 36 40	300,000. 0
West	W	1202	T.M.	90 10 1:17,000	90 10 36 40	700,000. 0
Indiana	IN					
East	E	1301	T.M.	85 40 1:30,000	85 40 37 30	100,000. 250,000.
West	W	1302	T.M.	87 05 1:30,000	87 05 37 30	900,000. 250,000.
Iowa	IA					
North	N	1401	L	42 04 43 16	93 30 41 30	1,500,000. 1,000,000.
South	S	1402	L	40 37 41 47	93 30 40 00	500,000. 0
Kansas	KS					
North	N	1501	L	38 43 39 47	98 00 38 20	400,000. 0
South	S	1502	L	37 16 38 34	98 30 36 40	400,000. 400,000.
Kentucky	KY					
North	N	1601	L	37 58 38 58	84 15 37 30	500,000. 0
South	S	1602	L	36 44 37 56	85 45 36 20	500,000. 500,000.

State/Zone/Code			Projection	Central Meridian and Scale Factor (T.M.) or Standard Parallels (L.)	Grid Origin Longitude Latitude	Easting Northing
Louisiana	LA					
North	N	1701	L	31 10 32 40	92 30 30 30*	1,000,000. 0
South	S	1702	L	29 18 30 42	91 20 28 30*	1,000,000. 0
Offshore	SH	1703	L	26 10 27 50	91 20 25 30*	1,000,000. 0
Maine	ME					
East	E	1801	T.M.	68 30 1:10,000	68 30 43 40*	300,000. 0
West	W	1802	T.M.	70 10 1:30,000	70 10 42 50	900,000. 0
Maryland	MD					
		1900	L	38 18 39 27	77 00 37 40*	400,000. 0
Massachusetts	MA					
Mainland	M	2001	L	41 43 42 41	71 30 41 00	200,000. 750,000.
Island	I	2002	L	41 17 41 29	70 30 41 00	500,000. 0
Michigan	MI					
North	N	2111	L	45 29 47 05	87 00 44 47	8,000,000. 0
Central	C	2112	L	44 11 45 42	84 22* 43 19	6,000,000. 0
South	S	2113	L	42 06 43 40	84 22* 41 30	4,000,000. 0
Minnesota	MN					
North	N	2201	L	47 02 48 38	93 06 46 30	800,000. 100,000.
Central	C	2202	L	45 37 47 03	94 15 45 00	800,000. 100,000.
South	S	2203	L	43 47 45 13	94 00 43 00	800,000. 100,000.

State/Zone/Code			Projection	Central Meridian and Scale Factor (T.M.) or Standard Parallels (L.)	Grid Origin Longitude Latitude	Easting Northing
Mississippi	MS					
East	E	2301	T.M.	88 50 1:20,000*	88 50 29 30*	300,000. 0
West	W	2302	T.M.	90 20 1:20,000*	90 20 29 30*	700,000. 0
Missouri	MO					
East	E	2401	T.M.	90 30 1:15,000	90 30 35 50	250,000. 0
Central	C	2402	T.M.	92 30 1:15,000	92 30 35 50	500,000. 0
West	W	2403	T.M.	94 30 1:17,000	94 30 36 10	850,000. 0
Montana	MT	2500	L	45 00* 49 00*	109 30 44 15*	600,000. 0
Nebraska	NE	2600	L	40 00* 43 00*	100 00* 39 50*	500,000. 0
Nevada	NV					
East	E	2701	T.M.	115 35 1:10,000	115 35 34 45	200,000. 8,000,000.
Central	C	2702	T.M.	116 40 1:10,000	116 40 34 45	500,000. 6,000,000.
West	W	2703	T.M.	118 35 1:10,000	118 35 34 45	800,000. 4,000,000.
New Hampshire	NH	2800	T.M.	71 40 1:30,000	71 40 42 30	300,000. 0
New Jersey (New York East)	NJ	2900	T.M.	74 30* 1:10,000*	74 30* 38 50	150,000. 0

State/Zone/Code			Projection	Central Meridian and Scale Factor (T.M.) or Standard Parallels (L.)	Grid Origin Longitude Latitude	Grid Origin Easting Northing
New Mexico	NM					
East	E	3001	T.M.	104 20 1:11,000	104 20 31 00	165,000. 0
Central	C	3002	T.M.	106 15 1:10,000	106 15 31 00	500,000. 0
West	W	3003	T.M.	107 50 1:12,000	107 50 31 00	830,000. 0
New York	NY					
East (New Jersey)	E	3101	T.M.	74 30* 1:10,000*	74 30* 38 50*	150,000. 0
Central	C	3102	T.M.	76 35 1:16,000	76 35 40 00	250,000. 0
West	W	3103	T.M.	78 35 1:16,000	78 35 40 00	350,000. 0
Long Island	L	3104	L	40 40 41 02	74 00 40 10*	300,000. 0
North Carolina	NC	3200	L	34 20 36 10	79 00 33 45	609,601.22 0
North Dakota	ND					
North	N	3301	L	47 26 48 44	100 30 47 00	600,000. 0
South	S	3302	L	46 11 47 29	100 30 45 40	600,000. 0
Ohio	OH					
North	N	3401	L	40 26 41 42	82 30 39 40	600,000. 0
South	S	3402	L	38 44 40 02	82 30 38 00	600,000. 0
Oklahoma	OK					
North	N	3501	L	35 34 36 46	98 00 35 00	600,000. 0
South	S	3502	L	33 56 35 14	98 00 33 20	600,000. 0

State/Zone/Code			Projection	Central Meridian and Scale Factor (T.M.) or Standard Parallels (L.)	Grid Origin Longitude Latitude	Grid Origin Easting Northing
Oregon	OR					
North	N	3601	L	44 20	120 30	2,500,000.
				46 00	43 40	0
South	S	3602	L	42 20	120 30	1,500,000.
				44 00	41 40	0
Pennsylvania	PA					
North	N	3701	L	40 53	77 45	600,000.
				41 57	40 10	0
South	S	3702	L	39 56	77 45	600,000.
				40 58	39 20	0
Rhode Island	RI					
		3800	T.M.	71 30	71 30	100,000.
				1:160,000	41 05	0
South Carolina	SC					
		3900	L	32 30*	81 00*	609,600.
				34 50*	31 50*	0
South Dakota	SD					
North	N	4001	L	44 25	100 00	600,000.
				45 41	43 50	0
South	S	4002	L	42 50	100 20	600,000.
				44 24	42 20	0
Tennessee	TN					
		4100	L	35 15	86 00	600,000.
				36 25	34 20*	0

State/Zone/Code			Projection	Central Meridian and Scale Factor (T.M.) or Standard Parallels (L.)	Grid Origin Longitude Latitude	Grid Origin Easting Northing
Texas	TX					
North	N	4201	L	34 39 36 11	101 30 34 00	200,000. 1,000,000.
North Central	NC	4202	L	32 08 33 58	98 30* 31 40	600,000. 2,000,000.
Central	C	4203	L	30 07 31 53	100 20 29 40	700,000. 3,000,000.
South Central	SC	4204	L	28 23 30 17	99 00 27 50	600,000. 4,000,000.
South	S	4205	L	26 10 27 50	98 30 25 40	300,000. 5,000,000.
Utah	UT					
North	N	4301	L	40 43 41 47	111 30 40 20	500,000. 1,000,000.
Central	C	4302	L	39 01 40 39	111 30 38 20	500,000. 2,000,000.
South	S	4303	L	37 13 38 21	111 30 36 40	500,000. 3,000,000.
Vermont	VT	4400	T.M.	72 30 1:28,000	72 30 42 30	500,000. 0
Virginia	VA					
North	N	4501	L	38 02 39 12	78 30 37 40	3,500,000. 2,000,000.
South	S	4502	L	36 46 37 58	78 30 36 20	3,500,000. 1,000,000.
Washington	WA					
North	N	4601	L	47 30 48 44	120 50 47 00	500,000. 0
South	S	4602	L	45 50 47 20	120 30 45 20	500,000. 0

State/Zone/Code			Projection	Central Meridian and Scale Factor (T.M.) or Standard Parallels (L.)	Grid Origin Longitude Latitude	Easting Northing
West Virginia	WV					
North	N	4701	L	39 00 40 15	79 30 38 30	600,000. 0
South	S	4702	L	37 29 38 53	81 00 37 00	600,000. 0
Wisconsin	WI					
North	N	4801	L	45 34 46 46	90 00 45 10	600,000. 0
Central	C	4802	L	44 15 45 30	90 00 43 50	600,000. 0
South	S	4803	L	42 44 44 04	90 00 42 00	600,000. 0
Wyoming	WY					
East	E	4901	T.M.	105 10 1:16,000*	105 10 40 30*	200,000. 0
East Central	EC	4902	T.M.	107 20 1:16,000*	107 20 40 30*	400,000. 100,000
West Central	WC	4903	T.M.	108 45 1:16,000*	108 45 40 30*	600,000. 0
West	W	4904	T.M.	110 05 1:16,000*	110 05 40 30*	800,000. 100,000.
Puerto Rico and Virgin Islands	PR	5200	L	18 02 18 26	66 26 17 50	200,000. 200,000.

* This represents a change from the defining constant used for the 1927 State Plane Coordinate System. All metric values assigned to the origins also are changes.

APPENDIX B. MODEL ACT FOR STATE PLANE COORDINATE SYSTEMS

An act to describe, define, and officially adopt a system of coordinates for designating the geographic position of points on the surface of the Earth within the State of ....

BE IT ENACTED BY THE LEGISLATURE OF THE STATE OF ....

Section 1. The systems of plane coordinates which have been established by the National Ocean Service/National Geodetic Survey (formerly the United States Coast and Geodetic Survey) or its successors for defining and stating the geographic positions or locations of points on the surface of the Earth within the State of .... are hereafter to be known and designated as the .... (name of State) Coordinate System of 1927 and the .... (name of State) Coordinate System of 1983.

For the purpose of the use of these systems, the State is divided into a ..... Zone and a ..... Zone (or as many zone identifications as now defined by the National Ocean Service.

The area now included in the following counties shall constitute the ..... Zone: (here enumerate the name of the counties included).

The area now included ..... (likewise for all zones).

Section 2. As established for use in the ..... Zone, the .... (name of State) Coordinate System of 1927 or the .... (name of State) Coordinate System of 1983 shall be named; and in any land description in which it is used, it shall be designated the ".......... (name of State) Coordinate System 1927 ..... Zone" or .... (name of State) Coordinate System of 1983 ..... Zone."

As established for use ..... (likewise for all zones).

Section 3. The plane coordinate values for a point on the Earth's surface, used to express the geographic position or location of such point in the appropriate zone of this system, shall consist of two distances expressed in U.S. Survey Feet and decimals of a foot when using the .... (name of State) Coordinate System of 1927 and expressed in meters and decimals of a meter when using the .... (name of State) Coordinate System of 1983. For SPCS 27, one of these distances, to be known as the "x-coordinate," shall give the position in an east-and-west direction; the other, to be known as the "y-coordinate," shall give the position in a north-and-south direction. For SPCS 83, one of the distances, to be known as the "northing" or "N", shall give the position in a north-and-south direction; the other, to be known as the "easting" or "E" shall give the position in an east-and-west direction. These coordinates shall be made to depend upon and conform to plane rectangular coordinate values for the monumented points of the North American National Geodetic Horizontal Network as published by the National Ocean Service/National Geodetic Survey (formerly the United States Coast and Geodetic Survey), or its successors, and whose plane coordinates have been computed on the systems defined in this chapter. Any such station may be used for establishing a survey connection to either .... (name of State) Coordinate System.

Section 4. For purposes of describing the location of any survey station or land boundary corner in the State of ...., it shall be considered a complete, legal, and satisfactory description of such location to give the position of said survey station or land boundary corner on the system of plane coordinates defined in this act.

Nothing contained in this act shall require a purchaser or mortgagee of real property to rely wholly on a land description, any part of which depends exclusively upon either .... (name of State) coordinate system.

Section 5. When any tract of land to be defined by a single description extends from one into the other of the above coordinate zones, the position of all points on its boundaries may be referred to either of the two zones, the zone which is used being specifically named in the description.

Section 6. (a) For purposes of more precisely defining the .... (name of State) Coordinate System of 1927, the following definition by the United States Coast and Geodetic Survey (now National Ocean Service/National Geodetic Survey) is adopted:

(For Lambert zones)

The ".......... (name of State) Coordinate System of 1927 ..... (Zone ID) Zone," is a Lambert conformal conic projection of the Clarke spheroid of 1866, having standard parallels at north latitudes ..... degrees ..... minutes and ..... degrees ..... minutes along which parallels the scale shall be exact. The origin of coordinates is at the intersection of the meridian ..... degrees ..... minutes west of Greenwich and the parallel ..... degrees ..... minutes north latitude. This origin is given the coordinates: x = ... feet and y = ... feet (as now defined).

(Use similar paragraphs for other Lambert zones on the 1927 Datum.)

(For transverse Mercator zones)

The ".......... (name of State) Coordinate System of 1927 ..... (Zone ID) zone," is a transverse Mercator projection of the Clarke spheroid of 1866, having a central meridian ..... degrees ..... minutes west of Greenwich, on which meridian the scale is set one part in ..... too small. The origin of coordinates is at the intersection of the meridian ..... degrees ..... minutes west of Greenwich and the parallel ..... degrees ..... minutes north latitude. This origin is given the coordinates: x = ... feet and y = ... feet (as now defined).

(Use similar paragraphs for other transverse Mercator zones on the 1927 Datum).

(b) For purposes of more precisely defining the .... (name of State) Coordinate System of 1983, the following definition by the National Ocean Service/National Geodetic Survey is adopted:

(For Lambert zones)

The ".......... (name of State) Coordinate System of 1983 ..... (Zone ID) Zone" is a Lambert conformal conic projection of the North American Datum of 1983, having standard parallels at north latitudes ..... degrees ..... minutes and

..... degrees ..... minutes along which parallels the scale shall be exact. The origin of coordinates is at the intersection of the meridian ..... degrees ..... minutes west of Greenwich and the parallel ..... degrees ..... minutes north latitude. This origin is given the coordinates: N = ... meters and E = ... meters.

(Use similar paragraphs for other Lambert zones on the 1983 Datum).

(For transverse Mercator zones)

The ".......... (name of State) Coordinate System of 1983 ..... (Zone ID) Zone," is a transverse Mercator projection of the North American Datum of 1983, having a central meridian ..... degrees ..... minutes west of Greenwich, on which meridian the scale is set one part in ..... too small. The origin of coordinates is at the intersection of the meridian ..... degrees ..... minutes west of Greenwich and the parallel ..... degrees ..... minutes north latitude. This origin is given the coordinates: N = ... meters and E = ... meters.

(Use similar paragraphs for other transverse Mercator zones on the 1983 Datum.)

Section 7. No coordinates based on either .... (name of State) coordinate system, purporting to define the position of a point on a land boundary, shall be presented to be recorded in any public land records or deed records unless such point is within 1 kilometer of a monumented horizontal control station established in conformity with the standards of accuracy and specifications for first- or second-order geodetic surveying as prepared and published by the Federal Geodetic Control Committee (FGCC) of the United States Department of Commerce. Standards and specifications of the FGCC or its successor in force on date of said survey shall apply. Publishing existing control stations, or the acceptance with intent to publish the newly established stations, by the Natinal Ocean Service/National Geodetic Survey will constitute evidence of adherence to the FGCC specifications. Above limitations may be modified by a duly authorized State agency to meet local conditions.

Section 8. The use of the term ".......... (name of State) Coordinate System of 1927 .... Zone" or ".......... (name of State) Coordinate System of 1983 .... Zone" on any map, report of survey, or other document shall be limited to coordinates based on the .... (name of State) coordinate system as defined in this act.

Section 9. If any provision of this act shall be declared invalid, such invalidity shall not affect any other portion of this act which can be given effect without the invalid provision; and to this end, the provisions of this act are declared severable.

Section 10. The .... (name of State) Coordinate System of 1927 shall not be used after .... (date); the .... (name of State) Coordinate System of 1983 will be the sole system after this date.

(Note: This model act was prepared in 1977. In light of GPS technology, the 1 kilometer limitation of Section 7 should be reevaluated.)

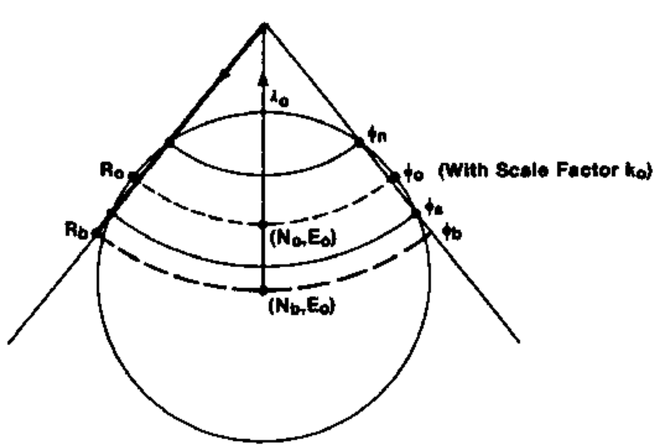
APPENDIX C.--CONSTANTS FOR THE LAMBERT PROJECTION BY THE POLYNOMIAL COEFFICIENT METHOD

Constants		Description
Bs	=	Southern standard parallel
Bn	=	Northern standard parallel
Bb	=	Latitude of grid origin
Lo	=	Longitude of the true and grid origin, the "central meridian"
Nb	=	Northing value at grid orgin "Bb"
Eo	=	Easting value at the origin "Lo"
Bo	=	Latitude of the true projection origin, the "central parallel"
SinBo	=	Sine of Bo
Rb	=	Mapping radius at Bb
Ro	=	Mapping radius at Bo
K	=	Mapping radius at the equator
No	=	Northing value at the true projection origin "Bo"
ko	=	Central parallel grid scale factor
Mo	=	Scaled radius of curvature in the meridian at "Bo"
ro	=	Geometric mean radius of curvature at Bo scaled to the grid

Bs, Bn, Bb, and Lo in degrees: minutes
Bo in decimal degrees
Linear units in meters

(See page 44 for equivalent notation of defining and derived constants used in the figure below.)

PARAMETERS OF A LAMBERT PROJECTION



AK 10 ALASKA 10 ZONE # 5010

Defining Constants

Bs	=	51:50
Bn	=	53:50
Bb	=	51:00
Lo	=	176:00
Nb	=	0.0000
Eo	=	1000000.0000

Computed Constants

Bo	=	52.8372090915
SinBo	=	0.796922389486
Rb	=	5048740.3829
Ro	=	4844318.3515
No	=	204422.0314
K	=	11499355.8664
ko	=	0.999848059991
Mo	=	6375089.0366
ro	=	6382923.

Coefficients for GP to PC

L(1)	=	111266.2938
L(2)	=	9.42762
L(3)	=	5.60521
L(4)	=	0.032566
L(5)	=	0.0008745

Coefficients for PC to GP

G(1)	=	8.987447734E-06
G(2)	=	-6.84405E-15
G(3)	=	-3.65605E-20
G(4)	=	-1.7676E-27
G(5)	=	-9.143E-36

Coefficients for Grid Scale Factor

F(1)	=	0.999848059991
F(2)	=	1.22755E-14
F(3)	=	8.34E-22

AR N ARKANSAS NORTH — ZONE # 0301

Defining Constants

Bs	=	34:56
Bn	=	36:14
Bb	=	34:20
Lo	=	92:00
Nb	=	0.0000
Eo	=	400000.0000

Computed Constants

Bo	=	35.5842283444
SinBo	=	0.581899128039
Rb	=	9062395.1981
Ro	=	8923619.0696
No	=	138776.1285
K	=	13112784.4998
ko	=	0.999935935348
Mo	=	6356634.6561
ro	=	6370786.

Coefficients for GP to PC

L(1)	=	110944.2037
L(2)	=	9.22246
L(3)	=	5.64616
L(4)	=	0.017597

Coefficients for PC to GP

G(1)	=	9.013539789E-06
G(2)	=	-6.75356E-15
G(3)	=	-3.72463E-20
G(4)	=	-9.0676E-28

Coefficients for Grid Scale Factor

F(1)	=	0.999935935348
F(2)	=	1.23195E-14
F(3)	=	4.52E-22

AR S ARKANSAS SOUTH — ZONE # 0302

Defining Constants

Bs	=	33:18
Bn	=	34:46
Bb	=	32:40
Lo	=	92:00
Nb	=	400000.0000
Eo	=	400000.0000

Computed Constants

Bo	=	34.0344093756
SinBo	=	0.559690686832
Rb	=	9604584.2290
Ro	=	9452884.9686
No	=	551699.2604
K	=	13438989.7695
ko	=	0.999918469533
Mo	=	6354902.0291
ro	=	6369591.

Coefficients for GP to PC

L(1)	=	110913.9635
L(2)	=	9.03498
L(3)	=	5.64949
L(4)	=	0.016534

Coefficients for PC to GP

G(1)	=	9.015997271E-06
G(2)	=	-6.62159E-15
G(3)	=	-3.73079E-20
G(4)	=	-8.5429E-28

Coefficients for Grid Scale Factor

F(1)	=	0.999918469533
F(2)	=	1.23240E-14
F(3)	=	4.26E-22

CA 01 CALIFORNIA 1

ZONE # 0401

Defining Constants

Bs	=	40:00
Bn	=	41:40
Bb	=	39:20
Lo	=	122:00
Nb	=	500000.0000
Eo	=	2000000.0000

Computed Constants

Bo	=	40.8351061249
SinBo	=	0.653884305400
Rb	=	7556554.6408
Ro	=	7389802.0597
No	=	666752.5811
K	=	12287826.3052
ko	=	0.999894636561
Mo	=	6362067.2798
ro	=	6374328.

Coefficients for GP to PC

L(1)	=	111039.0203
L(2)	=	9.65524
L(3)	=	5.63491
L(4)	=	0.021275

Coefficients for PC to GP

G(1)	=	9.005843038E-06
G(2)	=	-7.05240E-15
G(3)	=	-3.70393E-20
G(4)	=	-1.1142E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999894636561
F(2)	=	1.23062E-14
F(3)	=	5.47E-22

CA 02 CALIFORNIA 2

ZONE # 0402

Defining Constants

Bs	=	38:20
Bn	=	39:50
Bb	=	37:40
Lo	=	122:00
Nb	=	500000.0000
Eo	=	2000000.0000

Computed Constants

Bo	=	39.0846839219
SinBo	=	0.630468335285
Rb	=	8019788.9307
Ro	=	7862381.4027
No	=	657407.5280
K	=	12520351.6538
ko	=	0.999914672977
Mo	=	6360268.3937
ro	=	6373169.

Coefficients for GP to PC

L(1)	=	111007.6240
L(2)	=	9.54628
L(3)	=	5.63874
L(4)	=	0.019988

Coefficients for PC to GP

G(1)	=	9.008390180E-06
G(2)	=	-6.97872E-15
G(3)	=	-3.71084E-20
G(4)	=	-1.0411E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999914672977
F(2)	=	1.23106E-14
F(3)	=	5.14E-22

CA 03 CALIFORNIA 3 ZONE # 0403

Defining Constants

Bs	=	37:04
Bn	=	38:26
Bb	=	36:30
Lo	=	120:30
Nb	=	500000.0000
Eo	=	2000000.0000

Computed Constants

Bo	=	37.7510694363
SinBo	=	0.612232038295
Rb	=	8385775.1723
Ro	=	8246930.3684
No	=	638844.8039
K	=	12724574.9735
ko	=	0.999929178853
Mo	=	6358909.6841
ro	=	6372292.

Coefficients for GP to PC

L(1)	=	110983.9104
L(2)	=	9.43943
L(3)	=	5.64142
L(4)	=	0.019048

Coefficients for PC to GP

G(1)	=	9.010315015E-06
G(2)	=	-6.90503E-15
G(3)	=	-3.71614E-20
G(4)	=	-9.8819E-28

Coefficients for Grid Scale Factor

F(1)	=	0.999929178853
F(2)	=	1.23137E-14
F(3)	=	4.89E-22

CA 04 CALIFORNIA 4 ZONE # 0404

Defining Constants

Bs	=	36:00
Bn	=	37:15
Bb	=	35:20
Lo	=	119:00
Nb	=	500000.0000
Eo	=	2000000.0000

Computed Constants

Bo	=	36.6258593071
SinBo	=	0.596587149880
Rb	=	8733227.3793
Ro	=	8589806.8935
No	=	643420.4858
K	=	12916986.0281
ko	=	0.999940761703
Mo	=	6357772.8978
ro	=	6371557.

Coefficients for GP to PC

L(1)	=	110964.0696
L(2)	=	9.33334
L(3)	=	5.64410
L(4)	=	0.018382

Coefficients for PC to GP

G(1)	=	9.011926076E-06
G(2)	=	-6.83121E-15
G(3)	=	-3.72043E-20
G(4)	=	-9.4223E-28

Coefficients for Grid Scale Factor

F(1)	=	0.999940761703
F(2)	=	1.23168E-14
F(3)	=	4.70E-22

CA 05 CALIFORNIA 5

ZONE # 0405

Defining Constants

Bs	=	34:02
Bn	=	35:28
Bb	=	33:30
Lo	=	118:00
Nb	=	500000.0000
Eo	=	2000000.0000

Computed Constants

Bo	=	34.7510553142
SinBo	=	0.570011896174
Rb	=	9341756.1389
Ro	=	9202983.1099
No	=	638773.0290
K	=	13282624.8345
ko	=	0.999922127209
Mo	=	6355670.9697
ro	=	6370113.

Coefficients for GP to PC

L(1)	=	110927.3840
L(2)	=	9.12439
L(3)	=	5.64805
L(4)	=	0.017445

Coefficients for PC to GP

G(1)	=	9.014906468E-06
G(2)	=	-6.68534E-15
G(3)	=	-3.72796E-20
G(4)	=	-8.6394E-28

Coefficients for Grid Scale Factor

F(1)	=	0.999922127209
F(2)	=	1.23221E-14
F(3)	=	4.41E-22

CA 06 CALIFORNIA 6

ZONE # 0406

Defining Constants

Bs	=	32:47
Bn	=	33:53
Bb	=	32:10
Lo	=	116:15
Nb	=	500000.0000
Eo	=	2000000.0000

Computed Constants

Bo	=	33.3339229447
SinBo	=	0.549517575763
Rb	=	9836091.7896
Ro	=	9706640.0762
No	=	629451.7134
K	=	13602026.7133
ko	=	0.999954142490
Mo	=	6354407.2007
ro	=	6369336.

Coefficients for GP to PC

L(1)	=	110905.3274
L(2)	=	8.94188
L(3)	=	5.65087
L(4)	=	0.016171

Coefficients for PC to GP

G(1)	=	9.016699372E-06
G(2)	=	-6.55499E-15
G(3)	=	-3.73318E-20
G(4)	=	-8.2753E-28

Coefficients for Grid Scale Factor

F(1)	=	0.999954142490
F(2)	=	1.23251E-14
F(3)	=	4.15E-22

CO N COLORADO NORTH ZONE # 0501

Defining Constants

Bs	=	39:43
Bn	=	40:47
Bb	=	39:20
Lo	=	105:30
Nb	=	304800.6096
Eo	=	914401.8289

Computed Constants

Bo	=	40.2507114537
SinBo	=	0.646133456811
Rb	=	7646051.6244
Ro	=	7544194.6172
No	=	406657.6168
K	=	12361909.8309
ko	=	0.999956846063
Mo	=	6361817.5470
ro	=	6374293.

Coefficients for GP to PC

L(1)	=	111034.6624
L(2)	=	9.62324
L(3)	=	5.63555
L(4)	=	0.021040

Coefficients for PC to GP

G(1)	=	9.006196586E-06
G(2)	=	-7.02998E-15
G(3)	=	-3.70588E-20
G(4)	=	-1.0841E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999956846063
F(2)	=	1.23060E-14
F(3)	=	5.37E-22

CO C COLORADO CENTRAL ZONE # 0502

Defining Constants

Bs	=	38:27
Bn	=	39:45
Bb	=	37:50
Lo	=	105:30
Nb	=	304800.6096
Eo	=	914401.8289

Computed Constants

Bo	=	39.1010150117
SinBo	=	0.630689555225
Rb	=	7998699.7391
Ro	=	7857977.9317
No	=	445522.4170
K	=	12518269.8410
ko	=	0.999935909777
Mo	=	6360421.3434
ro	=	6373316.

Coefficients for GP to PC

L(1)	=	111010.2938
L(2)	=	9.54770
L(3)	=	5.63848
L(4)	=	0.019957

Coefficients for PC to GP

G(1)	=	9.008173565E-06
G(2)	=	-6.97922E-15
G(3)	=	-3.71064E-20
G(4)	=	-1.0428E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999935909777
F(2)	=	1.23099E-14
F(3)	=	5.14E-22

CO S COLORADO SOUTH ZONE # 0503

Defining Constants

Bs	=	37:14
Bn	=	38:26
Bb	=	36:40
Lo	=	105:30
Nb	=	304800.6096
Eo	=	914401.8289

Computed Constants

Bo	=	37.8341602703
SinBo	=	0.613378042371
Rb	=	8352015.4059
Ro	=	8222442.4013
No	=	434373.6143
K	=	12711335.3256
ko	=	0.999945398499
Mo	=	6359102.7444
ro	=	6372455.

Coefficients for GP to PC

L(1)	=	110987.2800
L(2)	=	9.44685
L(3)	=	5.64118
L(4)	=	0.019105

Coefficients for PC to GP

G(1)	=	9.010041469E-06
G(2)	=	-6.90983E-15
G(3)	=	-3.71567E-20
G(4)	=	-9.9134E-28

Coefficients for Grid Scale Factor

F(1)	=	0.999945398499
F(2)	=	1.23131E-14
F(3)	=	4.91E-22

CT CONNECTICUT ZONE # 0600

Defining Constants

Bs	=	41:12
Bn	=	41:52
Bb	=	40:50
Lo	=	72:45
Nb	=	152400.3048
Eo	=	304800.6096

Computed Constants

Bo	=	41.5336239347
SinBo	=	0.663059457532
Rb	=	7288924.5189
Ro	=	7211151.4122
No	=	230173.4115
K	=	12206545.8602
ko	=	0.999983140478
Mo	=	6363404.7042
ro	=	6375409.

Coefficients for GP to PC

L(1)	=	111062.3637
L(2)	=	9.68962
L(3)	=	5.63247
L(4)	=	0.021924

Coefficients for PC to GP

G(1)	=	9.003950270E-06
G(2)	=	-7.07309E-15
G(3)	=	-3.70044E-20
G(4)	=	-1.1414E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999983140478
F(2)	=	1.23017E-14
F(3)	=	5.61E-22

FL N FLORIDA NORTH

ZONE # 0903

Defining Constants

Bs	=	29:35
Bn	=	30:45
Bb	=	29:00
Lo	=	84:30
Nb	=	0.0000
Eo	=	600000.0000

Computed Constants

Bo	=	30.1672535540
SinBo	=	0.502525902671
Rb	=	11111265.2070
Ro	=	10981878.2256
No	=	129386.9814
K	=	14473086.8984
ko	=	0.999948432740
Mo	=	6351211.3497
ro	=	6367189.

Coefficients for GP to PC

L(1)	=	110849.5492
L(2)	=	8.45478
L(3)	=	5.65723
L(4)	=	0.014285

Coefficients for PC to GP

G(1)	=	9.021236462E-06
G(2)	=	-6.20727E-15
G(3)	=	-3.74501E-20
G(4)	=	-7.2421E-28

Coefficients for Grid Scale Factor

F(1)	=	0.999948432740
F(2)	=	1.23332E-14
F(3)	=	3.67E-22

IA N IOWA NORTH — ZONE # 1401

Defining Constants

Bs	=	42:04
Bn	=	43:16
Bb	=	41:30
Lo	=	93:30
Nb	=	1000000.0000
Eo	=	1500000.0000

Computed Constants

Bo	=	42.6676459541
SinBo	=	0.677744566795
Rb	=	7059740.0263
Ro	=	6930042.0331
No	=	1129697.9931
K	=	12083972.0985
ko	=	0.999945367870
Mo	=	6364426.3661
ro	=	6376011.

Coefficients for GP to PC

L(1)	=	111080.1947
L(2)	=	9.73155
L(3)	=	5.63034
L(4)	=	0.022691

Coefficients for PC to GP

G(1)	=	9.002504885E-06
G(2)	=	-7.10023E-15
G(3)	=	-3.69607E-20
G(4)	=	-1.1953E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999945367870
F(2)	=	1.22997E-14
F(3)	=	5.83E-22

IA S IOWA SOUTH — ZONE # 1402

Defining Constants

Bs	=	40:37
Bn	=	41:47
Bb	=	40:00
Lo	=	93:30
Nb	=	0.0000
Eo	=	500000.0000

Computed Constants

Bo	=	41.2008797613
SinBo	=	0.658701013169
Rb	=	7429044.5139
Ro	=	7295688.5838
No	=	133355.9301
K	=	12244655.5752
ko	=	0.999948369709
Mo	=	6362814.2760
ro	=	6374941.

Coefficients for GP to PC

L(1)	=	111052.0582
L(2)	=	9.67367
L(3)	=	5.63393
L(4)	=	0.021895

Coefficients for PC to GP

G(1)	=	9.004785763E-06
G(2)	=	-7.06375E-15
G(3)	=	-3.70197E-20
G(4)	=	-1.1221E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999948369709
F(2)	=	1.23041E-14
F(3)	=	5.56E-22

KS N KANSAS NORTH ZONE # 1501

Defining Constants

Bs	=	38:43
Bn	=	39:47
Bb	=	38:20
Lo	=	98:00
Nb	=	0.0000
Eo	=	400000.0000

Computed Constants

Bo	=	39.2506869474
SinBo	=	0.632714613092
Rb	=	7918239.4709
Ro	=	7816402.7262
No	=	101836.7447
K	=	12497179.1821
ko	=	0.999956851054
Mo	=	6360718.3963
ro	=	6373559.

Coefficients for GP to PC

L(1)	=	111015.4786
L(2)	=	9.55844
L(3)	=	5.63780
L(4)	=	0.020306

Coefficients for PC to GP

G(1)	=	9.007752883E-06
G(2)	=	-6.98626E-15
G(3)	=	-3.70994E-20
G(4)	=	-1.0424E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999956851054
F(2)	=	1.23088E-14
F(3)	=	5.18E-22

KS S KANSAS SOUTH ZONE # 1502

Defining Constants

Bs	=	37:16
Bn	=	38:34
Bb	=	36:40
Lo	=	98:30
Nb	=	400000.0000
Eo	=	400000.0000

Computed Constants

Bo	=	37.9176400609
SinBo	=	0.614528111936
Rb	=	8336559.0467
Ro	=	8197720.0530
No	=	538838.9936
K	=	12697806.8013
ko	=	0.999935918480
Mo	=	6359132.8597
ro	=	6372455.

Coefficients for GP to PC

L(1)	=	110987.8057
L(2)	=	9.45414
L(3)	=	5.64091
L(4)	=	0.018964

Coefficients for PC to GP

G(1)	=	9.009998800E-06
G(2)	=	-6.91489E-15
G(3)	=	-3.71545E-20
G(4)	=	-1.0003E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999935918480
F(2)	=	1.23130E-14
F(3)	=	4.91E-22

KY N KENTUCKY NORTH ZONE # 1601

Defining Constants

Bs	=	37:58
Bn	=	38:58
Bb	=	37:30
Lo	=	84:15
Nb	=	0.0000
Eo	=	500000.0000

Computed Constants

Bo	=	38.4672539691
SinBo	=	0.622067254038
Rb	=	8145306.4712
Ro	=	8037943.9917
No	=	107362.4795
K	=	12612341.7840
ko	=	0.999962079530
Mo	=	6359896.1212
ro	=	6373021.

Coefficients for GP to PC

L(1)	=	111001.1272
L(2)	=	9.49969
L(3)	=	5.63960
L(4)	=	0.019624

Coefficients for PC to GP

G(1)	=	9.008917501E-06
G(2)	=	-6.94594E-15
G(3)	=	-3.71303E-20
G(4)	=	-1.0140E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999962079530
F(2)	=	1.23109E-14
F(3)	=	5.03E-22

KY S KENTUCKY SOUTH ZONE # 1602

Defining Constants

Bs	=	36:44
Bn	=	37:56
Bb	=	36:20
Lo	=	85:45
Nb	=	500000.0000
Eo	=	500000.0000

Computed Constants

Bo	=	37.3341456532
SinBo	=	0.606462358287
Rb	=	8483079.4552
Ro	=	8372015.2303
No	=	611064.2249
K	=	12793783.0812
ko	=	0.999945401603
Mo	=	6358562.7562
ro	=	6372094.

Coefficients for GP to PC

L(1)	=	110977.8556
L(2)	=	9.40195
L(3)	=	5.64201
L(4)	=	0.018759

Coefficients for PC to GP

G(1)	=	9.010806634E-06
G(2)	=	-6.87874E-15
G(3)	=	-3.71775E-20
G(4)	=	-9.7208E-28

Coefficients for Grid Scale Factor

F(1)	=	0.999945401603
F(2)	=	1.23142E-14
F(3)	=	4.82E-22

LA N LOUISIANA NORTH ZONE # 1701

Defining Constants

Bs	=	31:10
Bn	=	32:40
Bb	=	30:30
Lo	=	92:30
Nb	=	0.0000
Eo	=	1000000.0000

Computed Constants

Bo	=	31.9177055892
SinBo	=	0.528700659421
Rb	=	10405759.0459
Ro	=	10248571.1515
No	=	157187.8944
K	=	13961752.4737
ko	=	0.999914740906
Mo	=	6352722.0540
ro	=	6368127.

Coefficients for GP to PC

L(1)	=	110875.9156
L(2)	=	8.73673
L(3)	=	5.65399
L(4)	=	0.015313

Coefficients for PC to GP

G(1)	=	9.019091156E-06
G(2)	=	-6.40970E-15
G(3)	=	-3.73877E-20
G(4)	=	-7.8031E-28

Coefficients for Grid Scale Factor

F(1)	=	0.999914740906
F(2)	=	1.23296E-14
F(3)	=	3.93E-22

LA S LOUISIANA SOUTH ZONE # 1702

Defining Constants

Bs	=	29:18
Bn	=	30:42
Bb	=	28:30
Lo	=	91:20
Nb	=	0.0000
Eo	=	1000000.0000

Computed Constants

Bo	=	30.0008395428
SinBo	=	0.500012689631
Rb	=	11221678.1079
Ro	=	11055318.6368
No	=	166359.4711
K	=	14525497.0844
ko	=	0.999925744553
Mo	=	6350906.2899
ro	=	6366937.

Coefficients for GP to PC

L(1)	=	110844.2246
L(2)	=	8.42633
L(3)	=	5.65782
L(4)	=	0.014018

Coefficients for PC to GP

G(1)	=	9.021669771E-06
G(2)	=	-6.18701E-15
G(3)	=	-3.74568E-20
G(4)	=	-7.2616E-28

Coefficients for Grid Scale Factor

F(1)	=	0.999925744553
F(2)	=	1.23343E-14
F(3)	=	3.64E-22

LA SH LOUISIANA OFFSHORE ZONE # 1703

Defining Constants

Bs	=	26:10
Bn	=	27:50
Bb	=	25:30
Lo	=	91:20
Nb	=	0.0000
Eo	=	1000000.0000

Computed Constants

Bo	=	27.0010512832
SinBo	=	0.454006848165
Rb	=	12690863.7281
Ro	=	12524558.0674
No	=	166305.6607
K	=	15621596.5270
ko	=	0.999894794114
Mo	=	6347907.1071
ro	=	6364866.

Coefficients for GP to PC

L(1)	=	110791.8786
L(2)	=	7.86506
L(3)	=	5.66365
L(4)	=	0.012775

Coefficients for PC to GP

G(1)	=	9.025932193E-06
G(2)	=	-5.78388E-15
G(3)	=	-3.75631E-20
G(4)	=	-6.1764E-28

Coefficients for Grid Scale Factor

F(1)	=	0.999894794114
F(2)	=	1.23421E-14
F(3)	=	3.23E-22

MD MARYLAND ZONE # 1900

Defining Constants

Bs	=	38:18
Bn	=	39:27
Bb	=	37:40
Lo	=	77:00
Nb	=	0.0000
Eo	=	400000.0000

Computed Constants

Bo	=	38.8757880051
SinBo	=	0.627634132356
Rb	=	8055622.7373
Ro	=	7921405.1556
No	=	134217.5816
K	=	12551136.6396
ko	=	0.999949847842
Mo	=	6360263.7936
ro	=	6373240.

Coefficients for GP to PC

L(1)	=	111007.5442
L(2)	=	9.53130
L(3)	=	5.63889
L(4)	=	0.019736

Coefficients for PC to GP

G(1)	=	9.008396710E-06
G(2)	=	-6.96769E-15
G(3)	=	-3.71144E-20
G(4)	=	-1.0352E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999949847842
F(2)	=	1.23102E-14
F(3)	=	5.09E-22

MA M MASS MAINLAND

ZONE # 2001

Defining Constants

Bs	=	41:43
Bn	=	42:41
Bb	=	41:00
Lo	=	71:30
Nb	=	750000.0000
Eo	=	200000.0000

Computed Constants

Bo	=	42.2006252872
SinBo	=	0.671728673921
Rb	=	7177701.7404
Ro	=	7044348.7021
No	=	883353.0384
K	=	12132804.7336
ko	=	0.999964550086
Mo	=	6364028.0516
ro	=	6375786.

Coefficients for GP to PC

L(1)	=	111073.2431
L(2)	=	9.71650
L(3)	=	5.63098
L(4)	=	0.021759

Coefficients for PC to GP

G(1)	=	9.003068344E-06
G(2)	=	-7.09026E-15
G(3)	=	-3.69789E-20
G(4)	=	-1.1855E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999964550086
F(2)	=	1.23003E-14
F(3)	=	5.69E-22

MA I MASS ISLAND

ZONE # 2002

Defining Constants

Bs	=	41:17
Bn	=	41:29
Bb	=	41:00
Lo	=	70:30
Nb	=	0.0000
Eo	=	500000.0000

Computed Constants

Bo	=	41.3833593510
SinBo	=	0.661093979591
Rb	=	7291990.4498
Ro	=	7249415.2230
No	=	42575.2267
K	=	12223979.6222
ko	=	0.999998482670
Mo	=	6363335.5426
ro	=	6375395.

Coefficients for GP to PC

L(1)	=	111061.1569
L(2)	=	9.68480
L(3)	=	5.62745

Coefficients for PC to GP

G(1)	=	9.004048113E-06
G(2)	=	-7.06961E-15
G(3)	=	-3.69799E-20

Coefficients for Grid Scale Factor

F(1)	=	0.999998482670
F(2)	=	1.23015E-14
F(3)	=	5.56E-22

MI N MICHIGAN NORTH ZONE # 2111

Defining Constants

Bs	=	45:29
Bn	=	47:05
Bb	=	44:47
Lo	=	87:00
Nb	=	0.0000
Eo	=	8000000.0000

Computed Constants

Bo	=	46.2853056176
SinBo	=	0.722789934733
Rb	=	6275243.8434
Ro	=	6108308.6036
No	=	166935.2398
K	=	11779843.7720
ko	=	0.999902834466
Mo	=	6368201.9117
ro	=	6378442.

Coefficients for GP to PC

L(1)	=	111146.0908
L(2)	=	9.76397
L(3)	=	5.62053
L(4)	=	0.025777
L(5)	=	0.0007325

Coefficients for PC to GP

G(1)	=	8.997167538E-06
G(2)	=	-7.11123E-15
G(3)	=	-3.68190E-20
G(4)	=	-1.3725E-27
G(5)	=	8.019E-35

Coefficients for Grid Scale Factor

F(1)	=	0.999902834466
F(2)	=	1.22919E-14
F(3)	=	6.70E-22

MI C MICHIGAN CENTRAL ZONE # 2112

Defining Constants

Bs	=	44:11
Bn	=	45:42
Bb	=	43:19
Lo	=	84:22
Nb	=	0.0000
Eo	=	6000000.0000

Computed Constants

Bo	=	44.9433587575
SinBo	=	0.706407406862
Rb	=	6581660.2321
Ro	=	6400902.4399
No	=	180757.7922
K	=	11878338.0174
ko	=	0.999912706253
Mo	=	6366762.5687
ro	=	6377502.

Coefficients for GP to PC

L(1)	=	111120.9691
L(2)	=	9.77091
L(3)	=	5.62494
L(4)	=	0.023788

Coefficients for PC to GP

G(1)	=	8.999201531E-06
G(2)	=	-7.12032E-15
G(3)	=	-3.68711E-20
G(4)	=	-1.3161E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999912706253
F(2)	=	1.22939E-14
F(3)	=	6.25E-22

MI S MICHIGAN SOUTH ZONE # 2113

Defining Constants

Bs	=	42:06
Bn	=	43:40
Bb	=	41:30
Lo	=	84:22
Nb	=	0.0000
Eo	=	4000000.0000

Computed Constants

Bo	=	42.8850151357
SinBo	=	0.680529259912
Rb	=	7031167.2907
Ro	=	6877323.4058
No	=	153843.8848
K	=	12061671.8385
ko	=	0.999906878420
Mo	=	6364423.8607
ro	=	6375928.

Coefficients for GP to PC

L(1)	=	111080.1507
L(2)	=	9.73761
L(3)	=	5.63002
L(4)	=	0.022802

Coefficients for PC to GP

G(1)	=	9.002508421E-06
G(2)	=	-7.10459E-15
G(3)	=	-3.69552E-20
G(4)	=	-1.2067E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999906878420
F(2)	=	1.23000E-14
F(3)	=	5.87E-22

MN N MINNESOTA NORTH ZONE # 2201

Defining Constants

Bs	=	47:02
Bn	=	48:38
Bb	=	46:30
Lo	=	93:06
Nb	=	100000.0000
Eo	=	800000.0000

Computed Constants

Bo	=	47.8354141053
SinBo	=	0.741219640371
Rb	=	5934713.4739
Ro	=	5786251.1143
No	=	248462.3596
K	=	11685145.4281
ko	=	0.999902816593
Mo	=	6369933.6096
ro	=	6379598.

Coefficients for GP to PC

L(1)	=	111176.3136
L(2)	=	9.72967
L(3)	=	5.61897
L(4)	=	0.027729

Coefficients for PC to GP

G(1)	=	8.994721600E-06
G(2)	=	-7.08107E-15
G(3)	=	-3.67535E-20
G(4)	=	-1.4515E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999902816593
F(2)	=	1.22867E-14
F(3)	=	7.04E-22

MN C MINNESOTA CENTRAL ZONE # 2202

Defining Constants

Bs	=	45:37
Bn	=	47:03
Bb	=	45:00
Lo	=	94:15
Nb	=	100000.0000
Eo	=	800000.0000

Computed Constants

Bo	=	46.3349188114
SinBo	=	0.723388068681
Rb	=	6246233.9437
Ro	=	6097862.9029
No	=	248371.0408
K	=	11776732.4900
ko	=	0.999922022624
Mo	=	6368379.6277
ro	=	6378602.

Coefficients for GP to PC

L(1)	=	111149.1920
L(2)	=	9.76378
L(3)	=	5.62196
L(4)	=	0.025568

Coefficients for PC to GP

G(1)	=	8.996916454E-06
G(2)	=	-7.11028E-15
G(3)	=	-3.68130E-20
G(4)	=	-1.3780E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999922022624
F(2)	=	1.22899E-14
F(3)	=	6.61E-22

MN S MINNESOTA SOUTH ZONE # 2203

Defining Constants

Bs	=	43:47
Bn	=	45:13
Bb	=	43:00
Lo	=	94:00
Nb	=	100000.0000
Eo	=	800000.0000

Computed Constants

Bo	=	44.5014884140
SinBo	=	0.700927792688
Rb	=	6667126.8494
Ro	=	6500294.5043
No	=	266832.3451
K	=	11914387.7514
ko	=	0.999922039553
Mo	=	6366327.3480
ro	=	6377231.

Coefficients for GP to PC

L(1)	=	111113.3724
L(2)	=	9.76742
L(3)	=	5.62679
L(4)	=	0.024208

Coefficients for PC to GP

G(1)	=	8.999816728E-06
G(2)	=	-7.12002E-15
G(3)	=	-3.68868E-20
G(4)	=	-1.2821E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999922039553
F(2)	=	1.22957E-14
F(3)	=	6.22E-22

MT MONTANA ZONE # 2500

Defining Constants

Bs	=	45:00
Bn	=	49:00
Bb	=	44:15
Lo	=	109:30
Nb	=	0.0000
Eo	=	600000.0000

Computed Constants

Bo	=	47.0126454240
SinBo	=	0.731504203765
Rb	=	6259119.5655
Ro	=	5952137.2048
No	=	306982.3608
K	=	11726990.9793
ko	=	0.999392636277
Mo	=	6365765.4708
ro	=	6375730.

Coefficients for GP to PC

L(1)	=	111103.5668
L(2)	=	9.74667
L(3)	=	5.61611
L(4)	=	0.026479
L(5)	=	0.0007162

Coefficients for PC to GP

G(1)	=	9.000611125E-06
G(2)	=	-7.10687E-15
G(3)	=	-3.68456E-20
G(4)	=	-1.4141E-27
G(5)	=	7.257E-35

Coefficients for Grid Scale Factor

F(1)	=	0.999392636277
F(2)	=	1.23001E-14
F(3)	=	6.75E-22

NE NEBRASKA ZONE # 2600

Defining Constants

Bs	=	40:00
Bn	=	43:00
Bb	=	39:50
Lo	=	100:00
Nb	=	0.0000
Eo	=	500000.0000

Computed Constants

Bo	=	41.5058803333
SinBo	=	0.662696910933
Rb	=	7401530.8340
Ro	=	7215835.9104
No	=	185694.9237
K	=	12205748.1618
ko	=	0.999658595062
Mo	=	6361308.6623
ro	=	6373319.

Coefficients for GP to PC

L(1)	=	111025.7809
L(2)	=	9.68528
L(3)	=	5.63025
L(4)	=	0.021792
L(5)	=	0.0006372

Coefficients for PC to GP

G(1)	=	9.006917060E-06
G(2)	=	-7.07688E-15
G(3)	=	-3.70427E-20
G(4)	=	-1.1443E-27
G(5)	=	1.251E-34

Coefficients for Grid Scale Factor

F(1)	=	0.999658595062
F(2)	=	1.23079E-14
F(3)	=	5.62E-22

NY L NEW YORK LONG ISLAND ZONE # 3104

Defining Constants

Bs	=	40:40
Bn	=	41:02
Bb	=	40:10
Lo	=	74:00
Nb	=	0.0000
Eo	=	300000.0000

Computed Constants

Bo	=	40.8500858421
SinBo	=	0.654082091204
Rb	=	7462536.3011
Ro	=	7386645.0143
No	=	75891.2868
K	=	12287232.6151
ko	=	0.999994900400
Mo	=	6362721.8083
ro	=	6374978.

Coefficients for GP to PC

L(1)	=	111050.4466
L(2)	=	9.66003
L(3)	=	5.62096

Coefficients for PC to GP

G(1)	=	9.004916524E-06
G(2)	=	-7.05345E-15
G(3)	=	-3.69553E-20

Coefficients for Grid Scale Factor

F(1)	=	0.999994900400
F(2)	=	1.23032E-14
F(3)	=	5.44E-22

NC NORTH CAROLINA ZONE # 3200

Defining Constants

Bs	=	34:20
Bn	=	36:10
Bb	=	33:45
Lo	=	79:00
Nb	=	0.0000
Eo	=	609601.2199

Computed Constants

Bo	=	35.2517586002
SinBo	=	0.577170255241
Rb	=	9199785.5932
Ro	=	9033195.6010
No	=	166589.9922
K	=	13178320.6222
ko	=	0.999872591882
Mo	=	6355881.3611
ro	=	6370148.

Coefficients for GP to PC

L(1)	=	110931.0558
L(2)	=	9.18403
L(3)	=	5.64691
L(4)	=	0.017289

Coefficients for PC to GP

G(1)	=	9.014608051E-06
G(2)	=	-6.72767E-15
G(3)	=	-3.72650E-20
G(4)	=	-8.9805E-28

Coefficients for Grid Scale Factor

F(1)	=	0.999872591882
F(2)	=	1.23215E-14
F(3)	=	4.46E-22

ND N NORTH DAKOTA NORTH ZONE # 3301

Defining Constants

Bs	=	47:26
Bn	=	48:44
Bb	=	47:00
Lo	=	100:30
Nb	=	0.0000
Eo	=	600000.0000

Computed Constants

Bo	=	48.0847188415
SinBo	=	0.744133404458
Rb	=	5856720.4592
Ro	=	5736120.4804
No	=	120599.9788
K	=	11672088.5605
ko	=	0.999935842096
Mo	=	6370421.8763
ro	=	6379995.

Coefficients for GP to PC

L(1)	=	111184.8361
L(2)	=	9.72243
L(3)	=	5.61786
L(4)	=	0.027700

Coefficients for PC to GP

G(1)	=	8.994032200E-06
G(2)	=	-7.07375E-15
G(3)	=	-3.67405E-20
G(4)	=	-1.4677E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999935842096
F(2)	=	1.22846E-14
F(3)	=	7.08E-22

ND S NORTH DAKOTA SOUTH ZONE # 3302

Defining Constants

Bs	=	46:11
Bn	=	47:29
Bb	=	45:40
Lo	=	100:30
Nb	=	0.0000
Eo	=	600000.0000

Computed Constants

Bo	=	46.8346602257
SinBo	=	0.729382600558
Rb	=	6122339.5950
Ro	=	5992509.2670
No	=	129830.3280
K	=	11744429.2917
ko	=	0.999935851558
Mo	=	6369026.6161
ro	=	6379063.

Coefficients for GP to PC

L(1)	=	111160.4842
L(2)	=	9.75568
L(3)	=	5.62076
L(4)	=	0.026264

Coefficients for PC to GP

G(1)	=	8.996002517E-06
G(2)	=	-7.10242E-15
G(3)	=	-3.67913E-20
G(4)	=	-1.4014E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999935851558
F(2)	=	1.22881E-14
F(3)	=	6.75E-22

OH N OHIO NORTH ZONE # 3401

Defining Constants

Bs	=	40:26
Bn	=	41:42
Bb	=	39:40
Lo	=	82:30
Nb	=	0.0000
Eo	=	600000.0000

Computed Constants

Bo	=	41.0676989228
SinBo	=	0.656950312341
Rb	=	7485451.5983
Ro	=	7329872.6916
No	=	155578.9068
K	=	12260321.3670
ko	=	0.999939140422
Mo	=	6362607.9595
ro	=	6374783.

Coefficients for GP to PC

L(1)	=	111048.4575
L(2)	=	9.66786
L(3)	=	5.63397
L(4)	=	0.021060

Coefficients for PC to GP

G(1)	=	9.005077760E-06
G(2)	=	-7.05943E-15
G(3)	=	-3.70266E-20
G(4)	=	-1.1329E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999939140422
F(2)	=	1.23043E-14
F(3)	=	5.48E-22

OH S OHIO SOUTH ZONE # 3402

Defining Constants

Bs	=	38:44
Bn	=	40:02
Bb	=	38:00
Lo	=	82:30
Nb	=	0.0000
Eo	=	600000.0000

Computed Constants

Bo	=	39.3843585118
SinBo	=	0.634519536788
Rb	=	7932869.0374
Ro	=	7779186.9467
No	=	153682.0906
K	=	12478096.2534
ko	=	0.999935907680
Mo	=	6360731.6589
ro	=	6373523.

Coefficients for GP to PC

L(1)	=	111015.7097
L(2)	=	9.56783
L(3)	=	5.63800
L(4)	=	0.020081

Coefficients for PC to GP

G(1)	=	9.007734087E-06
G(2)	=	-6.99281E-15
G(3)	=	-3.70945E-20
G(4)	=	-1.0564E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999935907680
F(2)	=	1.23093E-14
F(3)	=	5.18E-22

OK N OKLAHOMA NORTH ZONE # 3501

Defining Constants

Bs	=	35:34
Bn	=	36:46
Bb	=	35:00
Lo	=	98:00
Nb	=	0.0000
Eo	=	600000.0000

Computed Constants

Bo	=	36.1674456022
SinBo	=	0.590147072450
Rb	=	8864259.9258
Ro	=	8734728.4814
No	=	129531.4444
K	=	13001040.2487
ko	=	0.999945408786
Mo	=	6357313.3855
ro	=	6371260.

Coefficients for GP to PC

L(1)	=	110956.0498
L(2)	=	9.28617
L(3)	=	5.64482
L(4)	=	0.017979

Coefficients for PC to GP

G(1)	=	9.012577476E-06
G(2)	=	-6.79804E-15
G(3)	=	-3.72229E-20
G(4)	=	-9.2812E-28

Coefficients for Grid Scale Factor

F(1)	=	0.999945408786
F(2)	=	1.23177E-14
F(3)	=	4.62E-22

OK S OKLAHOMA SOUTH ZONE # 3502

Defining Constants

Bs	=	33:56
Bn	=	35:14
Bb	=	33:20
Lo	=	98:00
Nb	=	0.0000
Eo	=	600000.0000

Computed Constants

Bo	=	34.5841961094
SinBo	=	0.567616677812
Rb	=	9399243.5141
Ro	=	9260493.6985
No	=	138749.8157
K	=	13318364.4294
ko	=	0.999935942436
Mo	=	6355584.5004
ro	=	6370084.

Coefficients for GP to PC

L(1)	=	110925.8751
L(2)	=	9.10472
L(3)	=	5.64812
L(4)	=	0.016766

Coefficients for PC to GP

G(1)	=	9.015029132E-06
G(2)	=	-6.67047E-15
G(3)	=	-3.72859E-20
G(4)	=	-8.7733E-28

Coefficients for Grid Scale Factor

F(1)	=	0.999935942436
F(2)	=	1.23220E-14
F(3)	=	4.34E-22

OR N OREGON NORTH ZONE # 3601

Defining Constants

Bs	=	44:20
Bn	=	46:00
Bb	=	43:40
Lo	=	120:30
Nb	=	0.0000
Eo	=	2500000.0000

Computed Constants

Bo	=	45.1687259619
SinBo	=	0.709186016884
Rb	=	6517624.6963
Ro	=	6350713.9300
No	=	166910.7663
K	=	11860484.1452
ko	=	0.999894582577
Mo	=	6366899.4862
ro	=	6377555.

Coefficients for GP to PC

L(1)	=	111123.3583
L(2)	=	9.77067
L(3)	=	5.62487
L(4)	=	0.024544

Coefficients for PC to GP

G(1)	=	8.999007999E-06
G(2)	=	-7.12020E-15
G(3)	=	-3.68630E-20
G(4)	=	-1.3188E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999894582577
F(2)	=	1.22939E-14
F(3)	=	6.35E-22

OR S OREGON SOUTH ZONE # 3602

Defining Constants

Bs	=	42:20
Bn	=	44:00
Bb	=	41:40
Lo	=	120:30
Nb	=	0.0000
Eo	=	1500000.0000

Computed Constants

Bo	=	43.1685887665
SinBo	=	0.684147361010
Rb	=	6976289.2382
Ro	=	6809452.2816
No	=	166836.9566
K	=	12033772.6984
ko	=	0.999894607592
Mo	=	6364662.2994
ro	=	6376061.

Coefficients for GP to PC

L(1)	=	111084.3129
L(2)	=	9.74486
L(3)	=	5.62774
L(4)	=	0.023107
L(5)	=	0.0006671

Coefficients for PC to GP

G(1)	=	9.002171179E-06
G(2)	=	-7.10916E-15
G(3)	=	-3.69482E-20
G(4)	=	-1.2185E-27
G(5)	=	1.111E-34

Coefficients for Grid Scale Factor

F(1)	=	0.999894607592
F(2)	=	1.23002E-14
F(3)	=	5.97E-22

PA N PENNSYLVANIA NORTH ZONE # 3701

Defining Constants

Bs	=	40:53
Bn	=	41:57
Bb	=	40:10
Lo	=	77:45
Nb	=	0.0000
Eo	=	600000.0000

Computed Constants

Bo	=	41.4174076242
SinBo	=	0.661539733811
Rb	=	7379348.3668
Ro	=	7240448.7701
No	=	138899.5967
K	=	12219540.4665
ko	=	0.999956840202
Mo	=	6363108.3386
ro	=	6375155.

Coefficients for GP to PC

L(1)	=	111057.1908
L(2)	=	9.68441
L(3)	=	5.63320
L(4)	=	0.021500

Coefficients for PC to GP

G(1)	=	9.004369625E-06
G(2)	=	-7.07004E-15
G(3)	=	-3.70106E-20
G(4)	=	-1.1439E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999956840202
F(2)	=	1.23030E-14
F(3)	=	5.56E-22

PA S PENNSYLVANIA SOUTH ZONE # 3702

Defining Constants

Bs	=	39:56
Bn	=	40:58
Bb	=	39:20
Lo	=	77:45
Nb	=	0.0000
Eo	=	600000.0000

Computed Constants

Bo	=	40.4506723597
SinBo	=	0.648793151619
Rb	=	7615193.7581
Ro	=	7491129.9649
No	=	124063.7931
K	=	12336392.1867
ko	=	0.999959500101
Mo	=	6362055.0747
ro	=	6374457.

Coefficients for GP to PC

L(1)	=	111038.8080
L(2)	=	9.63502
L(3)	=	5.63528
L(4)	=	0.020898

Coefficients for PC to GP

G(1)	=	9.005860337E-06
G(2)	=	-7.03760E-15
G(3)	=	-3.70500E-20
G(4)	=	-1.0995E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999959500101
F(2)	=	1.23055E-14
F(3)	=	5.38E-22

SC SOUTH CAROLINA ZONE # 3900

Defining Constants

Bs	=	32:30
Bn	=	34:50
Bb	=	31:50
Lo	=	81:00
Nb	=	0.0000
Eo	=	609600.0000

Computed Constants

Bo	=	33.6693534716
SinBo	=	0.554399350127
Rb	=	9786198.7935
Ro	=	9582591.5259
No	=	203607.2676
K	=	13520786.8598
ko	=	0.999793656965
Mo	=	6353731.8876
ro	=	6368544.

Coefficients for GP to PC

L(1)	=	110893.5412
L(2)	=	8.98578
L(3)	=	5.64832
L(4)	=	0.016390
L(5)	=	0.0005454

Coefficients for PC to GP

G(1)	=	9.017657737E-06
G(2)	=	-6.58928E-15
G(3)	=	-3.73407E-20
G(4)	=	-8.3932E-28
G(5)	=	1.748E-34

Coefficients for Grid Scale Factor

F(1)	=	0.999793656965
F(2)	=	1.23274E-14
F(3)	=	4.21E-22

SD N SOUTH DAKOTA NORTH ZONE # 4001

Defining Constants

Bs	=	44:25
Bn	=	45:41
Bb	=	43:50
Lo	=	100:00
Nb	=	0.0000
Eo	=	600000.0000

Computed Constants

Bo	=	45.0511846016
SinBo	=	0.707738185595
Rb	=	6512395.0582
Ro	=	6377064.4907
No	=	135330.5675
K	=	11870154.6246
ko	=	0.999939111894
Mo	=	6367051.4253
ro	=	6377751.

Coefficients for GP to PC

L(1)	=	111126.0105
L(2)	=	9.77054
L(3)	=	5.62503
L(4)	=	0.024765

Coefficients for PC to GP

G(1)	=	8.998793259E-06
G(2)	=	-7.11994E-15
G(3)	=	-3.68635E-20
G(4)	=	-1.3078E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999939111894
F(2)	=	1.22932E-14
F(3)	=	6.35E-22

SD S SOUTH DAKOTA SOUTH ZONE # 4002

Defining Constants

Bs	=	42:50
Bn	=	44:24
Bb	=	42:20
Lo	=	100:20
Nb	=	0.0000
Eo	=	600000.0000

Computed Constants

Bo	=	43.6183915831
SinBo	=	0.689851962794
Rb	=	6846221.9383
Ro	=	6703463.3332
No	=	142758.6051
K	=	11991572.8665
ko	=	0.999906870345
Mo	=	6365242.9133
ro	=	6376475.

Coefficients for GP to PC

L(1)	=	111094.4459
L(2)	=	9.75472
L(3)	=	5.62829
L(4)	=	0.023597

Coefficients for PC to GP

G(1)	=	9.001350018E-06
G(2)	=	-7.11454E-15
G(3)	=	-3.69253E-20
G(4)	=	-1.2373E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999906870345
F(2)	=	1.22979E-14
F(3)	=	6.04E-22

TN TENNESSEE ZONE # 4100

Defining Constants

Bs	=	35:15
Bn	=	36:25
Bb	=	34:20
Lo	=	86:00
Nb	=	0.0000
Eo	=	600000.0000

Computed Constants

Bo	=	35.8340607459
SinBo	=	0.585439726459
Rb	=	9008631.3113
Ro	=	8842127.1422
No	=	166504.1691
K	=	13064326.2967
ko	=	0.999948401424
Mo	=	6356978.3321
ro	=	6371042.

Coefficients for GP to PC

L(1)	=	110950.2019
L(2)	=	9.25072
L(3)	=	5.64572
L(4)	=	0.017374

Coefficients for PC to GP

G(1)	=	9.013052490E-06
G(2)	=	-6.77268E-15
G(3)	=	-3.72351E-20
G(4)	=	-9.2828E-28

Coefficients for Grid Scale Factor

F(1)	=	0.999948401424
F(2)	=	1.23188E-14
F(3)	=	4.54E-22

TX N TEXAS NORTH ZONE # 4201

Defining Constants

```
Bs    =         34:39
Bn    =         36:11
Bb    =         34:00
Lo    =        101:30
Nb    =   1000000.0000
Eo    =    200000.0000
```

Computed Constants

```
Bo    =  35.4179042823
SinBo= 0.579535862261
Rb    =   9135570.8896
Ro    =   8978273.3931
No    =   1157297.4965
K     =  13145417.7356
ko    = 0.999910875663
Mo    =   6356299.7601
ro    =    6370509.
```

Coefficients for GP to PC

```
L(1)  = 110938.3584
L(2)  =      9.20339
L(3)  =      5.64670
L(4)  =      0.017491
```

Coefficients for PC to GP

```
G(1)   =  9.014014675E-06
G(2)   = -6.74066E-15
G(3)   = -3.72545E-20
G(4)   = -9.0079E-28
```

Coefficients for Grid Scale Factor

```
F(1)  = 0.999910875663
F(2)  = 1.23205E-14
F(3)  = 4.49E-22
```

TX NC TEXAS NORTH CENTRAL ZONE # 4202

Defining Constants

```
Bs    =         32:08
Bn    =         33:58
Bb    =         31:40
Lo    =         98:30
Nb    =   2000000.0000
Eo    =    600000.0000
```

Computed Constants

```
Bo    =  33.0516205542
SinBo= 0.545394412971
Rb    =   9964225.7538
Ro    =   9810648.6091
No    =   2153577.1446
K     =  13669256.3042
ko    = 0.999872622628
Mo    =   6353600.5552
ro    =    6368624.
```

Coefficients for GP to PC

```
L(1)  = 110891.2484
L(2)  =      8.90195
L(3)  =      5.65144
L(4)  =      0.016070
```

Coefficients for PC to GP

```
G(1)   =  9.017844103E-06
G(2)   = -6.52831E-15
G(3)   = -3.73499E-20
G(4)   = -8.1560E-28
```

Coefficients for Grid Scale Factor

```
F(1)  = 0.999872622628
F(2)  = 1.23272E-14
F(3)  = 4.11E-22
```

TX C TEXAS CENTRAL ZONE # 4203

Defining Constants

Bs	=	30:07
Bn	=	31:53
Bb	=	29:40
Lo	=	100:20
Nb	=	3000000.0000
Eo	=	700000.0000

Computed Constants

Bo	=	31.0013908377
SinBo	=	0.515058882235
Rb	=	10770561.1034
Ro	=	10622600.3250
No	=	3147960.7784
K	=	14219009.8813
ko	=	0.999881743629
Mo	=	6351602.5419
ro	=	6367308.

Coefficients for GP to PC

L(1)	=	110856.3764
L(2)	=	8.59215
L(3)	=	5.65568
L(4)	=	0.015131

Coefficients for PC to GP

G(1)	=	9.020680826E-06
G(2)	=	-6.30758E-15
G(3)	=	-3.74251E-20
G(4)	=	-7.3651E-28

Coefficients for Grid Scale Factor

F(1)	=	0.999881743629
F(2)	=	1.23324E-14
F(3)	=	3.81E-22

TX SC TEXAS SOUTH CENTRAL ZONE # 4204

Defining Constants

Bs	=	28:23
Bn	=	30:17
Bb	=	27:50
Lo	=	99:00
Nb	=	4000000.0000
Eo	=	600000.0000

Computed Constants

Bo	=	29.3348388416
SinBo	=	0.489912625143
Rb	=	11523512.5584
Ro	=	11357106.1291
No	=	4166406.4293
K	=	14743501.7826
ko	=	0.999863243591
Mo	=	6349870.7242
ro	=	6366112.

Coefficients for GP to PC

L(1)	=	110826.1504
L(2)	=	8.30885
L(3)	=	5.65894
L(4)	=	0.013811

Coefficients for PC to GP

G(1)	=	9.023141055E-06
G(2)	=	-6.10399E-15
G(3)	=	-3.74868E-20
G(4)	=	-6.9867E-28

Coefficients for Grid Scale Factor

F(1)	=	0.999863243591
F(2)	=	1.23368E-14
F(3)	=	3.55E-22

TX S TEXAS SOUTH ZONE # 4205

Defining Constants

Bs	=	26:10
Bn	=	27:50
Bb	=	25:40
Lo	=	98:30
Nb	=	5000000.0000
Eo	=	300000.0000

Computed Constants

Bo	=	27.0010512832
SinBo	=	0.454006848165
Rb	=	12672396.4573
Ro	=	12524558.0674
No	=	5147838.3899
K	=	15621596.5270
ko	=	0.999894794114
Mo	=	6347907.1071
ro	=	6364866.

Coefficients for GP to PC

L(1)	=	110791.8791
L(2)	=	7.86549
L(3)	=	5.66316
L(4)	=	0.012519

Coefficients for PC to GP

G(1)	=	9.025932226E-06
G(2)	=	-5.78365E-15
G(3)	=	-3.75655E-20
G(4)	=	-6.2913E-28

Coefficients for Grid Scale Factor

F(1)	=	0.999894794114
F(2)	=	1.23417E-14
F(3)	=	3.21E-22

UT N UTAH NORTH ZONE # 4301

Defining Constants

```
Bs    =        40:43
Bn    =        41:47
Bb    =        40:20
Lo    =       111:30
Nb    =  1000000.0000
Eo    =   500000.0000
```

Computed Constants

```
Bo    =  41.2507366798
SinBo= 0.659355481817
Rb    =   7384852.1452
Ro    =   7282974.6766
No    =   1101877.4686
K     =  12238904.9538
ko    = 0.999956841041
Mo    =   6362923.4572
ro    =    6375032.
```

Coefficients for GP to PC

```
L(1)  = 111053.9642
L(2)  =       9.67638
L(3)  =       5.63329
L(4)  =       0.021795
```

Coefficients for PC to GP

```
G(1)   =  9.004631262E-06
G(2)   = -7.06511E-15
G(3)   = -3.70181E-20
G(4)   = -1.1272E-27
```

Coefficients for Grid Scale Factor

```
F(1)  = 0.999956841041
F(2)  = 1.23032E-14
F(3)  = 5.56E-22
```

UT C UTAH CENTRAL ZONE # 4302

Defining Constants

```
Bs    =        39:01
Bn    =        40:39
Bb    =        38:20
Lo    =       111:30
Nb    =  2000000.0000
Eo    =   500000.0000
```

Computed Constants

```
Bo    =  39.8349774741
SinBo= 0.640578595825
Rb    =   7822240.6085
Ro    =   7655530.3911
No    =   2166710.2174
K     =  12415886.8989
ko    = 0.999898820765
Mo    =   6360990.5575
ro    =    6373617.
```

Coefficients for GP to PC

```
L(1)  = 111020.2282
L(2)  =       9.59755
L(3)  =       5.63694
L(4)  =       0.020325
```

Coefficients for PC to GP

```
G(1)   =  9.007367459E-06
G(2)   = -7.01354E-15
G(3)   = -3.70800E-20
G(4)   = -1.0771E-27
```

Coefficients for Grid Scale Factor

```
F(1)  = 0.999898820765
F(2)  = 1.23087E-14
F(3)  = 5.26E-22
```

UT S UTAH SOUTH ZONE # 4303

Defining Constants

```
Bs    =        37:13
Bn    =        38:21
Bb    =        36:40
Lo    =       111:30
Nb    =   3000000.0000
Eo    =    500000.0000
```

Computed Constants

```
Bo    =  37.7840696241
SinBo= 0.612687337234
Rb    =   8361336.2313
Ro    =   8237322.9910
No    =   3124013.2403
K     =  12719504.1729
ko    = 0.999951297078
Mo    =   6359086.0437
ro    =    6372457.
```

Coefficients for GP to PC

```
L(1)  = 110986.9886
L(2)  =      9.44259
L(3)  =      5.64118
L(4)  =      0.018991
```

Coefficients for PC to GP

```
G(1)   =  9.010065135E-06
G(2)   = -6.90671E-15
G(3)   = -3.71585E-20
G(4)   = -9.9163E-28
```

Coefficients for Grid Scale Factor

```
F(1)  = 0.999951297078
F(2)  = 1.23130E-14
F(3)  = 4.89E-22
```

VA N VIRGINIA NORTH ZONE # 4501

Defining Constants

Bs	=	38:02
Bn	=	39:12
Bb	=	37:40
Lo	=	78:30
Nb	=	2000000.0000
Eo	=	3500000.0000

Computed Constants

Bo	=	38.6174703154
SinBo	=	0.624117864647
Rb	=	8100315.6826
Ro	=	7994777.9034
No	=	2105537.7792
K	=	12589455.5135
ko	=	0.999948385156
Mo	=	6359972.6472
ro	=	6373043.

Coefficients for GP to PC

L(1)	=	111002.4628
L(2)	=	9.51137
L(3)	=	5.63918
L(4)	=	0.019770

Coefficients for PC to GP

G(1)	=	9.008809102E-06
G(2)	=	-6.95425E-15
G(3)	=	-3.71258E-20
G(4)	=	-1.0190E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999948385156
F(2)	=	1.23106E-14
F(3)	=	5.06E-22

VA S VIRGINIA SOUTH ZONE # 4502

Defining Constants

Bs	=	36:46
Bn	=	37:58
Bb	=	36:20
Lo	=	78:30
Nb	=	1000000.0000
Eo	=	3500000.0000

Computed Constants

Bo	=	37.3674799550
SinBo	=	0.606924846589
Rb	=	8476701.8059
Ro	=	8361937.6230
No	=	1114764.1829
K	=	12788171.0476
ko	=	0.999945401397
Mo	=	6358598.6747
ro	=	6372118.

Coefficients for GP to PC

L(1)	=	110978.4824
L(2)	=	9.40495
L(3)	=	5.64206
L(4)	=	0.018900

Coefficients for PC to GP

G(1)	=	9.010755731E-06
G(2)	=	-6.88091E-15
G(3)	=	-3.71758E-20
G(4)	=	-9.6990E-28

Coefficients for Grid Scale Factor

F(1)	=	0.999945401397
F(2)	=	1.23143E-14
F(3)	=	4.83E-22

WA N WASHINGTON NORTH ZONE # 4601

Defining Constants

Bs	=	47:30
Bn	=	48:44
Bb	=	47:00
Lo	=	120:50
Nb	=	0.0000
Eo	=	500000.0000

Computed Constants

Bo	=	48.1179151437
SinBo	=	0.744520326553
Rb	=	5853778.6038
Ro	=	5729486.2170
No	=	124292.3869
K	=	11670409.5559
ko	=	0.999942253481
Mo	=	6370499.7054
ro	=	6380060.

Coefficients for GP to PC

L(1)	=	111186.1944
L(2)	=	9.72145
L(3)	=	5.61785
L(4)	=	0.027630

Coefficients for PC to GP

G(1)	=	8.993922319E-06
G(2)	=	-7.07270E-15
G(3)	=	-3.67384E-20
G(4)	=	-1.4705E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999942253481
F(2)	=	1.22844E-14
F(3)	=	7.08E-22

WA S WASHINGTON SOUTH ZONE # 4602

Defining Constants

Bs	=	45:50
Bn	=	47:20
Bb	=	45:20
Lo	=	120:30
Nb	=	0.0000
Eo	=	500000.0000

Computed Constants

Bo	=	46.5850847865
SinBo	=	0.726395784020
Rb	=	6183952.2755
Ro	=	6044820.3632
No	=	139131.9123
K	=	11760132.9643
ko	=	0.999914597644
Mo	=	6368612.1773
ro	=	6378741.

Coefficients for GP to PC

L(1)	=	111153.2505
L(2)	=	9.75921
L(3)	=	5.62165
L(4)	=	0.026539

Coefficients for PC to GP

G(1)	=	8.996587928E-06
G(2)	=	-7.10693E-15
G(3)	=	-3.68032E-20
G(4)	=	-1.3823E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999914597644
F(2)	=	1.22897E-14
F(3)	=	6.73E-22

WV N WEST VIRGINIA NORTH ZONE # 4701

Defining Constants

Bs	=	39:00
Bn	=	40:15
Bb	=	38:30
Lo	=	79:30
Nb	=	0.0000
Eo	=	600000.0000

Computed Constants

Bo	=	39.6259559060
SinBo	=	0.637772979172
Rb	=	7837787.7954
Ro	=	7712787.3235
No	=	125000.4720
K	=	12444726.9475
ko	=	0.999940741388
Mo	=	6361027.5180
ro	=	6373731.

Coefficients for GP to PC

L(1)	=	111020.8737
L(2)	=	9.58417
L(3)	=	5.63702
L(4)	=	0.020271

Coefficients for PC to GP

G(1)	=	9.007315138E-06
G(2)	=	-7.00383E-15
G(3)	=	-3.70855E-20
G(4)	=	-1.0658E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999940741388
F(2)	=	1.23081E-14
F(3)	=	5.23E-22

WV S WEST VIRGINIA SOUTH ZONE # 4702

Defining Constants

Bs	=	37:29
Bn	=	38:53
Bb	=	37:00
Lo	=	81:00
Nb	=	0.0000
Eo	=	600000.0000

Computed Constants

Bo	=	38.1844729967
SinBo	=	0.618195407531
Rb	=	8250940.5496
Ro	=	8119477.8143
No	=	131462.7353
K	=	12655491.0285
ko	=	0.999925678359
Mo	=	6359357.1532
ro	=	6372583.

Coefficients for GP to PC

L(1)	=	110991.7203
L(2)	=	9.47644
L(3)	=	5.64030
L(4)	=	0.019308

Coefficients for PC to GP

G(1)	=	9.009681018E-06
G(2)	=	-6.93061E-15
G(3)	=	-3.71449E-20
G(4)	=	-1.0063E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999925678359
F(2)	=	1.23124E-14
F(3)	=	4.97E-22

WI N WISCONSIN NORTH ZONE # 4801

Defining Constants

Bs	=	45:34
Bn	=	46:46
Bb	=	45:10
Lo	=	90:00
Nb	=	0.0000
Eo	=	600000.0000

Computed Constants

Bo	=	46.1677715519
SinBo	=	0.721370788570
Rb	=	6244929.5105
Ro	=	6133662.3561
No	=	111267.1544
K	=	11788334.3169
ko	=	0.999945345317
Mo	=	6368341.1351
ro	=	6378625.

Coefficients for GP to PC

L(1)	=	111148.5205
L(2)	=	9.76579
L(3)	=	5.62201
L(4)	=	0.025652

Coefficients for PC to GP

G(1)	=	8.996970839E-06
G(2)	=	-7.11207E-15
G(3)	=	-3.68179E-20
G(4)	=	-1.3661E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999945345317
F(2)	=	1.22894E-14
F(3)	=	6.59E-22

WI C WISCONSIN CENTRAL ZONE # 4802

Defining Constants

Bs	=	44:15
Bn	=	45:30
Bb	=	43:50
Lo	=	90:00
Nb	=	0.0000
Eo	=	600000.0000

Computed Constants

Bo	=	44.8761466967
SinBo	=	0.705576614409
Rb	=	6531967.9926
Ro	=	6416091.9604
No	=	115876.0322
K	=	11884020.9704
ko	=	0.999940704902
Mo	=	6366865.5955
ro	=	6377630.

Coefficients for GP to PC

L(1)	=	111122.7674
L(2)	=	9.76998
L(3)	=	5.62509
L(4)	=	0.024632

Coefficients for PC to GP

G(1)	=	8.999055911E-06
G(2)	=	-7.12016E-15
G(3)	=	-3.68710E-20
G(4)	=	-1.2987E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999940704902
F(2)	=	1.22933E-14
F(3)	=	6.31E-22

WI S WISCONSIN SOUTH ZONE # 4803

Defining Constants

Bs	=	42:44
Bn	=	44:04
Bb	=	42:00
Lo	=	90:00
Nb	=	0.0000
Eo	=	600000.0000

Computed Constants

Bo	=	43.4012400263
SinBo	=	0.687103235566
Rb	=	6910290.1546
Ro	=	6754625.8558
No	=	155664.2988
K	=	12012072.0457
ko	=	0.999932547079
Mo	=	6365163.6776
ro	=	6376476.

Coefficients for GP to PC

L(1)	=	111093.0630
L(2)	=	9.75085
L(3)	=	5.62892
L(4)	=	0.023110

Coefficients for PC to GP

G(1)	=	9.001462070E-06
G(2)	=	-7.11165E-15
G(3)	=	-3.69314E-20
G(4)	=	-1.2326E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999932547079
F(2)	=	1.22981E-14
F(3)	=	5.97E-22

PR VI PUERTO RICO & VIRGIN I ZONE # 5200

Defining Constants

Bs	=	18:02
Bn	=	18:26
Bb	=	17:50
Lo	=	66:26
Nb	=	200000.0000
Eo	=	200000.0000

Computed Constants

Bo	=	18.2333725907
SinBo	=	0.312888187729
Rb	=	19411706.1974
Ro	=	19367429.4540
No	=	244276.7435
K	=	21418025.2279
ko	=	0.999993944472
Mo	=	6341634.1470
ro	=	6360883.

Coefficients for GP to PC

L(1)	=	110682.3958
L(2)	=	5.76845
L(3)	=	5.67659
L(4)	=	0.008098

Coefficients for PC to GP

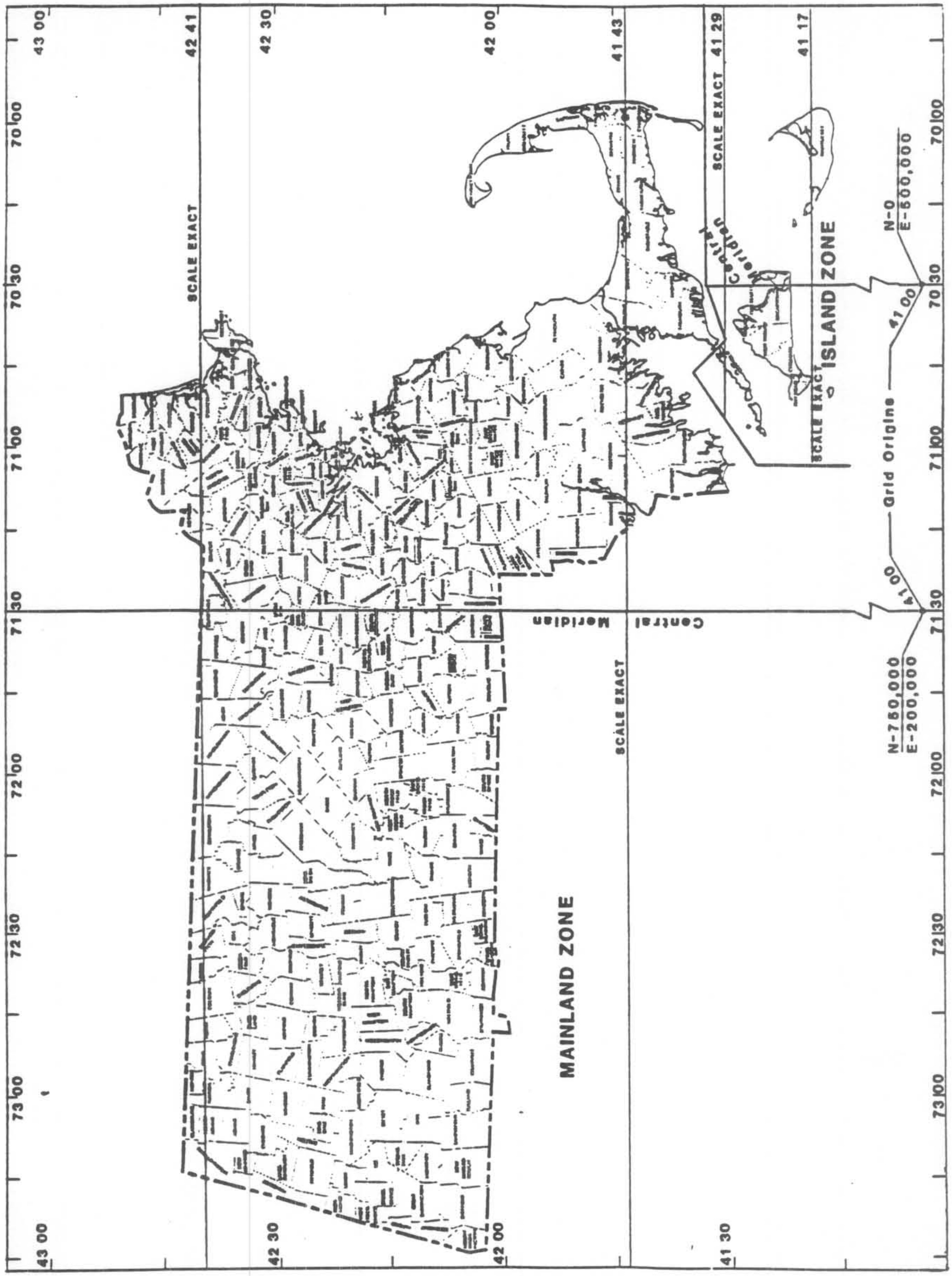
G(1)	=	9.034860445E-06
G(2)	=	-4.25426E-15
G(3)	=	-3.78192E-20
G(4)	=	-3.9493E-28

Coefficients for Grid Scale Factor

F(1)	=	0.999993944472
F(2)	=	1.23576E-14
F(3)	=	2.08E-22

APPENDIX A-10

- MASSACHUSETTS STATE PLANE COORDINATE ZONES DEFINITION MAP (NAD 83)



MASS HIGHWAY SURVEY MANUAL	MASSACHUSETTS STATE PLANE COORDINATE ZONES DEFINITION MAP (NAD 83)	DATE OF ISSUE SEPTEMBER 1996
		APPENDIX **A-10**

APPENDIX A-11

* STANDARDS AND SPECIFICATIONS FOR GEODETIC CONTROL NETWORKS

Standards and Specifications for Geodetic Control Networks

Federal Geodetic Control Committee

Rear Adm. John D. Bossler, Chairman

Rockville, Maryland

September 1984

Reprinted August 1993

FGCC

Standards and Specifications for Geodetic Control Networks

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For information write:
Chairman
Federal Geodetic Control Committee
6001 Executive Boulevard
Rockville, Maryland 20852

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NOAA, Rockville, MD 20852

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Standard and specifications for geodetic control networks.

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Preface

This single publication is designed to replace both "Classification, Standards of Accuracy and General Specifications of Geodetic Control Surveys," issued February 1974, and "Specifications to Support Classification, Standards of Accuracy, and General Specifications of Geodetic Control Surveys," issued June 1980. Because requirements and methods for acquisition of geodetic control are changing rapidly, this publication is being released in loose-leaf format so that it can be updated more conveniently and efficiently. Recipients of this publication wishing to receive updated information should complete and mail the form below. Comments on the contents and format of the publication are welcomed and should be addressed to:

FGCC Secretariat, Code N/CG1x5
National Geodetic Survey, NOAA
Rockville, Maryland 20852

- -

(Detach and mail to: National Geodetic Information Branch, code N/CG174 , NOAA, Rockville, Maryland 20852)

Please inform me of updated information for "Standards and Specifications for Geodetic Control Networks."

Name: ______________________________

Address: ______________________________

____

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____
(signature)

____
(date)

Contents

1. Introduction

The Government of the United States makes nationwide surveys, maps, and charts of various kinds. These are necessary to support the conduct of public business at all levels of government, for planning and carrying out national and local projects, the development and utilization of natural resources, national defense, land management, and monitoring crustal motion. Requirements for geodetic control surveys are most critical where intense development is taking place, particularly offshore areas, where surveys are used in the exploration and development of natural resources, and in delineation of state and international boundaries.

State and local governments and industry regularly cooperate in various parts of the total surveying and mapping program. In surveying and mapping large areas, it is first necessary to establish frameworks of horizontal, vertical, and gravity control. These provide a common basis for all surveying and mapping operations to ensure a coherent product. A reference system, or datum, is the set of numerical quantities that serves as a common basis. Three National Geodetic Control Networks have been created by the Government to provide the datums. It is the responsibility of the National Geodetic Survey (NGS) to actively maintain the National Geodetic Control Networks (appendix A).

These control networks consist of stable, identifiable points tied together by extremely accurate observations. From these observations, datum values (coordinates or gravity) are computed and published. These datum values provide the common basis that is so important to surveying and mapping activities.

As stated, the United States maintains three control networks. A horizontal network provides geodetic latitudes and longitudes in the North American Datum reference system; a vertical network furnishes elevations in the National Geodetic Vertical Datum reference system; and a gravity network supplies gravity values in the U.S. absolute gravity reference system. A given station may be a control point in one, two, or all three control networks.

It is not feasible for all points in the control networks to be of the highest possible accuracy. Different levels of accuracy are referred to as the "order" of a point. Orders are often subdivided further by a "class" designation. Datum values for a station are assigned an order (and class) based upon the appropriate classification standard for each of the three control networks. Horizontal and vertical standards are defined in reasonable conformance with past practice. The recent development of highly accurate absolute gravity instrumentation now allows a gravity reference standard. In the section on "Standards," the classification standards for each of the control networks are described, sample computations performed, and monumentation requirements given.

Control networks can be produced only by making very accurate measurements which are referred to identifiable control points. The combination of survey design, instrumentation, calibration procedures, observational techniques, and data reduction methods is known as a measurement system. The section on "Specifications" describes important components and states permissible tolerances for a variety of measurement systems.

Clearly, the control networks would be of little use if the datum values were not published. The section entitled "Information" describes the various products and formats of available geodetic data.

Upon request, the National Geodetic Survey will accept data submitted in the correct formats with the proper supporting documentation (appendix C) for incorporation into the national networks. When a survey is submitted for inclusion into the national networks, the survey measurements are processed in a quality control procedure that leads to their classification of accuracy and storage in the National Geodetic Survey data base. To fully explain the process we shall trace a survey from the planning stage to admission into the data base. This example will provide an overview of the standards and specifications, and how they work together.

The user should first compare the distribution and accuracy of current geodetic control with both immediate and long-term needs. From this basis, requirements for the extent and accuracy of the planned survey are determined. The classification standards of the control networks will help in this formulation. Hereafter, the requirements for the accuracy of the planned survey will be referred to as the "intended accuracy" of the survey. A measurement system is then chosen, based on various factors such as: distribution and accuracy of present control; region of the country; extent, distribution, and accuracy of the desired control; terrain and accessibility of control; and economic factors.

Upon selection of the measurement system, a survey design can be started. The design will be strongly dependent upon the "Network Geometry" specifications for that measurement system. Of particular importance is the requirement to connect to previously established control points. If this is not done, then the survey cannot be placed on the national datum. An adequate number of existing control point connections are often required in the specifications in order to ensure strong network geometry for other users of the control, and to provide several closure checks to help measure accuracy. NGS can certify the results of a survey only if it is connected to the national network.

Situations will arise where one cannot, or prefers not to, conform to the specifications. NGS may downgrade the classification of a survey based upon failure to adhere to the measurement system specifications if the departure degrades the precision, accuracy, or utility of the survey. On the other hand, if specification requirements for the desired level of accuracy are exceeded, it may be possible to upgrade a survey to a higher classification.

Depending upon circumstances, one may wish to go into the field to recover old control and perform reconnaissance and site inspection for the new survey. Monumentation may be performed at this stage. Instruments should be checked to conform to the "Instrumentation" specifications, and to meet the "Calibration Procedures" specifications. Frequent calibration is an excellent method to help ensure accurate surveys.

In the field, the "Field Procedures" specifications are used to guide the methods for taking survey measurements. It must be stressed that the "Field Procedures" section is not an exhaustive account of how to perform observations. Reference should be made also to the appropriate manuals of observation methods and instruments.

Computational checks can be found in the "Field Procedures" as well as in the "Office Procedures" specifications, since one will probably want to perform some of the computations in the field to detect blunders. It is not necessary for the user to do the computations described in the "Office Procedures" specifications, since they will be done by NGS. However, it is certainly in the interest of the user to compute these checks before leaving the field, in case reobservations are necessary. With the tremendous increase in programmable calculator and small computer technology, any of the computations in the "Office Procedures" specifications could be done with ease in the field.

At this point the survey measurements have been collected, together with the new description and recovery notes of the stations in the new survey. They are then placed into the formats specified in the Federal Geodetic Control Committee (FGCC) publications *Input Formats and Specifications of the National Geodetic Survey Data Base*. Further details of this process can be found in appendix C, "Procedures for Submitting Data to the National Geodetic Survey."

The data and supporting documentation, after being received at NGS, are processed through a quality control procedure to make sure that all users may place confidence in the new survey points. First, the data and documentation are examined for compliance with the measurement system specifications for the intended accuracy of the new survey. Then office computations are performed, including a minimally constrained least squares adjustment. (See appendix B for details.) From this adjustment, accuracy measures can be computed by error propagation. The accuracy classification thus computed is called the "provisional accuracy" of the survey.

The provisional accuracy is compared to the intended accuracy. The difference indicates the departure of the accuracy of the survey from the specifications. If the difference is small, the intended accuracy has precedence because a possible shift in classification is not warranted. However, if the difference is substantial, the provisional accuracy will supersede the intended accuracy, either as a downgrade or an upgrade.

As the final step in the quality control procedure, the variance factor ratio computation using established control, as explained in the section on "Standards," is determined for the new survey. If this result meets the criteria stated there, then the survey is classified in accordance with the provisional accuracy (or intended accuracy, whichever has precedence).

Cases arise where the variance factor ratio is significantly larger than expected. Then the control network is at fault, or the new survey is subject to some unmodeled error source which degrades its accuracy. Both the established control measurements and the new survey measurements will be scrutinized by NGS to determine the source of the problem. In difficult cases, NGS may make diagnostic measurements in the field.

Upon completion of the quality control check, the survey measurements and datum values are placed into the data base. They become immediately available for electronic retrieval, and will be distributed in the next publication cycle by the National Geodetic Information Branch of NGS.

A final remark bears on the relationship between the classification standards and measurement system specifications. Specifications are combinations of rules of thumb and studies of error propagation, based upon experience, of how to best achieve a desired level of quality. Unfortunately, there is no guarantee that a particular standard will be met if the associated specifications are followed. However, the situation is ameliorated by a safety factor of two incorporated in the standards and specifications. Because of this safety factor, it is possible that one may fail to meet the specifications and still satisfy the desired standard. This is why the geodetic control is not automatically downgraded when one does not adhere to the specifications. Slight departures from the specifications can be accommodated. In practice, one should always strive to meet the measurement system specifications when extending a National Geodetic Control Network.

2. Standards

The classification standards of the National Geodetic Control Networks are based on accuracy. This means that when control points in a particular survey are classified, they are certified as having datum values consistent with all other points in the network, not merely those within that particular survey. It is not observation closures within a survey which are used to classify control points, but the ability of that survey to duplicate already established control values. This comparison takes into account models of crustal motion, refraction, and any other systematic effects known to influence the survey measurements.

The NGS procedure leading to classification covers four steps:

1. The survey measurements, field records, sketches, and other documentation are examined to verify compliance with the specifications for the intended accuracy of the survey. This examination may lead to a modification of the intended accuracy.
2. Results of a minimally constrained least squares adjustment of the survey measurements are examined to ensure correct weighting of the observations and freedom from blunders.
3. Accuracy measures computed by random error propagation determine the provisional accuracy. If the provisional accuracy is substantially different from the intended accuracy of the survey, then the provisional accuracy supersedes the intended accuracy.
4. A variance factor ratio for the new survey combined with network data is computed by the Iterated Almost Unbiased Estimator (IAUE) method (appendix B). If the variance factor ratio is reasonably close to 1.0 (typically less than 1.5), then the survey is considered to check with the network, and the survey is classified with the provisional (or intended) accuracy. If the variance factor ratio is much greater than 1.0 (typically 1.5 or greater), then the survey is considered to not check with the network, and both the survey and network measurements will be scrutinized for the source of the problem.

2.1 Horizontal Control Network Standards

When a horizontal control point is classified with a particular order and class, NGS certifies that the geodetic latitude and longitude of that control point bear a relation of specific accuracy to the coordinates of all other points in the horizontal control network. This relation is expressed as a distance accuracy, 1:a. A distance accuracy is the ratio of the relative positional error of a pair of control points to the horizontal separation of those points.

Table 2.1—Distance accuracy standards

Classification	*Minimum distance accuracy*
First-order	1:100,000
Second-order, class I	1: 50,000
Second-order, class II	1: 20,000
Third-order, class I	1: 10,000
Third-order, class II	1: 5,000

A distance accuracy, 1:a, is computed from a minimally constrained, correctly weighted, least squares adjustment by:

$$a = d/s$$

where

a = distance accuracy denominator

s = propagated standard deviation of distance between survey points obtained from the least squares adjustment

d = distance between survey points

The distance accuracy pertains to all pairs of points (but in practice is computed for a sampling of pairs of points). The worst distance accuracy (smallest denominator) is taken as the provisional accuracy. If this is substantially larger or smaller than the intended accuracy, then the provisional accuracy takes precedence.

As a test for systematic errors, the variance factor ratio of the new survey is computed by the Iterated Almost Unbiased Estimator (IAUE) method described in appendix B. This computation combines the new survey measurements with existing network data, which are assumed to be correctly weighted and free of systematic error. If the variance factor ratio is substantially greater than unity then the survey does not check with the network, and both the survey and the network data will be examined by NGS.

Computer simulations performed by NGS have shown that a variance factor ratio greater than 1.5 typically indicates systematic errors between the survey and the network. Setting a cutoff value higher than this could allow undetected systematic error to propagate into the national network. On the other hand, a higher cutoff value might be considered if the survey has only a small number of connections to the network, because this circumstance would tend to increase the variance factor ratio.

In some situations, a survey has been designed in which different sections provide different orders of control. For these multi-order surveys, the computed distance accuracy denominators should be grouped into sets appropriate to the different parts of the survey. Then, the smallest value of a in each set is used to classify the control points of that portion, as discussed above. If there are sufficient connections to the network, several variance factor ratios, one for each section of the survey, should be computed.

Horizontal Example

Suppose a survey with an intended accuracy of first-order (1:100,000) has been performed. A series of propagated distance accuracies from a minimally constrained adjustment is now computed.

Line	*s* (m)	*d* (m)	*1:a*
1-2	0.141	17,107	1:121,326
1-3	0.170	20,123	1:118,371
2-3	0.164	15,505	1: 94,543
....	.	.	.
....	.	.	.
....	.	.	.

Suppose that the worst distance accuracy is 1:94,543. This is not substantially different from the intended accuracy of 1:100,000, which would therefore have precedence for classification. It is not feasible to precisely quantify "substantially different." Judgment and experience are determining factors.

Now assume that a solution combining survey and network data has been obtained (as per appendix B), and that a variance factor ratio of 1.2 was computed for the survey. This would be reasonably close to unity, and would indicate that the survey checks with the network. The survey would then be classified as first-order using the intended accuracy of 1:100,000.

However, if a variance factor of, say, 1.9 was computed, the survey would not check with the network. Both the survey and network measurements then would have to be scrutinized to find the problem.

Monumentation

Control points should be part of the National Geodetic Horizontal Network only if they possess permanence, horizontal stability with respect to the Earth's crust, and a horizontal location which can be defined as a point. A 30-centimeter-long wooden stake driven into the ground, for example, would lack both permanence and horizontal stability. A mountain peak is difficult to define as a point. Typically, corrosion resistant metal disks set in a large concrete mass have the necessary qualities. First-order and second-order, class I, control points should have an underground mark, at least two monumented reference marks at right angles to one another, and at least one monumented azimuth mark no less than 400 m from the control point. Replacement of a temporary mark by a more permanent mark is not acceptable unless the two marks are connected in timely fashion by survey observations of sufficient accuracy. Detailed information may be found in C&GS *Special Publication* 247, "Manual of geodetic triangulation."

2.2 Vertical Control Network Standards

When a vertical control point is classified with a particular order and class, NGS certifies that the orthometric elevation at that point bears a relation of specific accuracy to the elevations of all other points in the vertical control network. That relation is expressed as an elevation difference accuracy, b. An elevation difference accuracy is the relative elevation error between a pair of control points that is scaled by the square root of their horizontal separation traced along existing level routes.

Table 2.2—Elevation accuracy standards

Classification	*Maximum elevation difference accuracy*
First-order, class I	0.5
First-order, class II	0.7
Second-order, class I	1.0
Second-order, class II	1.3
Third-order	2.0

An elevation difference accuracy, b, is computed from a minimally constrained, correctly weighted, least squares adjustment by

$$b = S/\sqrt{d}$$

where

d = approximate horizontal distance in kilometers between control point positions traced along existing level routes.

S = propagated standard deviation of elevation difference in millimeters between survey control points obtained from the least squares adjustment. Note that the units of b are $(mm)/\sqrt{(km)}$.

The elevation difference accuracy pertains to all pairs of points (but in practice is computed for a sample). The worst elevation difference accuracy (largest value) is taken as the provisional accuracy. If this is substantially larger or smaller than the intended accuracy, then the provisional accuracy takes precedence.

As a test for systematic errors, the variance factor ratio of the new survey is computed by the Iterated Almost Unbiased Estimator (IAUE) method described in appendix B. This computation combines the new survey measurements with existing network data, which are assumed to be correctly weighted and free of systematic error. If the variance factor ratio is substantially greater than unity, then the survey does not check with the network, and both the survey and the network data will be examined by NGS.

Computer simulations performed by NGS have shown that a variance factor ratio greater than 1.5 typically indicates systematic errors between the survey and the network. Setting a cutoff value higher than this could allow undetected systematic error to propagate into the national network. On the other hand, a higher cutoff value might be considered if the survey has only a small number of connections to the network, because this circumstance would tend to increase the variance factor ratio.

In some situations, a survey has been designed in which different sections provide different orders of control. For these multi-order surveys, the computed elevation difference accuracies should be grouped into sets appropriate to the different parts of the survey. Then, the largest value of b in each set is used to classify the control points of that portion, as discussed above. If there are sufficient connections to the network, several variance factor ratios, one for each section of the survey, should be computed.

Vertical Example

Suppose a survey with an intended accuracy of second-order, class II has been performed. A series of propagated elevation difference accuracies from a minimally constrained adjustment is now computed.

Line	*S* *(mm)*	*d* *(km)*	*b* *(mm)/$\sqrt{}$ (km)*
1-2	1.574	1.718	1.20
1-3	1.743	2.321	1.14
2-3	2.647	4.039	1.32
....	.	.	.
....	.	.	.
....	.	.	.

Suppose that the worst elevation difference accuracy is 1.32. This is not substantially different from the intended accuracy of 1.3 which would therefore have precedence for classification. It is not feasible to precisely quantify "substantially different." Judgment and experience are determining factors.

Now assume that a solution combining survey and network data has been obtained (as per appendix B), and that a variance factor ratio of 1.2 was computed for the survey. This would be reasonably close to unity and would indicate that the survey checks with the network. The survey would then be classified as second-order, class II, using the intended accuracy of 1.3.

However, if a survey variance factor ratio of, say, 1.9 was computed, the survey would not check with the network. Both the survey and network measurements then would have to be scrutinized to find the problem.

Monumentation

Control points should be part of the National Geodetic Vertical Network only if they possess permanence, vertical stability with respect to the Earth's crust, and a vertical location that can be defined as a point. A 30-centimeter-long wooden stake driven into the ground, for example, would lack both permanence and vertical stability. A rooftop lacks stability and is difficult to define as a point. Typically, corrosion resistant metal disks set in large rock outcrops or long metal rods driven deep into the ground have the necessary qualities. Replacement of a temporary mark by a more permanent mark is not acceptable unless the two marks are connected in timely fashion by survey observations of sufficient accuracy. Detailed information may be found in *NOAA Manual NOS NGS* 1, "Geodetic bench marks."

2.3 Gravity Control Network Standards

When a gravity control point is classified with a particular order and class, NGS certifies that the gravity value at that control point possesses a specific accuracy.

Gravity is commonly expressed in units of milligals (mGal) or microgals (μGal) equal, respectively, to (10^{-5}) meters/sec^2, and (10^{-8}) meters/sec^2. Classification order refers to measurement accuracies and class to site stability.

Table 2.3—Gravity accuracy standards

Classification	*Gravity accuracy* *(μGal)*
First-order, class I	20 (subject to stability verification)
First-order, class II	20
Second-order	50
Third-order	100

When a survey establishes only new points, and where only absolute measurements are observed, then each survey point is classified independently. The standard deviation from the mean of measurements observed at that point is corrected by the error budget for noise sources in accordance with the following formula:

$$c^2 = \sum_{i+1}^{n} \frac{(x_i - x_m)^2}{n-1} + e^2$$

where
c = gravity accuracy
x_i = gravity measurement
n = number of measurements

$$x_m = (\sum_{i=1}^{n} x_i)/n$$

e = external random error

The value obtained for c is then compared directly against the gravity accuracy standards table.

When a survey establishes points at which both absolute and relative measurements are made, the absolute determination ordinarily takes precedence and the point is classified accordingly. (However, see Example D below for an exception.)

When a survey establishes points where only relative measurements are observed, and where the survey is tied to the National Geodetic Gravity Network, then the gravity accuracy is identified with the propagated gravity standard deviation from a minimally constrained, correctly weighted, least squares adjustment.

The worst gravity accuracy of all the points in the survey is taken as the provisional accuracy. If the provisional accuracy exceeds the gravity accuracy limit set for the intended survey classification, then the survey is classified using the provisional accuracy.

As a test for systematic errors, the variance factor ratio of the new survey is computed by the Iterated Almost Unbiased Estimator (IAUE) method described in appendix B. This computation combines the new survey measurements with existing network data which are assumed to be correctly weighted and free of systematic error. If the variance factor ratio is substantially greater than unity, then the survey does not check with the network, and both the survey and the network data will be examined by NGS.

Computer simulations performed by NGS have shown that a variance factor ratio greater than 1.5 typically indicates systematic errors between the survey and the network. Setting a cutoff value higher than this could allow undetected systematic error to propagate into the national network. On the other hand, a higher cutoff value might be considered if the survey has only a minimal number of connections to the network, because this circumstance would tend to increase the variance factor ratio.

In some situations, a survey has been designed in which different sections provide different orders of control. For these multi-order surveys, the computed gravity accuracies should be grouped into sets appropriate to the different parts of the survey. Then, the largest value of c in each set is used to classify the control points of that portion, as discussed above. If there are sufficient connections to the network, several variance factor ratios, one for each part of the survey, should be computed.

Gravity Examples

Example A. Suppose a gravity survey using absolute measurement techniques has been performed. These points are then unrelated. Consider one of these survey points.

Assume n = 750

$$\sum_{i=1}^{750} (x_i - x_m)^2 = .169 \text{ mGal}^2$$

$$e = 5\ \mu\text{Gal}$$

$$c^2 = \frac{0.169}{750-1} + (.005)^2$$

$$c = 16\ \mu\text{Gal}$$

The point is then classified as first-order, class II.

Example B. Suppose a relative gravity survey with an intended accuracy of second-order (50 μGal) has been performed. A series of propagated gravity accuracies from a minimally constrained adjustment is now computed.

Station	*Gravity standard deviation (μGal)*
1	38
2	44
3	55
.	.
.	.
.	.

Suppose that the worst gravity accuracy was 55 μGal. This is worse than the intended accuracy of 50 μGal. Therefore, the provisional accuracy of 55 μGal would have precedence for classification, which would be set to third-order.

Now assume that a solution combining survey and network data has been obtained (as per appendix B) and that a variance factor of 1.2 was computed for the survey. This would be reasonably close to unity, and would indicate that the survey checks with the network. The survey would then be classified as third-order using the provisional accuracy of 55 μGal.

However, if a variance factor of, say, 1.9 was computed, the survey would not check with the network. Both the survey and network measurements then would have to be scrutinized to find the problem.

Example C. Suppose a survey consisting of both absolute and relative measurements has been made at the same points. Assume the absolute observation at one of the points yielded a classification of first-order, class II, whereas the relative measurements produced a value to second-order standards. The point in question would be classified as first-order, class II, in accordance with the absolute observation.

Example D. Suppose we have a survey similar to Case C, where the absolute measurements at a particular point yielded a third-order classification due to an unusually noisy observation session, but the relative measurements still satisfied the second-order standard. The point in question would be classified as second-order, in accordance with the relative measurements.

Monumentation

Control points should be part of the National Geodetic Gravity Network only if they possess permanence, horizontal and vertical stability with respect to the Earth's crust, and a horizontal and vertical location which can be defined as a point. For all orders of accuracy, the mark should be imbedded in a stable platform such as flat, horizontal concrete. For first-order, class I stations, the platform should be imbedded in stable, hard rock, and checked at least twice for the first year to ensure stability. For first-order, class II stations, the platform should be located in an extremely stable environment, such as the concrete floor of a mature structure. For second and third-order stations, standard bench mark monumentation is adequate. Replacement of a temporary mark by a more permanent mark is not acceptable unless the two marks are connected in timely fashion by survey observations of sufficient accuracy. Detailed information is given in *NOAA Manual NOS NGS* 1, "Geodetic bench marks." Monuments should not be near sources of electromagnetic interference.

It is recommended, but not necessary, to monument third-order stations. However, the location associated with the gravity value should be recoverable, based upon the station description.

3. Specifications

3.1 Introduction

All measurement systems regardless of their nature have certain common qualities. Because of this, the measurement system specifications follow a prescribed structure as outlined below. These specifications describe the important components and state permissible tolerances used in a general context of accurate surveying methods. The user is cautioned that these specifications are not substitutes for manuals that detail recommended field operations and procedures.

The observations will have spatial or temporal relationships with one another as given in the "Network Geometry" section. In addition, this section specifies the frequency of incorporation of old control into the survey. Computer simulations could be performed instead of following the "Network Geometry" and "Field Procedures" specifications. However, the user should consult the National Geodetic Survey before undertaking such a departure from the specifications.

The "Instrumentation" section describes the types and characteristics of the instruments used to make observations. An instrument must be able to attain the precision requirements given in "Field Procedures."

The section "Calibration Procedures" specifies the nature and frequency of instrument calibration. An instrument must be calibrated whenever it has been damaged or repaired.

The "Field Procedures" section specifies particular rules and limits to be met while following an appropriate method of observation. For a detailed account of how to perform observations, the user should consult the appropriate manuals.

Since NGS will perform the computations described under "Office Procedures," it is not necessary for the user to do them. However, these computations provide valuable checks on the survey measurements that could indicate the need for some reobservations. This section specifies commonly applied corrections to observations, and computations which monitor the precision and accuracy of the survey. It also discusses the correctly weighted, minimally constrained least squares adjustment used to ensure that the survey work is free from blunders and able to achieve the intended accuracy. Results of the least squares adjustment are used in the quality control and accuracy classification procedures. The adjustment performed by NGS will use models of error sources, such as crustal motion, when they are judged to be significant to the level of accuracy of the survey.

3.2 Triangulation

Triangulation is a measurement system comprised of joined or overlapping triangles of angular observations supported by occasional distance and astronomic observations. Triangulation is used to extend horizontal control.

Network Geometry

Order *Class*	*First*	*Second* *I*	*Second* *II*	*Third* *I*	*Third* *II*
Station spacing not less than (km)	15	10	5	0.5	0.5
Average minimum distance angle† of figures not less than	40°	35°	30°	30°	25°
Minimum distance angle† of all figures not less than	30°	25°	25°	20°	20°
Base line spacing not more than (triangles)	5	10	12	15	15
Astronomic azimuth spacing not more than (triangles)	8	10	10	12	15

† Distance angle is angle opposite the side through which distance is propagated.

The new survey is required to tie to at least four network control points spaced well apart. These network points must have datum values equivalent to or better than the intended order (and class) of the new survey. For example, in an arc of triangulation, at least two network control points should be occupied at each end of the arc. Whenever the distance between two new unconnected survey points is less than 20 percent of the distance between those points traced along existing or new connections, then a direct connection should be made between those two survey points. In addition, the survey should tie into any sufficiently accurate network control points within the station spacing distance of the survey. These network stations should be occupied and sufficient observations taken to make these stations integral parts of the survey. Nonredundant geodetic connections to the network stations are not considered sufficient ties. Nonredundantly determined stations are not allowed. Control stations should not be determined by intersection or resection methods. Simultaneous reciprocal vertical angles or geodetic leveling are observed along base lines. A base line need not be observed if other base lines of sufficient accuracy were observed within the base line spacing specification in the network, and similarly for astronomic azimuths.

Instrumentation

Only properly maintained theodolites are adequate for observing directions and azimuths for triangulation. Only precisely marked targets, mounted stably on tripods or supported towers, should be employed. The target should have a clearly defined center, resolvable at the minimum control spacing. Optical plummets or collimators are required to ensure that the theodolites and targets are centered over the marks. Microwave-type electronic distance measurement (EDM) equipment is not sufficiently accurate for measuring higher-order base lines.

Order *Class*	*First*	*Second* *I*	*Second* *II*	*Third* *I*	*Third* *II*
Theodolite, least count	0.2"	0.2"	1.0"	1.0"	1.0"

Calibration Procedures

Each year and whenever the difference between direct and reverse readings of the theodolite depart from 180° by more than 30", the instrument should be adjusted for collimation error. Readjustment of the cross hairs and the level bubble should be done whenever their misadjustments affect the instrument reading by the amount of the least count.

All EDM devices and retroreflectors should be serviced regularly and checked frequently over lines of known distances. The National Geodetic Survey has established specific calibration base lines for this purpose. EDM instruments should be calibrated annually, and frequency checks made semiannually.

Field Procedures

Theodolite observations for first-order and second-order, class I surveys may only be made at night. Reciprocal vertical angles should be observed at times of best atmospheric conditions (between noon and late afternoon) for all orders of accuracy. Electronic distance measurements need a record at both ends of the line of wet and dry bulb temperatures to ±1°C, and barometric pressure to ±5 mm of mercury. The theodolite and targets should be centered to within 1 mm over the survey mark or eccentric point.

Order *Class*	*First*	*Second* *I*	*Second* *II*	*Third* *I*	*Third* *II*
Directions					
Number of positions	16	16	8 or 12†	4	2
Standard deviation of mean not to exceed	0.4"	0.5"	0.8"	1.2"	2.0"
Rejection limit from the mean	4"	4"	5"	5"	5"
Reciprocal Vertical Angles (along distance sight path)					
Number of independent observations direct/reverse	3	3	2	2	2
Maximum spread	10"	10"	10"	10"	20"
Maximum time interval between reciprocal angles (hr)	1	1	1	1	1
Astronomic Azimuths					
Observations per night	16	16	16	8	4
Number of nights	2	2	1	1	1
Standard deviation of mean not to exceed	0.45"	0.45"	0.6"	1.0"	1.7"
Rejection limit from the mean	5"	5"	5"	6"	6"
Electro-Optical Distances					
Minimum number of days	2*	2*	1	1	1
Minimum number of measurements/day	2§	2§	2§	1	1
Minimum number of concentric observations/measurement	2	2	1	1	1
Minimum number of offset observations/measurement	2	2	2	1	1
Maximum difference from mean of observations (mm)	40	40	50	60	60
Minimum number of readings/observation (or equivalent)	10	10	10	10	10
Maximum difference from mean of readings (mm)	‡	‡	‡	‡	‡
Infrared Distances					
Minimum number of days	—	2*	1	1	1
Minimum number of measurements	—	2§	2§	1	1
Minimum number of concentric observations/measurement	—	1	1	1	1
Minimum number of offset observations/measurement	—	2	1	1	1
Maximum difference from mean of observations (mm)	—	5	5	10	10
Minimum number of readings/observation (or equivalent)	—	10	10	10	10
Maximum difference from mean of readings (mm)	—	‡	‡	‡	‡
Microwave Distances					
Minimum number of measurements	—	—	—	2	1
Minimum time span between measurements (hr)	—	—	—	8	—

Order Class	First	Second I	Second II	Third I	Third II
Maximum difference between measurements (mm)	—	—	—	100	—
Minimum number of concentric observations/measurement	—	—	—	2**	1**
Maximum difference from mean of observations (mm)	—	—	—	100	150
Minimum number of readings/observation (or equivalent)	—	—	—	20	20
Maximum difference from mean of readings (mm)	—	—	—	‡	‡

† 8 if 0.2", 12 if 1.0" resolution.
* two or more instruments.
§ one measurement at each end of the line.
‡ as specified by manufacturer.
** carried out at both ends of the line.

Measurements of astronomic latitude and longitude are not required in the United States, except perhaps for first-order work, because sufficient information for determining deflections of the vertical exists. Detailed procedures can be found in Hoskinson and Duerksen (1952).

Office Procedures

Order Class	First	Second I	Second II	Third I	Third II
Triangle Closure					
Average not to exceed	1.0"	1.2"	2.0"	3.0"	5.0"
Maximum not to exceed	3"	3"	5"	5"	10"
Side Checks					
Mean absolute correction by side equation not to exceed	0.3"	0.4"	0.6"	0.8"	2.0"

A minimally constrained least squares adjustment will be checked for blunders by examining the normalized residuals. The observation weights will be checked by inspecting the postadjustment estimate of the variance of unit weight. Distance standard errors computed by error propagation in this correctly weighted least squares adjustment will indicate the provisional accuracy classification. A survey variance factor ratio will be computed to check for systematic error. The least squares adjustment will use models which account for the following:

semimajor axis of the ellipsoid ....(a = 6378137 m)
reciprocal flattening of the ellipsoid ....(1/f = 298.257222)
mark elevation above mean sea level ....(known to ± 1 m)
geoid heights ....(known to ± 6 m)
deflections of the vertical ....(known to ± 3")
geodesic correction
skew normal correction
height of instrument
height of target
sea level correction
arc correction
geoid height correction
second velocity correction
crustal motion

3.3 Traverse

Traverse is a measurement system comprised of joined distance and theodolite observations supported by occasional astronomic observations. Traverse is used to densify horizontal control.

Network Geometry

Order Class	First	Second I	Second II	Third I	Third II
Station spacing not less than (km)	10	4	2	0.5	0.5
Maximum deviation of main traverse from straight line	20°	20°	25°	30°	40°
Minimum number of bench mark ties	2	2	2	2	2
Bench mark tie spacing not more than (segments)	6	8	10	15	20
Astronomic azimuth spacing not more than (segments)	6	12	20	25	40
Minimum number of network control points	4	3	2	2	2

The new survey is required to tie to a minimum number of network control points spaced well apart. These network points must have datum values equivalent to or better than the intended order (and class) of the new survey. Whenever the distance between two new unconnected survey points is less than 20 percent of the distance between those points traced along existing or new connections, then a direct connection must be made between those two survey points. In addition, the survey should tie into any sufficiently accurate network control points within the station spacing distance of the survey. These ties must include EDM or taped distances. Nonredundant geodetic connections to the network stations are not considered sufficient ties. Nonredundantly determined stations are not allowed. Reciprocal vertical angles or geodetic leveling are observed along all traverse lines.

Instrumentation

Only properly maintained theodolites are adequate for observing directions and azimuths for traverse. Only precisely marked targets, mounted stably on tripods or supported towers, should be employed. The target should have a clearly defined center, resolvable at the minimum control spacing. Optical plummets or collimators are required to ensure that the theodolites and targets are centered over the marks. Microwave-type electronic distance measurement equipment is not sufficiently accurate for measuring first-order traverses.

Order Class	First	Second I	Second II	Third I	Third II
Theodolite, least count	0.2"	1.0"	1.0"	1.0"	1.0"

Calibration Procedures

Each year and whenever the difference between direct and reverse readings of the theodolite depart from 180° by more than 30", the instrument should be adjusted for collimation error. Readjustment of the cross hairs and the level bubble should be done whenever their misadjustments affect the instrument reading by the amount of the least count.

All electronic distance measuring devices and retroreflectors should be serviced regularly and checked frequently over lines of known distances. The National Geodetic Survey has established specific calibration base lines for this purpose. EDM instruments should be calibrated annually, and frequency checks made semiannually.

Field Procedures

Theodolite observations for first-order and second-order, class I surveys may be made only at night. Electronic distance measurements need a record at both ends of the line of wet and dry bulb temperatures to ±1°C and barometric pressure to ±5 mm of mercury. The theodolite, EDM, and targets should be centered to within 1 mm over the survey mark or eccentric point.

Order Class	First	Second I	Second II	Third I	Third II
Directions					
Number of positions	16	8 or 12†	6 or 8*	4	2
Standard deviation of mean not to exceed	0.4"	0.5"	0.8"	1.2"	2.0"
Rejection limit from the mean	4"	5"	5"	5"	5"
Reciprocal Vertical Angles (along distance sight path)					
Number of independent observations direct/reverse	3	3	2	2	2
Maximum spread	10"	10"	10"	10"	20"
Maximum time interval between reciprocal angles (hr)	1	1	1	1	1
Astronomic Azimuths					
Observations per night	16	16	12	8	4
Number of nights	2	2	1	1	1
Standard deviation of mean not to exceed	0.45"	0.45"	0.6"	1.0"	1.7"
Rejection limit from the mean	5"	5"	5"	6"	6"
Electro-Optical Distances					
Minimum number of measurements	1	1	1	1	1
Minimum number of concentric observations/measurement	1	1	1	1	1
Minimum number of offset observations/measurement	1	1	—	—	—
Maximum difference from mean of observations (mm)	60	60	—	—	—
Minimum number of readings/observation (or equivalent)	10	10	10	10	10
Maximum difference from mean of readings (mm)	§	§	§	§	§
Infrared Distances					
Minimum number of measurements	1	1	1	1	1
Minimum number of concentric observations/measurement	1	1	1	1	1
Minimum number of offset observations/measurement	1	1	1‡	—	—
Maximum difference from mean of observations (mm)	10	10	10‡	—	—
Minimum number of readings/observation	10	10	10	10	10
Maximum difference from mean of readings (mm)	§	§	§	§	§
Microwave Distances					
Minimum number of measurements	—	1	1	1	1
Minimum number of concentric observations/measurement	—	2**	1**	1**	1**
Maximum difference from mean of observations (mm)	—	150	150	200	200
Minimum number of readings/observation	—	20	20	10	10
Maximum difference from mean of readings (mm)	—	§	§	§	§

† 8 if 0.2", 12 if 1.0" resolution.
* 6 if 0.2", 8 if 1.0" resolution.
§ as specified by manufacturer.
‡ only if decimal reading near 0 or high 9's.
** carried out at both ends of the line.

Measurements of astronomic latitude and longitude are not required in the United States, except perhaps for first-order work, because sufficient information for determining deflections of the vertical exists. Detailed procedures can be found in Hoskinson and Duerksen (1952).

Office Procedures

Order Class	First	Second I	Second II	Third I	Third II
Azimuth closure at azimuth check point (seconds of arc)	$1.7\sqrt{N}$	$3.0\sqrt{N}$	$4.5\sqrt{N}$	$10.0\sqrt{N}$	$12.0\sqrt{N}$
Position closure after azimuth adjustment†	$0.04\sqrt{K}$ or 1:100,000	$0.08\sqrt{K}$ or 1:50,000	$0.20\sqrt{K}$ or 1:20,000	$0.40\sqrt{K}$ or 1:10,000	$0.80\sqrt{K}$ or 1:5,000

(N is number of segments, K is route distance in km)

† The expression containing the square root is designed for longer lines where higher proportional accuracy is required. Use the formula that gives the smallest permissible closure. The closure (e.g., 1:100,000) is obtained by computing the difference between the computed and fixed values, and dividing this difference by K.

Note: Do not confuse closure with distance accuracy of the survey.

A minimally constrained least squares adjustment will be checked for blunders by examining the normalized residuals. The observation weights will be checked by inspecting the postadjustment estimate of the variance of unit weight. Distance standard errors computed by error propagation in a correctly weighted least squares adjustment will indicate the provisional accuracy classification. A survey variance factor ratio will be computed to check for systematic error. The least squares adjustment will use models which account for the following:

semimajor axis of the ellipsoid	(a = 6378137 m)
reciprocal flattening of the ellipsoid	(1/f = 298.257222)
mark elevation above mean sea level	(known to ± 1 m)
geoid heights	(known to ± 6 m)
deflections of the vertical	(known to ± 3")
geodesic correction	
skew normal correction	
height of instrument	
height of target	
sea level correction	
arc correction	
geoid height correction	
second velocity correction	
crustal motion	

3.4 Inertial Surveying

Inertial surveying is a measurement system comprised of lines, or a grid, of Inertial Surveying System (ISS) observations. These specifications cover use of inertial systems only for horizontal control.

Network Geometry

Order / *Class*	*Second* *I*	*Second* *II*	*Third* *I*	*Third* *II*
Station spacing not less than (km)	10	4	2	1
Maximum deviation from straight line connecting endpoints	20°	25°	30°	35°

Each inertial survey line is required to tie into a minimum of four horizontal network control points spaced well apart and should begin and end at network control points. These network control points must have horizontal datum values better than the intended order (and class) of the new survey. Whenever the shortest distance between two new unconnected survey points is less than 20 percent of the distance between those points traced along existing or new connections, then a direct connection should be made between those two survey points. In addition, the survey should connect to any sufficiently accurate network control points within the distance specified by the station spacing. The connections may be measured by EDM or tape traverse, or by another ISS line. If an ISS line is used, then these lines should follow the same specifications as all other ISS lines in the survey.

For extended area surveys by ISS, a grid of intersecting lines that satisfies the 20 percent rule stated above can be designed. There must be a mark at each intersection of the lines. This mark need not be a permanent monument; it may be a stake driven into the ground. For a position to receive an accuracy classification, it must be permanently monumented.

A grid of intersecting lines should contain a minimum of eight network points, and should have a network control point at each corner. The remaining network control points may be distributed about the interior or the periphery of the grid. However, there should be at least one network control point at an intersection of the grid lines near the center of the grid. If the required network points are not available, then they should be established by some other measurement system. Again, the horizontal datum values of these network control points must have an order (and class) better than the intended order (and class) of the new survey.

Instrumentation

ISS equipment falls into two types: analytic (or strapdown) and semianalytic. Analytic inertial units are not considered to possess geodetic accuracy. Semianalytic units are either "space stable" or "local level." Space stable systems maintain the orientation of the platform with respect to inertial space. Local level systems continuously torque the accelerometers to account for Earth rotation and movement of the inertial unit, and also torque the platform to coincide with the local level. This may be done on command at a coordinate update, or whenever the unit achieves zero velocity (Zero velocity UPdaTe, or "ZUPT"). Independently of the measurement technique, the recorded data may be filtered by an onboard computer. Because of the variable quality of individual ISS instruments, the user should test an instrument with existing geodetic control beforehand.

An offset measurement device accurate to within 5 mm should be affixed to the inertial unit or the vehicle.

Calibration Procedures

A static calibration should be performed yearly and immediately after repairs affecting the platform, gyroscopes, or accelerometers.

A dynamic or field calibration should be performed prior to each project or subsequent to a static calibration. The dynamic calibration should be performed only between horizontal control points of first-order accuracy and in each cardinal direction. The accelerometer scale factors from this calibration should be recorded and, if possible, stored in the onboard computer of the inertial unit.

Before each project or after repairs affecting the offset measurement device or the inertial unit, the relation between the center of the inertial unit and the zero point of the offset measurement device should be established.

Field Procedures

When surveying in a helicopter, the helicopter must come to rest on the ground for all ZUPT's and all measurements.

Order Class	Second I	Second II	Third I	Third II
Minimum number of complete runs per line	2	1	1	1
Maximum deviation from a uniform rate of travel (including ZUPT)	15%	20%	25%	30%
Maximum ZUPT interval (ZUPT to ZUPT) (sec)	200	240	300	300

A complete ISS measurement consists of measurement of the line while traveling in one direction, followed by measurement of the same line while traveling in the reverse direction (double-run). A coordinate update should not be performed at the far point or at midpoints of a line, even though those coordinates may be known.

The mark offset should be measured to the nearest 5 mm.

Office Procedures

Order Class	Second I	Second II	Third I	Third II
Maximum difference of smoothed coordinates between forward and reverse run (cm)	60	60	70	80

A minimally constrained least squares adjustment of the raw or filtered survey data will be checked for blunders by examining the normalized residuals. The observation weights will be checked by inspecting the postadjustment estimate of the variance of unit weight. Distance standard errors computed by error propagation in this correctly weighted least squares adjustment will indicate the provisional accuracy classification. A survey variance factor ratio will be computed to check for systematic error. The least squares adjustment will use the best available model for the particular inertial system. Weighted averages of individually smoothed lines are not considered substitutes for a combined least squares adjustment to achieve geodetic accuracy.

3.5 Geodetic Leveling

Geodetic leveling is a measurement system comprised of elevation differences observed between nearby rods. Leveling is used to extend vertical control.

Network Geometry

Order Class	First I	First II	Second I	Second II	Third
Bench mark spacing not more than (km)	3	3	3	3	3
Average bench mark spacing not more than (km)	1.6	1.6	1.6	3.0	3.0

Order Class	First I	First II	Second I	Second II	Third
Line length between network control points not more than (km)	300	100	50	50 (double-run) 25 (single-run)	25 10

New surveys are required to tie to existing network bench marks at the beginning and end of the leveling line. These network bench marks must have an order (and class) equivalent to or better than the intended order (and class) of the new survey. First-order surveys are required to perform check connections to a minimum of six bench marks, three at each end. All other surveys require a minimum of four check connections, two at each end. "Check connection" means that the observed elevation difference agrees with the adjusted elevation difference within the tolerance limit of the new survey. Checking the elevation difference between two bench marks located on the same structure, or so close together that both may have been affected by the same localized disturbance, is not considered a proper check. In addition, the survey is required to connect to any network control points within 3 km of its path. However, if the survey is run parallel to existing control, then the following table specifies the maximum spacing of extra connections between the survey and the control. At least one extra connection should always be made.

Distance, survey to network	*Maximum spacing of extra connections (km)*
0.5 km or less	5
0.5 km to 2.0 km	10
2.0 km to 3.0 km	20

Instrumentation

Order Class	First I	First II	Second I	Second II	Third
Leveling instrument					
Minimum repeatability of line of sight	0.25"	0.25"	0.50"	0.50"	1.00"
Leveling rod construction	IDS	IDS	IDS† or ISS	ISS	Wood or Metal
Instrument and rod resolution (combined)					
Least count (mm)	0.1	0.1	0.5-1.0*	1.0	1.0

(IDS—Invar, double scale)
(ISS—Invar, single scale)
† if optional micrometer is used.
* 1.0 mm if 3-wire method, 0.5 mm if optical micrometer.

Only a compensator or tilting leveling instrument with an optical micrometer should be used for first-order leveling. Leveling rods should be one piece. Wooden or metal rods may be employed only for third-order work. A turning point consisting of a steel turning pin with a driving cap should be utilized. If a steel pin cannot be driven, then a turning plate ("turtle") weighing at least 7 kg should be substituted. In situations allowing neither turning pins nor turning plates (sandy or marshy soils), a long wooden stake with a double-headed nail should be driven to a firm depth.

Calibration Procedures

Order Class	*First I*	*First II*	*Second I*	*Second II*	*Third*
Leveling instrument					
Maximum collimation error, single line of sight (mm/m)	0.05	0.05	0.05	0.05	0.10
Maximum collimation error, reversible compensator type instruments, mean of two lines of sight (mm/m)	0.02	0.02	0.02	0.02	0.04
Time interval between collimation error determinations not longer than (days)					
Reversible compensator	7	7	7	7	7
Other types	1	1	1	1	7
Maximum angular difference between two lines of sight, reversible compensator	40"	40"	40"	40"	60"
Leveling rod					
Minimum scale calibration standard	N	N	N	M	M
Time interval between scale calibrations (yr)	1	1	—	—	—
Leveling rod bubble verticality maintained to within	10'	10'	10'	10'	10'

(N—National standard)
(M—Manufacturer's standard)

Compensator-type instruments should be checked for proper operation at least every 2 weeks of use. Rod calibration should be repeated whenever the rod is dropped or damaged in any way. Rod levels should be checked for proper alignment once a week. The manufacturer's calibration standard should, as a minimum, describe scale behavior with respect to temperature.

Field Procedures

Order Class	*First I*	*First II*	*Second I*	*Second II*	*Third*
Minimal observation method	micro-meter	micro-meter	micro-meter or 3-wire	3-wire	center wire
Section running	SRDS or DR or SP	SRDS or DR or SP	SRDS or DR† or SP	SRDS or DR*	SRDS or DR§

Field Procedures—Continued

Order Class	*First I*	*First II*	*Second I*	*Second II*	*Third*
Difference of forward and backward sight lengths never to exceed					
per setup (m)	2	5	5	10	10
per section (m)	4	10	10	10	10
Maximum sight length (m)	50	60	60	70	90
Minimum ground clearance of line of sight (m)	0.5	0.5	0.5	0.5	0.5
Even number of setups when not using leveling rods with detailed calibration	yes	yes	yes	yes	—
Determine temperature gradient for the vertical range of the line of sight at each setup	yes	yes	yes	—	—
Maximum section misclosure (mm)	$3\sqrt{D}$	$4\sqrt{D}$	$6\sqrt{D}$	$8\sqrt{D}$	$12\sqrt{D}$
Maximum loop misclosure (mm)	$4\sqrt{E}$	$5\sqrt{E}$	$6\sqrt{E}$	$8\sqrt{E}$	$12\sqrt{E}$
Single-run methods					
Reverse direction of single runs every half day	yes	yes	yes	—	—
Nonreversible compensator leveling instruments					
Off-level/relevel instrument between observing the high and low rod scales	yes	yes	yes	—	—
3-wire method					
Reading check (difference between top and bottom intervals) for one setup not to exceed (tenths of rod units)	—	—	2	2	3
Read rod 1 first in alternate setup method	—	—	yes	yes	yes
Double scale rods					
Low-high scale elevation difference for one setup not to exceed (mm)					
With reversible compensator	0.40	1.00	1.00	2.00	2.00
Other instrument types:					
Half-centimeter rods	0.25	0.30	0.60	0.70	1.30
Full-centimeter rods	0.30	0.30	0.60	0.70	1.30

(SRDS—Single-Run, Double Simultaneous procedure)
(DR—Double-Run)
(SP—SPur, less than 25 km, double-run)
D—shortest length of section (one-way) in km
E—perimeter of loop in km
† Must double-run when using 3-wire method.
* May single-run if line length between network control points is less than 25 km.
§ May single-run if line length between network control points is less than 10 km.

Double-run leveling may always be used, but single-run leveling done with the double simultaneous procedure may be used only where it can be evaluated by loop closures. Rods should be leap-frogged between setups (alternate setup method). The date, beginning and ending times, cloud coverage, air temperature (to the nearest degree), temperature scale, and average wind speed should be recorded for each section plus any changes in the date, instrumentation, observer or time zone. The instrument need not be off-leveled/releveled between observing the high and low scales when using an instrument with a reversible compensator. The low-high scale difference tolerance for a reversible compensator is used only for the control of blunders.

With double scale rods, the following observing sequence should be used:

backsight, low-scale
backsight, stadia
foresight, low-scale
foresight, stadia
off-level/relevel or reverse compensator
foresight, high-scale
backsight, high-scale

Office Procedures

Order / *Class*	*First* *I*	*First* *II*	*Second* *I*	*Second* *II*	*Third*
Section misclosures (backward and forward)					
Algebraic sum of all corrected section misclosures of a leveling line not to exceed (mm)	$3\sqrt{D}$	$4\sqrt{D}$	$6\sqrt{D}$	$8\sqrt{D}$	$12\sqrt{D}$
Section misclosure not to exceed (mm)	$3\sqrt{E}$	$4\sqrt{E}$	$6\sqrt{E}$	$8\sqrt{E}$	$12\sqrt{E}$
Loop misclosures					
Algebraic sum of all corrected misclosures not to exceed (mm)	$4\sqrt{F}$	$5\sqrt{F}$	$6\sqrt{F}$	$8\sqrt{F}$	$12\sqrt{F}$
Loop misclosure not to exceed (mm)	$4\sqrt{F}$	$5\sqrt{F}$	$6\sqrt{F}$	$8\sqrt{F}$	$12\sqrt{F}$

(D—shortest length of leveling line (one-way) in km)
(E—shortest one-way length of section in km)
(F—length of loop in km)

The normalized residuals from a minimally constrained least squares adjustment will be checked for blunders. The observation weights will be checked by inspecting the postadjustment estimate of the variance of unit weight. Elevation difference standard errors computed by error propagation in a correctly weighted least squares adjustment will indicate the provisional accuracy classification. A survey variance factor ratio will be computed to check for systematic error. The least squares adjustment will use models that account for:

gravity effect or orthometric correction
rod scale errors
rod (Invar) temperature
refraction—need latitude and longitude to 6" or vertical temperature difference observations between 0.5 and 2.5 m above the ground
earth tides and magnetic field
collimation error
crustal motion

3.6 Photogrammetry

Photogrammetry is a measurement system comprised of photographs taken by a precise metric camera and measured by a comparator. Photogrammetry is used for densification of horizontal control. The following specifications apply only to analytic methods.

Network Geometry

Order / *Class*	*Second* *I*	*Second* *II*	*Third* *I*	*Third* *II*
Forward overlap not less than	66%	66%	60%	60%
Side overlap not less than	66%	66%	20%	20%
Intersecting rays per point not less than (design criteria)	9	8	3	3

The photogrammetric survey should be areal: single strips of photography are not acceptable. The survey should encompass, ideally, a minimum of eight horizontal control points and four vertical points spaced about the perimeter of the survey. In addition, the horizontal control points should be spaced no farther apart than seven air bases. The horizontal control points should have an order (and class) better than the intended order (and class) of the survey. The vertical points need not meet geodetic control standards. If the required control points are not available, then they must be established by some other measurement system.

Instrumentation

Order / *Class*	*Second* *I*	*Second* *II*	*Third* *I*	*Third* *II*
Metric Camera				
Maximum warp of platen not more than (μm)	10	10	10	10
Dimensional control not less than	reseau with maximum spacing of 2 cm	8 fiducials	8 fiducials	8 fiducials
Comparator				
Least count (μm)	1	1	1	1

The camera should be of at least the quality of those employed for large-scale mapping. A platen should be included onto which the film must be satisfactorily flattened during exposure. Note that a reseau should be used for second-order, class I surveys.

Calibration Procedures

Order / *Class*	*Second* *I*	*Second* *II*	*Third* *I*	*Third* *II*
Metric camera				
Root mean square of calibrated radial distortion not more than (μm)	1	3	3	5

Calibration Procedures—Continued

Order *Class*	*Second* *I*	*Second* *II*	*Third* *I*	*Third* *II*
Root mean square of calibrated decentering distortion not more than (μm)	1	5†	5†	5†
Root mean square of reseau coordinates not more than (μm)	1	1	3	3
Root mean square of fiducial coordinates not more than (μm)	—	1	3	3

† not usually treated separately in camera calibration facilities; manufacturer's certification is satisfactory.

The metric camera should be calibrated every 2 years, and the comparator should be calibrated every 6 months. These instruments should also be calibrated after repair or modifications.

Characteristics of the camera's internal geometry (radial symmetric distortion, decentered lens distortion, principal point and point of symmetry coordinates, and reseau coordinates) should be determined using recognized calibration techniques, like those described in the current edition of the *Manual of Photogrammetry*. These characteristics will be applied as corrections to the measured image coordinates.

Field Procedures

Photogrammetry involves hybrid measurements: a metric camera photographs targets and features in the field, and a comparator measures these photographs in an office environment. Although this section is entitled "Field Procedures," it deals with the actual measurement process and thus includes comparator specifications.

Order *Class*	*Second* *I*	*Second* *II*	*Third* *I*	*Third* *II*
Targets				
Control points targeted	yes	yes	yes	yes
Pass points targeted	yes	yes	optional	optional
Comparator				
Pointings per target not less than	4	3	2	2
Pointings per reseau (or fiducial) not less than	4	3	2	2
Number of different reseau intersections per target not less than	4	—	—	—
Rejection limit from mean of pointings per target (μm)	3	3	3	3

Office Procedures

Order *Class*	*Second* *I*	*Second* *II*	*Third* *I*	*Third* *II*
Root mean square of adjusted photocoordinates not more than (μm)	4	6	8	12

A least squares adjustment of the photocoordinates, constrained by the coordinates of the horizontal and vertical control points, will be checked for blunders by examining the normalized residuals. The observation weights will be checked by inspecting the postadjustment estimate of the variance of unit weight. Distance standard errors computed by error propagation in this correctly weighted least squares adjustment will indicate the provisional accuracy classification. A survey variance factor ratio will be computed to check for systematic error. The least squares adjustment will use models that incorporate the quantities determined by calibration.

3.7 Satellite Doppler Positioning

Satellite Doppler positioning is a three-dimensional measurement system based on the radio signals of the U.S. Navy Navigational Satellite System (NNSS), commonly referred to as the TRANSIT system. Satellite Doppler positioning is used primarily to establish horizontal control.

The Doppler observations are processed to determine station positions in Cartesian coordinates, which can be transformed to geodetic coordinates (geodetic latitude and longitude and height above reference ellipsoid). There are two methods by which station positions can be derived: point positioning and relative positioning.

Point positioning, for geodetic applications, requires that the processing of the Doppler data be performed with the precise ephemerides that are supplied by the Defense Mapping Agency. In this method, data from a single station is processed to yield the station coordinates.

Relative positioning is possible when two or more receivers are operated together in the survey area. The processing of the Doppler data can be performed in four modes: simultaneous point positioning, translocation, semishort arc, and short arc. The specifications for relative positioning are valid only for data reduced by the semishort or short arc methods. The semishort arc mode allows up to 5 degrees of freedom in the ephemerides; the short arc mode allows 6 or more degrees of freedom. These modes allow the use of the broadcast ephemerides in place of the precise ephemerides.

The specifications quoted in the following sections are based on the experience gained from the analysis of Doppler surveys performed by agencies of the Federal government. Since the data are primarily from surveys performed within the continental United States, the precisions and related specifications may not be appropriate for other areas of the world.

Network Geometry

The order of a Doppler survey is determined by: the spacing between primary Doppler stations, the order of the base network stations from which the primaries are established, and the method of data reduction that is used. The order and class of a survey cannot exceed the lowest order (and class) of the base stations used to establish the survey.

The primary stations should be spaced at regular intervals which meet or exceed the spacing required for the desired accuracy of the survey. The primary stations will carry the same order as the survey.

Supplemental stations may be established in the same survey as the primary stations. The lowest order (and class) of a supplemental station is determined either by its spacing with, or by the order of, the nearest Doppler or other horizontal control station. The processing mode determines the allowable station spacing.

In carrying out a Doppler survey, one should occupy, using the same Doppler equipment and procedures, at least two existing horizontal network (base) stations of order (and class) equivalent to, or better than, the intended order (and class) of the Doppler survey. If the Doppler survey is to be first-order, at least three base stations must be occupied. If relative positioning is to be used, all base station base lines must be directly observed during the survey. Base stations should be selected near the perimeter of the survey, so as to encompass the entire survey.

Stations which have a precise elevation referenced by geodetic leveling to the National Geodetic Vertical Datum (NGVD) are preferred. This will allow geoidal heights to be determined. As many base stations as possible should be tied to the NGVD. If a selection is to be made, those stations should be chosen which span the largest portion of the survey.

If none of the selected base stations is tied to the NGVD, at least two, preferably more, bench marks of the NGVD should be occupied. An attempt should be made to span the entire survey area.

Datum shifts for transformation of point position solutions should be derived from the observations made on the base stations.

The minimum spacing, D, of the Doppler stations may be computed by a formula determined by the processing mode to be employed. This spacing is also used in conjunction with established control, and other Doppler control, to determine the order and class of the supplemental stations.

By using the appropriate formula, tables can be constructed showing station spacing as a function of point or relative one-sigma position precision (s_p or s_r) and desired survey (or station) order.

Point Positioning

$$D = 2\sqrt{2}\, s_p a$$

where

a = denominator of distance accuracy classification standard (e.g., a = 100,000 for first-order standard).

Order	*First*	*Second*	*Second*	*Third*	*Third*
Class		*I*	*II*	*I*	*II*
s_p (cm)			D (km)		
200	566	242	114	56	28
100	283	141	57	28	14
70	200	100	40	20	10
50	141	71	26	14	7

Relative Positioning

$$D = 2\, s_r a$$

where

a = denominator of distance accuracy classification standard (e.g., a = 100,000 for first-order standard).

Order	*First*	*Second*	*Second*	*Third*	*Third*
Class		*I*	*II*	*I*	*II*
s_r (cm)			D (km)		
50	100	50	20	10	5
35	70	35	14	7	4
20	40	20	8	4	2

However, the spacing for relative positioning should not exceed 500 km.

Instrumentation

The receivers should receive the two carrier frequencies transmitted by the NNSS. The receivers should record the Doppler count of the satellite, the receiver clock times, and the signal strength. The integration interval should be approximately 4.6 sec. Typically six or seven of these intervals are accumulated to form a 30-second Doppler count observation. The reference frequency should be stable to within $5.0(10^{-11})$ per 100 sec. The maximum difference from the average receiver delay should not exceed 50 μsec. The best estimate of the mean electrical center of the antenna should be marked. This mark will be the reference point for all height-of-antenna measurements.

Calibration Procedures

Receivers should be calibrated at least once a year, or whenever a modification to the equipment is made. It is desirable to perform a calibration before every project to verify that the equipment is operational. The two-receiver method explained next is preferred and should be used whenever possible.

Two-Receiver Method

The observations are made on a vector base line, of internal accuracy sufficient to serve as a comparison standard, 10 to 50 m in length. The base line should be located in an area free of radio interference in the 150 and 400 MHz frequencies. The procedures found in the table on relative positioning in "Field Procedures" under the 20 cm column heading will be used. The data are reduced by either shortarc or semishort arc methods. The receivers will be considered operational if the differences between the Doppler and the terrestrial base line components do not exceed 40 cm (along any coordinate axis).

Single-Receiver Method

Observations are made on a first-order station using the procedures found in the table on relative positioning in "Field Procedures" under the 50 cm column heading. The data are reduced with the precise ephemerides. The resultant position must agree within 1 m of the network position.

Field Procedures

The following tables of field procedures are valid only for measurements made with the Navy Navigational Satellite System (TRANSIT).

Point Positioning

s_p *(precise ephemerides)*	*50 cm*	*70 cm*	*100 cm*	*200 cm*
Max. standard deviation of mean of counts/pass (cm), broadcast ephemerides	25	25	25	25
Period of observation not less than (hr)	48	36	24	12
Number of observed passes not less than†	40	30	15	8
Number of acceptable passes (evaluated by on-site point processing) not less than	30	20	9	4
Minimum number of acceptable passes within each quadrant*	6	4	2	1
Frequency standard warm-up time (hr)				
crystal	48	48	24	24
atomic	1.5	1.5	1.0	1.0
Maximum interval between meteorological observations (hr)	6	§	§	§

† Number of passes refers to those for which the precise ephemerides are available for reduction.

* There should be a nearly equal number of northward and southward passes.

§ each setup, visit and takedown.

Relative positioning

s_r	*20 cm*	*35 cm*	*50 cm*
Maximum standard deviation of mean of counts/pass (cm), broadcast ephemerides	25	25	25
Period of observation not less than (hr)	48	36	24
Number of observed passes not less than†	40	30	15
Number of acceptable passes (evaluated by on-site point position processing) not less than	30	20	9
Minimum number of acceptable passes within each quadrant*	6	4	2
Frequency standard warm-up time (hr)			
crystal	48	48	48
atomic	1.5	1.5	1.5
Maximum interval between meteorological observations (hr)	6	6	§

† Number of observed passes refers to all satellites available for tracking and reduction with the broadcast or precise ephemerides.

* Number of northward and southward passes should be nearly equal.

§ Each setup, visit and takedown.

The antenna should be located where radio interference is minimal for the 150 and 400 MHz frequencies. Medium frequency radar, high voltage power lines, transformers, excessive noise from automotive ignition systems, and high power radio and television transmission antennas should be avoided. The horizon should not be obstructed above 7.5°.

The antenna should not be located near metal structures, or, when on the roof of a building, less than 2 m from the edge. The antenna must be stably located within 1 mm over the station mark for the duration of the observations. The height difference between the mark and the reference point for the antenna phase center should be measured to the nearest millimeter. If an antenna is moved while a pass is in progress, that pass is not acceptable. If moved, the antenna should be relocated within 5 mm of the original antenna height; otherwise the data may have to be processed as if two separate stations were established. In the case of a reoccupation of an existing Doppler station, the antenna should be relocated within 5 mm of the original observing height.

Long-term reference frequency drift should be monitored to ensure it does not exceed the manufacturer's specifications.

Observations of temperature and relative humidity should be collected, if possible, at or near the height of the phase center of the antenna. Observations of wet-bulb and dry-bulb temperature readings should be recorded to the nearest 0.5°C. Barometric readings at the station site should be recorded to the nearest millibar and corrected for difference in height between the antenna and barometer.

Office Procedures

The processing constants and criteria for determining the quality of point and relative positioning results are as follows:

1. For all passes for a given station occupation, the average number of Doppler counts per pass should be at least 20 (before processing).
2. The cutoff angle for both data points and passes should be 7.5°.
3. For a given pass, the maximum allowable rejection of counts, 3 sigma postprocessing, will be 10.
4. Counts rejected (excluding cutoff angle) for a solution should be less than 10 percent.
5. Depending on number of passes and quality of data, the standard deviation of the range residuals for all passes of a solution should range between:
   Point positioning—10 to 20 cm
   Relative positioning—5 to 20 cm

A minimally constrained least squares adjustment will be checked for blunders by examining the normalized residuals. The observation weights will be checked by inspecting the postadjustment estimate of the variance of unit weight. Distance standard errors computed by error propagation between points in this correctly weighted least squares adjustment will indicate the maximum achievable accuracy classification. The formula presented in "Standards" will be used to arrive at the actual classification. The least squares adjustment will use models which account for:

- tropospheric scale bias, 10 percent uncertainty
- receiver time delay
- satellite/receiver frequency offset
- precise ephemeris
- tropospheric refraction
- ionospheric refraction
- long-term ephemeris variations
- crustal motion

3.8 Absolute Gravimetry

Absolute gravimetry is a measurement system which determines the magnitude of gravity at a station at a specific time. Absolute gravity measurements are used to establish and extend gravity control. Within the context of a geodetic gravity network, as discussed in "Standards," a series of absolute measurements at a control point is in itself sufficient to establish an absolute gravity value for that location.

The value of gravity at a point is time dependent, being subject to dynamic effects in the Earth. The extent of gravimetric stability can be determined only by repeated observations over many years.

Network Geometry

Network geometry cannot by systematized since absolute observations at a specific location are discrete and uncorrelated with other points. In absolute gravimetry, a network may consist of a single point.

A first-order, class I station must possess gravimetric stability, which only repeated measurements can determine. This gravimetric stability should not be confused with the accuracy determined at a specific time. It is possible for a value to be determined very precisely at two different dates and for the values at each of these respective dates to differ. Although the ultimate stability of a point cannot be determined by a single observation session, an attempt should be made to select sites which are believed to be tectonically stable, and sufficiently distant from large bodies of water to minimize ocean tide coastal loading.

The classification of first-order, class I is reserved for network points which have demonstrated long-term stability. To ensure this stability, the point should be reobserved at least twice during the year of establishment and thereafter at sufficient intervals to ensure the continuing stability of the point. The long-term drift should indicate that the value will not change by more than 20 μGal for at least 5 years. A point intended as first-order, class I will initially be classified as first-order, class II until stability during the first year is demonstrated.

Instrumentation

The system currently being used is a ballistic-laser device and is the only one at the current state of technology considered sufficiently accurate for absolute gravity measurements. An absolute instrument measures gravity at a specific elevation above the surface, usually about 1 m. For this reason, the gravity value is referenced to that level. A measurement of the vertical gravity gradient, using a relative gravity meter and a tripod, must be made to transfer the gravity value to ground level. The accuracy of the relative gravimeter must satisfy the gravity gradient specifications found in "Field Procedures."

Calibration Procedures

Ballistic-laser instruments are extremely delicate and each one represents a unique entity with its own characteristics. It is impossible to identify common systematic errors for all instruments. Therefore, the manufacturer's recommendations for individual instrument calibration should be followed rigorously.

To identify any possible bias associated with a particular instrument, comparisons with other absolute devices are strongly recommended whenever possible. Comparisons with previously established first-order, class I network points, as well as first-order, class II network points tied to the class I points, are also useful.

Field Procedures

The following specifications were determined from results of a prototype device built by J. Faller and M. Zumberge (Zumberge, M., "A Portable Apparatus for Absolute Measurements of the Earth's Gravity," Department of Physics, University of Colorado, 1981) and are given merely as a guideline. It is possible that some of these values may be inappropriate for other instruments or models. Therefore, exceptions to these specifications are allowed on a case-by-case basis upon the recommendation of the manufacturer. Deviations from the specifications should be noted upon submission of data for classification.

Order *Class*	*First* *I*	*First* *II*	*Second*	*Third*
Absolute measurement				
Standard deviation of each accepted measurement set not to exceed (μGal)	20	20	50	100
Minimum number of sets/observation	5	5	5	5
Maximum difference of a measurement set from mean of all measurements (μGal)	12	12	37	48
Barometric pressure standard error (mbar)	4	4	—	—
Gradient measurement				
Standard deviation of measurement of vertical gravity gradient at time of observation (μGal/m)	5	5	5	5
Standard deviation of height of instrument above point (mm)	1	1	5	10

Office Procedures

The manufacturer of an absolute gravity instrument usually provides a reduction process which identifies and accounts for error sources and identifiable parameters. This procedure may be sufficient, making further office adjustments unnecessary.

A least squares adjustment will be checked for blunders by examining the normalized residuals. The observation weights will be checked by inspecting the postadjustment estimate of the variance of unit weight. Gravity value standard deviations computed by error propagation in a correctly weighted, least squares adjustment will indicate the provisional accuracy classification. The least squares adjustment, as well as digital filtering techniques and/or sampling, should use models which account for:

atmospheric mass attraction
microseismic activity
instrumental characteristics
lunisolar attraction
elastic and plastic response of the Earth (tidal loading)

3.9 Relative Gravimetry

Relative gravimetry is a measurement system which determines the difference in magnitude of gravity between two stations. Relative gravity measurements are used to extend and densify gravity control.

Network Geometry

A first-order, class I station must possess gravimetric stability, which only repeated measurements can determine. This gravimetric stability should not be confused with the accuracy determined at a specific time. It is possible for a value to be determined very precisely at two different dates, and for the values at each of these respective dates to differ. Although the ultimate stability of a point cannot be determined by a single observation session, an attempt should be made to select sites which are believed to be tectonically stable.

The classification of first-order, class I is reserved for network points that have demonstrated long-term stability. To ensure this stability, the point should be reobserved at least twice during the year of establishment and thereafter at sufficient intervals. The long-term drift should indicate that the value will not change by more than the 20 μGal for at least 5 years. A point intended as first-order, class I will initially be classified as first-order, class II until stability during the first year is demonstrated.

The new survey is required to tie at least two network points, which should have an order (and class) equivalent to or better than the intended order (and class) of the new survey. This is required to check the validity of existing network points as well as to ensure instrument calibration. Users are encouraged to exceed this minimal requirement. However, if one of the network stations is a first-order, class I mark, then that station alone can satisfy the minimum connecting requirement if the intended order of the new survey is less than first-order.

Instrumentation

Regardless of the type of a relative gravimeter, the internal error is of primary concern.

Order *Class*	*First* *I*	*First* *II*	*Second*	*Third*
Minimum instrument internal error (one-sigma), (μGal)	10	10	20	30

The instrument's internal accuracy may be determined by performing a relative survey over a calibration line (see below) and examining the standard deviation of a single reading. This determination should be performed after the instrument is calibrated using the latest calibration information. Thus the internal error is the measure of instrument uncertainty after all possible systematic error sources have been eliminated by calibration.

Calibration Procedures

An instrument should be properly calibrated before a geodetic survey is performed. The most important calibration item is the determination of the mathematical model that relates dial units, voltage, or some other observable to milligals. This may consist only of a scale factor. In other cases the model may demonstrate nonlinearity or periodicity. Most manufacturers provide tables or scale factors with each instrument. Care must be taken to ensure the validity of these data over time.

When performing first-order work, this calibration model should be determined by a combination of bench tests and field measurements. The bench tests are specified by the manufacturer. A field calibration should be performed over existing control points of first-order, class I or II. The entire usable gravimeter range interval should be sampled to ensure an uncertainty of less than 5 μGal. FGCC member agencies have established calibration lines for this specific purpose.

The response of an instrument to air pressure and temperature should be determined. The meter should be adjusted or calibrated for various pressures and temperatures so that the allowable uncertainty from these sources does not exceed the values in the table below.

The manufacturer's recommendations should be followed to ensure that all internal criteria, such as galvanometer sensitivity, long and cross level or tilt sensitivity, and reading line, are within the manufacturer's allowable tolerances.

The response of an instrument due to local orientation should also be determined. Systematic differences may be due to an instrument's sensitivity to local magnetic variations. Manufacturers attempt to limit or negate such a response. However, if a meter displays a variation with respect to orientation, then one must either have the instrument repaired by the manufacturer, or minimize the effect by fixing the orientation of the instrument throughout a survey.

Order Class	First I	First II	Second	Third
Necessary for user to determine calibration model	Yes	Yes	Yes	No
Allowable uncertainty of calibration model (μGal)	5	5	10	15
Allowable uncertainty due to external air temperature changes (μGal)	1	1	3	—
Maximum uncertainty due to external air pressure changes (μGal)	1	1	2	—
Allowable uncertainty due to other factors (μGal)	3	3	5	—

Field Procedures

A relative gravity survey is performed using a sequence of measurements known as a loop sequence. There are three common types: ladder, modified ladder, and line.

The ladder sequence begins and ends at the same network point, with the survey points being observed twice during the sequence: once in forward running and once in backward running. Of course, more than one network point may be present in a ladder sequence.

Order Class	First I	First II	Second	Third
Minimum number of instruments used in survey	2	2	2	1
Recommended number of instruments used in survey	3	3	2	1
Allowable loop sequence	a	a	a,b	a,b,c
Minimum number of readings at each observation/instrument	5	5	2†	1
Standard deviation of consecutive readings (unclamped) from mean* not to exceed (μGal)	2	2	5	—
Monitor external temperature and air pressure	Yes	Yes	No	No
Standard deviation of temperature measurements (°C)	0.1	0.1	—	—
Standard deviation of air pressure measurement (mbar)	1	1	—	—
Standard deviation of height of instrument above point (mm)	1	1	5	10

(a—ladder) (b—modified ladder) (c—line)
† Although two readings are required, only one reading need be recorded.
* corrected for lunisolar attraction.

The modified ladder sequence also begins and ends at the same network point. However, not all the survey points are observed twice during the sequence. Again, more than one network point may be observed in the sequence.

The line sequence begins at a network point and ends at a different network point. A survey point in a line sequence is usually observed only once.

One should always monitor the internal temperature of the instrument to ensure it does not fluctuate beyond the manufacturer's recommended limits. The time of each reading should be recorded to the nearest minute.

Office Procedures

Order	First I	First II	Second	Third
Rejection Limits				
Maximum standard error of a gravity value (μGal)	20	20	50	100
Total allowable instrument uncertainty (μGal)	10	10	20	30
Model Uncertainties				
Uncertainty of atmospheric mass model (μGal)	0.5	0.5	—	—
Uncertainty of lunisolar attraction (μGal)	1	1	5	5
Uncertainty of Earth elastic and plastic response to tidal loading (μGal)	2	2	5	—

A least squares adjustment, constrained by the network configuration and precision of established gravity control, will be checked for blunders by examining the normalized residuals. The observation weights will be checked by inspecting the postadjustment estimate of the variance of unit weight. Gravity standard errors computed by error propagation in a correctly weighted least squares adjustment will indicate the provisional accuracy classification. A survey variance factor ratio will be computed to check for systematic error. The least squares adjustment will use models which account for:

instrument calibrations
1) conversion factors (linear and higher order)
2) thermal response (if necessary)
3) atmospheric pressure response (if necessary)

instrument drift
1) static
2) dynamic

atmospheric mass attraction (if necessary)

Earth tides
1) lunisolar attraction
2) Earth elastic and plastic response (if necessary)

4. Information

Geodetic control data and cartographic information that pertain to the National Geodetic Control Networks are widely distributed by a component of the National Geodetic Survey, the National Geodetic Information Branch (NGIB). Users of this information include Federal, State, and local agencies, universities, private companies, and individuals. Data are furnished in response to individual orders, or by an automatic mailing service (the mechanism whereby users who maintain active geodetic files automatically receive newly published data for specified areas). Electronic retrieval of data can be carried out directly from the NGS data base by a user.

Geodetic control data for the national networks are primarily published as standard quadrangles of 30′ in latitude by 30′ in longitude. However, in congested areas, the standard quadrangles are 15′ in latitude by 15′ in longitude. In most areas of Alaska, because of the sparseness of control, quadrangle units are 1° in latitude by 1° in longitude. Data are now available in these formats for all horizontal control and approximately 65 percent of the vertical control. The remaining 35 percent are presented in the old formats; i.e., State leveling lines and description booklets. Until the old format data have been converted to the standard quadrangle formats, the vertical control data in the unconverted areas will be available only by complete county coverage. Field data and recently adjusted projects with data in manuscript form are available from NGS upon special request. The National Geodetic Control Networks are cartographically depicted on approximately 850 different control diagrams. NGS provides other related geodetic information: e.g., geoid heights, deflections of the vertical, calibration base lines, gravity values, astronomic positions, horizontal and vertical data for crustal movement studies, satellite-derived positions, UTM coordinates, computer programs, geodetic calculator programs, and reference materials from the NGS data bases.

The NGIB receives data from all NOAA geodetic field operations and mark-recovery programs. In addition, other Federal, State, and local governments, and private organizations contribute survey data from their field operations. These are incorporated into the NGS data base. NOAA has entered into formal agreements with several Federal and State Government agencies whereby NGIB publishes, maintains, and distributes geodetic data received from these organizations. Guidelines and formats have been established to standardize the data for processing and inclusion into the NGS data base. These formats are available to organizations interested in participating in the transfer of their files to NOAA (appendix C).

Upon completion of the geodetic data base management system, information generated from the data base will be automatically revised. A new data output format is being designed for both horizontal and vertical published control information. These formats, which were necessitated by the requirements of the new adjustments of the horizontal and vertical geodetic networks, will be more comprehensive than the present versions.

New micropublishing techniques are being introduced in the form of computer-generated microforms. Some geodetic data are available on magnetic tape, microfilm, and microfiche. These services will be expanded as the automation system is fully implemented. Charges for digital data are determined on the basis of the individual requests, and reflect processing time, materials, and postage. The booklets *Publications of the National Geodetic Survey* and *Products and Services of the National Geodetic Survey* are available from NGIB.

For additional information, write:

Chief, National Geodetic Information
Branch, N/CG17
National Oceanic and Atmospheric Administration
Rockville, MD 20852

To order by telephone:

data: 301-443-8631
publications: 301-443-8316
computer programs or digital data: 301-443-8623

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APPENDIX A

Governmental Authority

A.1 Authority

The U.S. Department of Commerce's National Oceanic and Atmospheric Administration (NOAA) is responsible for establishing and maintaining the basic national horizontal, vertical, and gravity geodetic control networks to meet the needs of the Nation. Within NOAA this task is assigned to the National Geodetic Survey, a Division of the Office of Charting and Geodetic Services within the National Ocean Service. This responsibility has evolved from legislation dating back to the Act of February 10, 1807 (2 Stat. 413), which created the first scientific Federal agency, known as the "Survey of the Coast." Current authority is contained in United States Code, Title 33, USC 883a, as amended, and specifically defined by Executive Directive, Bureau of the Budget (now the Office of Management and Budget) Circular No. A-16, Revised (Bureau of the Budget 1967).

To coordinate national mapping, charting, and surveying activities, the Board of Surveys and Maps of the Federal Government was formed December 30, 1919, by Executive Order No. 3206. "Specifications for Horizontal and Vertical Control" were agreed upon by Federal surveying and mapping agencies and approved by the Board on May 9, 1933. When the Board was abolished March 10, 1942, its functions were transferred to the Bureau of the Budget, now the Office of Management and Budget, by Executive Order No. 9094. The basic survey specifications continued in effect. Bureau of the Budget Circular No. A-16, published January 16, 1953, and revised May 6, 1967 (Bureau of the Budget 1967), provides for the coordination of Federal surveying and mapping activities. "Classification and Standards of Accuracy of Geodetic Control Surveys," published March 1, 1957, replaced the 1933 specifications. Exhibit C to Circular A-16, dated October 10, 1958 (Bureau of the Budget 1958), established procedures for the required coordination of Federal geodetic and control surveys performed in accordance with the Bureau of the Budget classifications and standards.

The Federal Geodetic Control Committee (FGCC) was chartered December 11, 1968, and a Federal Coordinator for Geodetic Control and Related Surveys was appointed April 4, 1969. The FGCC Circular No. 1, "Exchange of Information," dated October 16, 1972, prescribes reporting procedures for the committee (vice Exhibit C of Circular A-16) (Federal Geodetic Control Committee 1972).

The Federal Coordinator for Geodetic Control and Related Surveys, Department of Commerce, is responsible for coordinating, planning, and executing national geodetic control surveys and related survey activities of Federal agencies, financed in whole or in part by Federal funds. The Executive Directive (Bureau of the Budget 1967: p. 2) states:

(1) The geodetic control needs of Government agencies and the public at large are met in the most expeditious and economical manner possible with available resources; and

(2) all surveying activities financed in whole or in part by Federal funds contribute to the National Networks of Geodetic Control when it is practicable and economical to do so.

The Federal Geodetic Control Committee assists and advises the Federal Coordinator for Geodetic Control and Related Surveys.

A.2 References

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APPENDIX B
Variance Factor Estimation

B.1 Introduction

The classification accuracies for the National Geodetic Control Networks measure how well a survey can provide position, elevation, and gravity. (More specifically, a distance accuracy is used for horizontal networks, and an elevation difference accuracy is used for vertical networks.) The interpretation of what is meant by "how well" contains two parts. A survey must be precise, i.e., fairly free of random error; it must also be accurate, i.e., relatively free of systematic error. This leads to a natural question of how to test for random and systematic error.

Testing for random error is an extremely broad subject, and is not examined here. It is assumed that the standard deviation of distance, elevation difference, or gravity provides an adequate basis to describe the amount of random error in a survey. Further, it is assumed that the selection of the worst instance of the classification accuracy computed at all points (or between all pairs of points) provides a satisfactory means of classifying a new survey. This procedure may seem harsh, but it allows the user of geodetic control to rely better upon a minimum quality of survey work. The nominal quality of a survey could be much higher.

Consider the method of observation equations (see Mikhail (1976) for a general discussion):

$$L_a = F(X_a)$$

where

L_a is a vector of computed values for the observations of dimension n,

X_a is a vector of coordinate and model parameters of dimension u, and

F is a vector of functions that describes the observations in terms of the parameters.

The design matrix, A, is defined as

$$A = \left.\frac{\partial F}{\partial X_a}\right|_{X_a = X_o}$$

where A is a matrix of differential changes in the observation model F with respect to the parameters, X_a, evaluated at a particular set of parameter values, X_o. A vector of observation misclosures is

$$L = L_b - L_a$$

where L_b is the vector of actual observations and L_a is the vector described above.

Associated with the observation vector L_b is a symmetric variance-covariance matrix Σ_{L_b}, which contains information on observation precision and correlation.

The observation equation may now be written in linearized form

$$AX = L + V$$

where V is a vector of residual errors and X is a vector of corrections to the parameter vector X_a. The least squares estimate of X is

$$X = (A^t(\Sigma_{L_b})^{-1}A)^{-1}\, A^t(\Sigma_{L_b})^{-1}L$$

where the superscripts ^{t} and $^{-1}$ denote transpose and inverse (of a matrix) respectively.

The estimate provides a new set of values for the parameters by

$$X_a + X \rightarrow X_a$$

If the observation model $F(X_a)$ is nonlinear (that is, A is not constant for any set of X_a), then the entire process, starting with the first equation, must be iterated until the vector X reaches a stationary point.

Once convergence is achieved, L_a, computed from the first equation, is the vector of adjusted observations. The vector of observation residual errors, V, is

$$V = L_a - L_b$$

Estimates of parameter precision and correlations are given by the adjusted parameter variance-covariance matrix, Σ_{X_a} computed by

$$\Sigma_{X_a} = (A^t(\Sigma_{L_b})^{-1}A)^{-1}.$$

The precision of any other quantity that can be derived from the parameters may also be computed. Suppose one wishes to compute a vector of quantities, S,

$$S = S(X_a)$$

from the adjusted parameters, X_a. A geometry matrix, G, is defined as

$$G = \left.\frac{\partial S}{\partial X_a}\right| X_a = X_o$$

where G is a matrix of differential changes in the functions, S, with respect to the parameters, X_a, evaluated at a particular set of parameter values, X_o. By the principle of linear error propagation,

$$\Sigma_S = G\,\Sigma_{X_a} G^t$$

or

$$\Sigma_S = G(A^t(\Sigma_{L_b})^{-1}A)^{-1}\,G^t$$

where Σ_S is the variance-covariance matrix of the computed quantities.

This last equation is important since its terms are variances and covariances such as those for distance or height difference. Use of this equation assumes that the model is not too nonlinear, that the parameter vector X_a has been adequately estimated by the method of least squares, that the design matrix A, the geometry matrix G, and the variance-covariance matrix of the observations Σ_{L_b} are known. This last assumption is the focal point for the remainder of this appendix.

We must somehow estimate the n (n + 1)/2 elements of Σ_L. Usually, we know Σ_L subject to some global variance factor, f. We would then assume that

$$\Sigma_L = f\,\Sigma_L^o$$

where

Σ_L = the "true" variance-covariance matrix of the observations

Σ_L^o = initial estimate of variance-covariance matrix of the observations

Our assumption about the the structure of Σ_L^o relative to a single factor usually suffices. But this assumption can be improved if we generalize the idea. Consider a partition of the observations into k homogeneous groups. We now estimate k different local variance factors

$$\sum_L = \begin{pmatrix} f_1\Sigma_{L_1}^0 & & & 0 \\ & f_2\Sigma_{L_2}^0 & & \\ & & \ddots & \\ 0 & & & f_k\Sigma_{L_k}^0 \end{pmatrix}$$

As will be discussed later, we may also detect systematic error if one of the variance components is based on certified network observations.

B.2 Global Variance Factor Estimation (k = 1)

The global variance factor, f, is simply the a posteriori variance of unit weight, $\hat{\sigma}_0^2$, when given an a priori variance of unit weight, σ_0^2, equal to 1.

It can be shown that

$$E(V^t(\Sigma_L)^{-1}V) = n - u. \qquad \text{(Mikhail 1976: p. 287)}$$

For a single variance factor

$$\Sigma_L = f\,\Sigma_L^o$$

so that

$$\frac{1}{f}\Sigma(V^t(\Sigma_L^0)^{-1}V) = n - u$$

or for f to be unbiased (Hamilton 1964, p. 130)

$$f = \frac{E(V^t(\Sigma_L^0)^{-1}V)}{n-u} = \frac{V^t(\Sigma_L^0)^{-1}V}{n-u}.$$

This is identical to the form $\hat{\sigma}_0^2 = \frac{V^tPV}{n-u}$, where P is defined as $\sigma_0^2(\Sigma_L^0)^{-1}$

Since we are given that $\sigma_0^2 = 1$, then $P = (\Sigma_L^o)^{-1}$. Then $f = \hat{\sigma}_0^2$, as we wished to prove.

The derivation assumes that there is no bias in the residuals (Mikhail 1976), i.e.,

$$E(V) = 0.$$

However, outliers, as well as systematic errors, can produce a biased global variance factor. We must be satisfied that the observations contain no blunders, and that our mathematical model is satisfactory in order to use the global variance factor.

Particular types of systematic errors—global scale or orientation errors—are not detectable in a survey adjustment. They will not bias the residuals and will not influence the global variance factor. For example, to detect a global scale error, it must be transformed into a local scale error by addition of more data or measurements that can discriminate between global and local.

B.3 Local Variance Factor Estimation (k = 2,3,...)

Let us separate our observations into k homogeneous groups, and assume that we know the variance-covariance matrices of all k groups, $\Sigma_{L_i}^o$, subject to k local variance factors, f_i. Then

$$\sum_L = \begin{pmatrix} f_1\Sigma_{L_1}^0 & & & 0 \\ & f_2\Sigma_{L_2}^0 & & \\ & & \ddots & \\ 0 & & & f_k\Sigma_{L_k}^0 \end{pmatrix}$$

A variety of methods has been proposed that can be used to estimate local variance factors. Among them are MInimum Norm Quadratic Unbiased Estimation (MINQUE) (Rao 1971), Iterated MInimum Norm Quadratic Estimation (IMINQE) (Rao 1972), Almost Unbiased Estimation (AUE) (Horn et al. 1975), and Iterated Almost Unbiased Estimation (IAUE) (Lucas 1984). Underlying these methods is the assumption that there is no bias in any group of residuals; that is

$$E(V_k) = 0.$$

This assumption can be turned to our advantage in the detection of local systematic error.

Consider the partition of observations into a network group, subscript N, and a survey group, subscript S ($k = 2$). Then

$$\Sigma_L = \begin{pmatrix} f_N\Sigma_N^0 & 0 \\ 0 & f_S\Sigma_S^0 \end{pmatrix}$$

For an adjustment of the network only, we may estimate

$$\Sigma_N' = f_N'\Sigma_N^0$$

and for an adjustment of the survey only, we may estimate

$$\Sigma_S' = f_S'\Sigma_S^0$$

where f'_S is the global variance factor of the survey observations computed by a least squares adjustment free of outliers and known systematic errors.

With perfect information and an unbiased model we compute $f_N = f'_N$ and $f_S = f'_S$. On the other hand, if our model is biased, this may not be the case. In other words, we have a linkage between systematic error and consistent estimation of local variance factors.

Now assume that our network observations are certified as having no systematic error, and that we have perfect knowledge of their weights. Then $f'_N = 1$ and $\Sigma_N = \Sigma_N^0$. In the absence of residual bias in the survey, we should compute $f_N = 1$ and $f_S = f'_S$. In fact, we could impose a constraint on the computation, $f_N = 1$, to ensure this result. A survey systematic error could then manifest itself as an increase in f_S over f'_S.

There is no guarantee that systematic error in a survey will increase f_S over f'_S. For example, a survey may be connected to the network at only one control point. A scale error local to the survey would remain undetectable with combined variance factor estimation. With a second connection to the network, the survey scale error will begin to be detectable. As the survey is more closely connected to the network, the capability to detect a survey scale error becomes much better. We see that systematic error in a survey that is well-connected to a certified geodetic network can be discovered by local variance factor estimation. Of course a systematic error, such as a scale factor influencing both the network and the survey, would continue to remain hidden.

B.4 Iterated Almost Unbiased Estimation (IAUE)

The IAUE method (Lucas 1984) can be used to estimate covariance elements as well as the variance elements of Σ_L. However, in testing for systematic error we are concerned only with the survey and the network variance factors ($k = 2$).

As suggested by the title, the method is iterative. We start with the initial values

$$f_S^0 \text{ and } \Sigma_S^0, \text{ with } f_N^0 \text{ set to } 1 \ .$$

Let

$$\Sigma_L^0 = \begin{pmatrix} f_N^0\Sigma_N^0 & 0 \\ 0 & f_S^0\Sigma_S^0 \end{pmatrix}$$

$$P_L^0 = (\Sigma_L^0)^{-1} = \begin{pmatrix} P_N^0 & 0 \\ 0 & P_S^0 \end{pmatrix}$$

We now iterate from $i = 0$ to convergence

1) Perform least squares adjustment for

$$\hat{X} = (A^tP_L^iA)^{-1}\, A^tP_L^iL \ .$$

2) $\Sigma_{\hat{V}_S}^i = (P_S^i)^{-1} - A_S(A^tP_L^iA)^{-1}\, A_S^t$.

3) $$f_S^{i+1} = \frac{(V_S^i)^tP_S^iV_S^i}{tr(\Sigma_{\hat{V}_S}^iP_S^i)}$$

where tr is the trace function.

4) $\Sigma_S^{i+1} = f_S^{i+1}\Sigma_S^i$.

We test for convergence by

$$\frac{f_S^{i+1} - f_S^i}{f_S^i} < \epsilon$$

where ϵ is a preset quantity > 0. The local survey variance factor is

$$f_S = \prod_{i=0}^{m} f_S^i$$

where m is the number of iterations to convergence. We can then compute a survey variance factor ratio,

$$f_S/f'_S$$

Computer simulations have shown that when the survey variance factor ratio exceeds 1.5, then the survey contains systematic error. This rule becomes less reliable when a survey is minimally connected to a network.

We note that for k = 1, the third step of the method yields

$$f^{i+1} = \frac{(V'PV)^i}{n-u} .$$

It is immediately recognized as the a posteriori estimate of the variance of unit weight. In this special case, IAUE convergence is correct, immediate, and unbiased.

The IAUE method is particularly attractive from a computational point of view. If Σ_L is diagonal, or nearly so, then the requisite elements of Σ_L may be computed from elements of Σ_X that lie completely within the profile of the normal equations. Thus, the usual apparatus of sparse least squares adjustments can be retained.

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APPENDIX C
Procedures for Submitting Data to the National Geodetic Survey

The National Geodetic Survey (NGS) has determined that the value to the national network of geodetic observations performed by other Federal, State, and local organizations compensates for the costs of analyzing, adjusting, and publishing the associated data. Consequently, a procedure has been established for data from horizontal, vertical, and gravity control surveys to be submitted to NGS. Persons submitting data must adhere to the requirements stated herein, but in any event, the final decision of acceptance on data will be the responsibility of the Chief, NGS.

The survey data must be submitted in the format specified in the Federal Geodetic Control Committee (FGCC) publication, *Input Formats and Specifications of the National Geodetic Survey Data Base,* which describes the procedures for submission of data for adjustment and assimilation into the National Geodetic Survey data base. Volume I (Horizontal control data), volume II (Vertical control data) or volume III (Gravity control data) may be purchased from:

National Geodetic Information Branch (N/CG17x2)
National Oceanic and Atmospheric Administration
Rockville, MD 20852

Horizontal control surveys must be accomplished to at least third-order, class I standards and tied to the National Geodetic Horizontal Network. Vertical control surveys must be accomplished in accordance with third-order or higher standards and tied to the National Geodetic Vertical Network. Gravity control surveys must be accomplished to at least second-order standards and tied to the National Geodetic Gravity Network. Third-order gravity surveys ("detail" surveys) will be accepted by NGS for inclusion into the NGS Gravity Working Files only in accordance with the above mentioned FGCC publication. A clear and accurate station description should be provided for all control points.

The original field records (or acceptable copies), including sketches, record books, and project reports, are required. NGS will retain these records in the National Archives. This is necessary if questions arise concerning the surveys on which the adjusted data are based. In lieu of the original notes, high quality photo copies and microfilm are acceptable. The material in the original field books or sheets are needed, not the abstracts or intermediate computations.

Reconnaissance reports should be submitted before beginning the field measurements, describing proposed connections to the national network, the instrumentation, and the field procedures to be used. This will enable NGS to comment on the proposed survey, drawing on the information available in the NGS data base concerning the accuracy and condition of these points, and to determine if the proposed survey can meet its anticipated accuracy. This project review saves the submitting agency the expense of placing data that would fail to meet accuracy criteria into computer-readable form.

APPENDIX A-12

- GEOMETRIC GEODETIC ACCURACY STANDARDS AND SPECIFICATIONS FOR USING GPS RELATIVE POSITIONING TECHNIQUES

GEOMETRIC GEODETIC ACCURACY STANDARDS

AND

SPECIFICATIONS FOR USING GPS RELATIVE POSITIONING TECHNIQUES

FEDERAL GEODETIC CONTROL COMMITTEE

Rear Adm. Wesley V. Hull, Chairman

Version 5.0: May 11, 1988
Reprinted with corrections: August 1, 1989

Note: This is a preliminary document. Use only as a guideline for the planning and execution of geodetic surveys using GPS relative positioning techniques.

FOREWORD

This document was first prepared and distributed in draft form as version 1.0 in May 1, 1985. It supersedes all previous versions, including version 4.0, dated September 1, 1986.

The document is subject to frequent revisions as requirements for classification of geodetic control surveys change, as the definitions for accuracy standards are modified, as we gain experience in performing GPS surveys with an enhanced satellite system, as GPS surveying equipment are improved, as the field procedures are streamlined, and as refinements are made to processing software.

Questions and comments may be sent to:

Larry D. Hothem
N/CG144, Rockwall Building, Room 306
National Geodetic Survey, NOAA
Rockville, MD 20852
Telephone: 301-443-8171
FAX: 301-443-8328

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D I S C L A I M E R

Until this document is officially sanctioned by the Federal Geodetic Control Committee (FGCC), distribution does not constitute, in any way, an endorsement by the National Geodetic Survey, CGS, NOS, NOAA, or the FGCC. The "Geometric Geodetic Survey Standards and Specifications for Geodetic Surveys Using GPS Relative Positioning Techniques" is intended only for the purpose of providing the user, guidelines for planning, execution and classification of geodetic surveys performed by GPS satellite surveying relative positioning methods using carrier phase observations.

GEOMETRIC GEODETIC ACCURACY STANDARDS
AND
SPECIFICATIONS FOR GPS RELATIVE POSITIONING TECHNIQUES

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GEOMETRIC GEODETIC ACCURACY STANDARDS
AND
SPECIFICATIONS FOR USING GPS RELATIVE POSITIONING TECHNIQUES

Federal Geodetic Control Committee
Charting and Geodetic Services, N/CG
National Ocean Service, NOAA
Rockville, MD 20852

ABSTRACT. The practical experience gained in performing relative positioning geodetic surveys using Global Positioning System (GPS) satellite surveying techniques, the advancements in software developments, improvements in geodetic survey receiver systems, development of improved planning methods and observing strategies, and the results of tests by the Federal Geodetic Control Committee, provide the basis for development of geometric (three-dimensional) geodetic survey standards and specifications for GPS relative positioning surveys. The geometric standards are designed to meet classification requirements for a wide range of three-dimensional relative positional accuracy requirements. The GPS specifications cover network geometry, instrumentation, calibration procedures, field procedures, and office reduction procedures. Because application of GPS relative positioning techniques are relatively new, definitions for the accuracy standards and specifications for field procedures and data analysis will undergo rapid evolution. This will mean frequent revisions for the next several years or until at least a few years beyond deployment of the Block II satellites of the operational GPS constellation (presently 1991).

INTRODUCTION

The rapidly growing use of the Global Positioning System (GPS) for geodetic surveying applications has resulted in a critical need for development of acceptable accuracy standards and GPS survey specifications. Specifications are essential to promote efficiency in the conduct of field operations and to facilitate classification of surveys.

The extensive GPS survey field experience and numerous reports on the analysis of results for GPS survey projects are the basis for developing specifications for GPS geodetic surveys. These results are documented in unpublished National Geodetic Survey (NGS) GPS project reports, published NGS special reports, the FGCC GPS survey system test reports, papers in the proceedings of the First International Symposium on Precise Positioning with the Global Positioning System held in Rockville, Maryland, April 1985 (Goad 1985), and papers in the proceedings of the Fourth International Geodetic Symposium on Satellite Positioning held in Austin, Texas, April 1986 (Defense Mapping Agency (DMA) and NGS 1986).

The GPS specifications are for control surveys performed by relative positioning techniques where two or more receivers are collecting carrier phase measurement data simultaneously. They are a guide for determining how to meet requirements for horizontal, vertical, and azimuth accuracy standards.

Survey standards are defined as minimum accuracies that are necessary to meet specific objectives. Specifications are defined as field methods required to meet a particular survey standard. This document will complement the FGCC (Federal Geodetic Control Committee) Standards and Specifications for Geodetic Control Networks dated September 1984 (FGCC 1984).

The 1984 standards for horizontal coordinates are based on a "distance accuracy standard" which is the ratio of the relative positional error of a pair of control points to the horizontal separation of those points. As this ratio increases, the classification of the control survey degrades. If a relative positional error is constant, classification degrades as minimum separation between stations decreases. Thus, there is a minimum station spacing for the 1984 standards. The most stringent distance accuracy standard is 1:100,000 (10 ppm) which is classified as an order 1 standard. For example, if the relative positional error was ± 1 cm (2 sigma), the minimum distance between stations in a project would be 1 km.

The 1984 vertical control standards, which are based on elevation difference accuracies, is considerably more stringent. For example, the maximum elevation difference accuracies for a first-order, class I, survey range from 2.0 to 0.05 ppm for bench marks spaced 1 to 100 km apart. This is computed using the equation $b = S/\sqrt{d}$ where b is the maximum elevation difference accuracy, S is propagated elevation difference in mm between stations, and d is distance between stations in km. Thus, for a relative positional error of ± 1 cm (2 sigma), the minimum distance between stations in a project would be 100 km.

Experience has shown that it is possible to successfully measure base lines by GPS relative positioning techniques and obtain precisions routinely at the (1 cm + 1-2 ppm) level in each component or 10 times better than the FGCC 1984 order 1 standard. With careful planning, the use of appropriate observing strategies, and data processing with optimized software and procedures, precisions approaching (0.3 cm + 0.01 ppm) have been achieved. This is 1000 times better than the existing order 1 distance accuracy standard of 1:100,000.

Geometric or ellipsoidal height differences, when combined with geoid height differences, can give very useful orthometric height differences. Typical accuracies for orthometric height differences determined from the results of GPS relative positioning surveys range from a centimeter to several decimeters (depending on location of the survey project and spacing between stations). In most cases, the dominant error in the orthometric height differences is the error in estimating the geoidal slope or geoid undulation differences (Zilkoski and Hothem 1988).

In part 1 of this document, geometric (three-dimensional) accuracy standards for classifying relative positioning surveys by space measurement techniques are presented. These accuracy standards complement the terrestrial distance accuracy standards provided in the September 1984 document. In addition to three low orders, three high order standards are provided: 0.01, 0.1 and 1 ppm.

To classify elevation differences determined indirectly from use of space survey systems such as GPS, accuracy standards consistent with expected user requirements are proposed in Appendix E. These proposed elevation difference standards do not replace the present FGCC accuracy standards for elevation differences determined directly by precise differential or trigonometric leveling measurement techniques. They are to be used only for classifying or specifying the accuracy for elevation (orthometric height) differences determined from systems that measure height differences relative to a reference ellipsoid rather than a mean sea level datum or the National Geodetic Vertical Datum (NGVD) 1929.

The format for GPS relative positioning specifications is based on the current edition of the FGCC document for standards and specifications for geodetic control networks (FGCC 1984). The section on specifications includes network design and geometry, instrumentation, calibration procedures, field procedures, and office processing procedures.

These geometric accuracy standards and GPS relative positioning survey specifications are now under review by the Federal Geodetic Control Committee (FGCC). The FGCC, a U.S. interagency committee, is officially responsible for the adoption of standards and specifications for geodetic control networks. (See appendix A.)

BACKGROUND

GPS satellite surveying is a three-dimensional measurement system based on observations of the radio signals of the NAVSTAR Global Positioning System. The GPS observations are processed to determine station positions in Cartesian coordinates (X,Y,Z), which can be converted to geodetic coordinates (latitude, longitude, and height-above-reference ellipsoid). With adequate connections to vertical control network points and determination of the height of the geoid, orthometric heights or elevations can be computed for the points with unknown elevations.

The present GPS system is made up of the Block I satellites. The Block II system of 21 to 24 satellites is expected to be in full operation by about 1991. There are three primary modes of access to the GPS satellite signals: the "Standard Positioning Service" (SPS), the "Precise Positioning Service" (PPS), and codeless. The SPS is based on the Course/Acquisition Code (C/A Code) for the L1 frequency only while the PPS will be based on access to the P-code for the L1 and L2 frequency. With the proposed encryption of the PPS for the Block II system allowing only restricted access, SPS and codeless may be the only options for most users. Receiver designs that incorporate codeless technology can observe the two frequencies without access to either the SPS or PPS codes. Another receiver design combines SPS tracking capability for the L1 signal and codeless technology for the L2 frequency.

There are two methods by which station positions can be derived: point positioning and relative positioning. In the point positioning method, data from a single station are processed to determine three-dimensional coordinates (X,Y,Z) referenced to the WGS-84 earth-centered reference frame (datum). The present accuracy for GPS point position determinations ranges between 50 cm to 10 m (one sigma) depending on the accuracy of the ephemerides and period of the observations.

To perform geodetic surveys at the decimeter-level or better, one must employ GPS relative positioning techniques. In relative positioning, two or more GPS geodetic receivers receive signals simultaneously from the same set of satellites. These observations are processed to obtain the components of the base line vectors between observing stations (station coordinate differences (dX,dY,dZ)).

When the coordinates for one or more stations are known, the coordinates for new points can be determined after adjusting for the systematic differences between the reference system for the GPS satellites and local geodetic network control.

The specifications in this document are presently limited to fixed or static mode of relative positioning survey operations. In the static mode receiver/antennas are not moving while data is being collected. Future versions of this document will include specifications for kinematic modes of operation where one or more receiver/antennas are moving (possibly stopping only briefly at survey points) while one or more other receivers are continuously collecting data at fixed locations.

Proposed selective availability (sa) and encryption restrictions should have very little or no effect on static relative positioning techniques.

Since January 21, 1987, the orbital coordinate data for the GPS satellites are computed in the World Geodetic System 1984 (WGS-84), an Earth-centered and Earth-fixed coordinate system (DMA 1987).

There are at least four GPS signal measurement types that have been used for relative positioning techniques: pseudorange, code phase, integrated Doppler, and carrier phase. Although these observables have different characteristics, they are all functions of the instantaneous ranges between satellite and ground stations and their time derivatives. The most precise measurement type is the carrier phase.

Carrier phase measurements are made by "beating" the satellite carrier signal with the signal from the local receiver oscillator. The frequencies of these signals differ, primarily, by the amount of the Doppler frequency. Carrier phase observations are measurements of the phase difference of received signals emitted by the satellite's oscillator and the nominal carrier signal generated by the receiver's oscillator (Remondi 1985). There are several receivers capable of measuring the carrier phase of the L1 signal (1575.42 MHz) and or both the L1 and L2 (1227.6 MHz) signals (McDonald et al. 1987).

There are numerous approaches to processing carrier phase measurements. They are generally referred to as single, double, triple, or undifferenced methods. Each can be designed for either single- or multi-baseline processing (Goad 1985, and DMA and NGS 1986). In the multiple base line data processing mode, the data are processed for a single observing session or for multiple observing sessions in a single adjustment. The multiple session mode is also called a network solution and is only practical if there are adequate links or common stations between the observing sessions.

The major factors affecting accuracy of relative position determinations in the static (land survey) mode are: accuracy of the satellite positions, capability to model atmospheric (ionospheric and tropospheric) refraction errors, receiver timing bias, and field procedural errors (Beutler et al 1987 and Kinlyside 1988). Although stable weather conditions should not degrade the results substantially, severe storm fronts passing over one or more of the survey sites during an observing session can substantially degrade the results. Development of methods and techniques to bring these error sources under control will enhance survey capability in terms of accuracy, logistics, and, therefore, economy.

The present estimated accuracy of the precise ephemeris for the GPS satellites is 1 part-per-million (ppm) or better. The accuracy of the broadcast ephemeris is estimated to be 2 to 3 ppm. When the GPS orbit coordinates are fixed in the data processing, the errors in the orbits will propagate proportionately into each component of the base line determinations. To obtain precise base line vectors at the 0.01 or 0.1 ppm level, the average allowable orbital errors will have to be much smaller than are presently available. Should such accuracies be required and the post-computed orbit is not accurate enough, then data from fiducial stations (continuous tracking stations) will be processed with the project's GPS observations. In this method, the satellite orbital coordinates are adjusted while simultaneously solving for the station coordinate differences.

STANDARDS

Classification Standards

Six "orders" of geometric relative positioning accuracy standards are specified. These are summarized in table 1. These standards, reflecting a wide range of accuracy requirements, augment the present distance accuracy standards found in the 1984 FGCC document (FGCC 1984). Potential uses or applications for each of the orders are included in the table. The accuracy standards at the 95 percent confidence level for the six orders range from a very stringent standard in centimeters of $\pm\sqrt{((0.3)^2+(0.1d0.01)^2)}$ to $\pm\sqrt{((5.0)^2+(0.1d100)^2)}$ for the lowest order (d is the vector baseline length in kilometers). The three highest accuracy orders are called AA, A and B, respectively.

The highly stringent accuracy standard of order AA has been achieved for projects where data was processed in conjunction with continuous tracking data collected at stations of the Cooperative International GPS Network (CIGNET). The data were processed using orbital adjustment techniques. The distances generally ranged between 500 to 5000 km. The orders for 1 and lower accuracy standards are comparable (except for exclusion of Order 3, Class II) to the orders provided in the FGCC September 1984 document. Thus, the standards are defined in reasonable conformance with present GPS surveying capabilities.

Although the concept of "order/class" is retained, it should not be used for specifying the accuracy for a survey and for final station classification purposes. The user of these standards should determine the real accuracy needs and the cost implications. The accuracy needs should be specified in terms of accuracy values in distance units and parts per million. In specifying the accuracy values, the range of distances between adjacent stations should be included. Given this information, appropriate procedures for meeting these specified standards can be proposed.

Table 1. -- Geometric relative positioning accuracy standards for three-dimensional surveys using space system techniques.

Survey categories	Order	(95 percent confidence level) Minimum geometric Accuracy standard: Base error e (cm)	Line-length Dependent error p (ppm)	a (1:a)
Global-regional geodynamics; deformation measurements	AA	0.3	0.01	1:100,000,000
National Geodetic Reference System, "primary" networks; regional-local geodynamics; deformation measurements	A	0.5	0.1	1: 10,000,000
National Geodetic Reference System, "secondary" networks; connections to the "primary" NGRS network; local geodynamics; deformation measurements; high-precision engineering surveys	B	0.8	1	1: 1,000,000
National Geodetic Reference System (Terrestrial based); dependent control surveys to meet mapping, land information, property, and engineering requirements	(C)			
	1	1.0	10	1: 100,000
	2-I	2.0	20	1: 50,000
	2-II	3.0	50	1: 20,000
	3	5.0	100	1: 10,000

Note: For ease of computation and understanding, it is assumed that the accuracy for each component of a vector base line measurement is equal to the linear accuracy standard for a single-dimensional measurement at the 95 percent confidence level. Thus, the linear one-standard deviation (s) is computed by:

$$s = \pm[\sqrt{e^2 + (0.1d \cdot p)^2}]/1.96. \qquad \text{(See appendix B.)}$$

Where, d is the length of the baseline in kilometers. 5-26-88

In defining the accuracy standards, it was assumed that each component of the baseline determined by GPS relative positioning techniques are much alike, i.e. error sources that are highly correlated. Thus, no particular component has characteristics making it desirable to treat it differently from the other two components. It was also a premise that optimum accuracies achievable with GPS satellite surveying techniques are routinely and economically possible if the survey is carried out carefully and with adequate control of error sources.

The accuracy standards are not based on the technical training or ability of a surveyor, but instead they are based on the capabilities of the GPS measurement systems. As we approach the date when the Block II GPS satellites become fully operational, the cost of survey systems is expected to continue to decrease. Equipment costs as it relates to the economics of conducting a GPS survey will be an insignificant factor in determining overall project costs. Rather, the cost for a survey project will largely depend on costs for labor, logistical support, and other factors.

When specifying an accuracy standard for a survey there may be an "intended" standard that is substantially more stringent than a minimally "acceptable" accuracy standard. Today, GPS satellite geodetic survey systems (with carrier phase measurement capability) operated in the relative positioning static mode can yield vector baseline results with one-sigma uncertainties that are typically better than $\pm\sqrt{((1.0cm)^2+(0.1d2ppm)^2)}$ from data sets collected for periods of about 1 hour. Periods of less than 60 minutes can yield comparable results, but with lower reliability. Even with about 30 minutes of data consisting of 4 or more satellites, good geometric distribution, very few or no cycle slips, and an accurate ephemerides, it is possible to achieve results comparable to 60 minute data sets. Even though the present constellation is not optimized for getting reliable accuracies, the final classification for a GPS survey may still be within an "acceptable" standard.

In practice, scheduling the observing units to collect simultaneous data for less than 30 minutes can increase the risk of achieving unsuccessful observing sessions, particularly when there may be factors that would affect the quantity and/or quality of the observations. Furthermore, when operating in the static mode, the difference in operating costs between a 60 minute and 30 minute observing span is insignificant.

In developing the specifications for orders 1, 2, and 3, these orders were grouped with a single set of criteria. Thus, the specification criteria for design and field procedures were defined for four primary orders: AA, A, B and C (1, 2I, 2II, and 3). The only exception to this are the specifications for office procedures where a unique set of criteria was defined for each of the six orders.

There may be two "final" classifications for a GPS relative positioning survey project. The first, a "geometric" classification, would be determined by analysis of the internal consistency for a GPS relative positioning network. Data for this classification would be based on analysis of loop misclosures, repeat baseline results, and minimally constrained (free) least-squares network adjustments (independent of the local network control). The "geometric" classification is especially important for surveys that are designed to meet high-accuracy requirements such as for establishment of a high-precision primary networks, deformation measurement investigations (crustal motion, subsidence monitoring, motion of structures, etc.) and other special high precision engineering surveys.

The second classification for a GPS project would be based on the results of a constrained 3D adjustment where published coordinates for existing stations of the National Geodetic Reference System (NGRS) are either fixed or given weighted constraints. When a survey is adjusted into the local network control system, it would receive an "NGRS" classification that would depend on the accuracy of the existing horizontal network control. In the constrained adjustment, the existing network is "assumed to be correctly weighted and free of significant systematic error." The "NGRS" classification may also depend on the accuracy standard specified for the orthometric heights determined from the GPS relative positioning data. In turn, this would depend on the accuracy of the geoidal height differences.

Relative position accuracy denotes the relative accuracy of the various components between one station and other stations of a network. The concept of relative position accuracy can be applied to networks established by single-dimensional conventional measurements or by three-dimensional space system measurements. The accuracy standards in table 1 apply to both single-dimensional conventional terrestrial measurement techniques and three-dimensional GPS relative positioning techniques.

For each geometric relative position accuracy standard, the maximum allowable linear error in centimeters (at the 95 percent confidence level) can be computed for a corresponding station spacing by (see appendix B):

$$s = \sqrt{(e^2 + (0.1pd)^2)} \qquad (1)$$

where, s = maximum allowable error in centimeters at 95 percent confidence level
d = distance in kilometers between any two stations
p = the minimum geometric relative position accuracy standard in parts-per-million (ppm) at the 95 percent confidence level.
e = base error in centimeters (this includes station-dependent setup error)

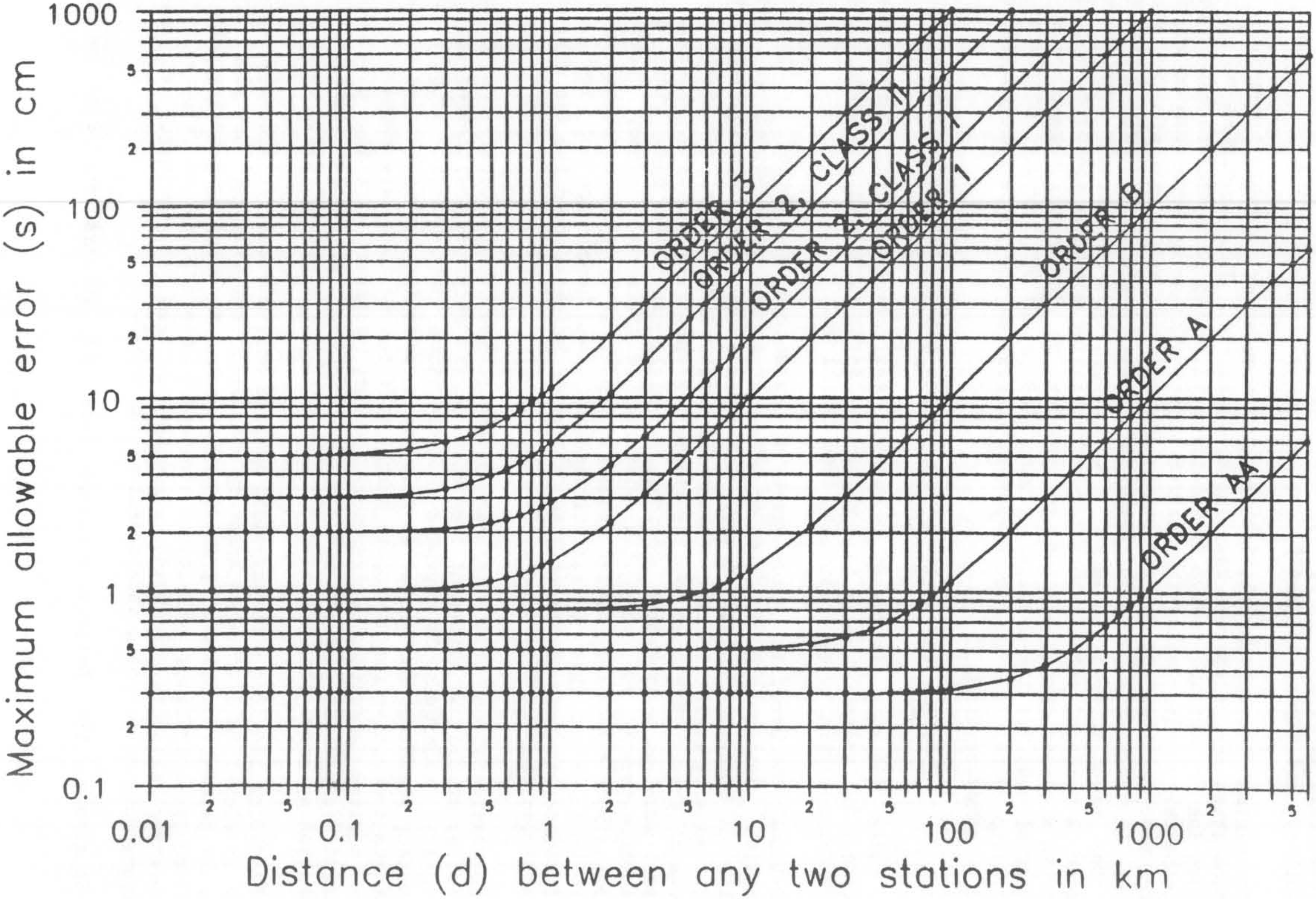
Figure 1 is a graph of the maximum spherical or linear error at the 95 percent confidence level for each order and class of the standards against the distance between any two stations.

Appendix C is a tabulation of one-sigma minimum standard errors computed from the minimum relative position accuracy standards given in table 1.

A survey station of a network is classified according to whether the propagated error at the 95 percent confidence region is less than or equal to the maximum allowable error "s" specified for the project. In the case of GPS determined baseline vectors, typically, the error propagation proceeds linearly for distances greater than about 20 km. The magnitude of the line-length dependent error will depend on the quality and quantity of the observations and the effectiveness of the baseline processing software for minimizing linearly dependent error sources.

For example, two stations are spaced 10 km apart and the accuracy standard for the baseline measurement is specified as order 1. The maximum allowable geometric relative error (at the 95 percent confidence level) between stations 1 and 2 is 10 cm. In this example, the value for s of 10 cm is 10 times greater than the base error of 1 cm. Thus, the base error (e) does not contribute significantly to the total value for s. On the other hand, if order B is specified, s = 1.3 cm. In this case, s is less than a factor of 2 greater than the value for e, thus the base error e is significant.

Geometric relative positioning accuracy standards
Maximum allowable error at 95% confidence level



This shows the importance of taking extra precautions to minimize the contribution to the base error caused by problems with antenna setup, antenna phase center stability, and signal multipath.

The minimum geometric relative position accuracies in table 1 represent present capabilities for making GPS baseline measurements. This includes any significant errors due to antenna setup (plumbing or centering, and measurement of height of antenna phase center above the station mark). The setup error can be the dominant error when establishing closely spaced stations for any of the accuracy standards. It may be the most significant error source when measuring widely spaced stations at the high accuracy orders. To control this potentially significant error source, a range of setup errors for corresponding accuracy standards and distances between stations are presented in appendix D. The errors were computed using a factor of 0.05 for the critical region (100 minus 95 percent confidence level). The setup error (k) in each component (N,E,U) at the 95 percent confidence level can be computed from:

$$k = 0.1pd(0.05), \text{ where, } k_{min} = 0.3 \text{ cm, and } k_{max} = 10 \text{ cm.}$$

The value for k_{min} is based on current realistic estimates for expected setup errors. The value for k_{max} is a worst-case setup error; in practice, it should be much smaller than 10 cm, typically less than 1 cm.

Although the accurate measurement of geometric quantities is important, in practice, orthometric heights or elevations may be desired in addition to ellipsoid heights. In many areas, the geoid slopes are usually less than most required accuracies for the orthometric height differences. For example, in most areas of the conterminous US, the slopes are well within 25 ppm. However, in some areas, such as mountainous regions, it might exceed 75 ppm. This may not be tolerable except for very low elevation difference accuracy standards. Most applications requiring either geometric order AA or A are concerned with changes with time rather than spatial differences, and hence are not sensitive to the difference between orthometric and ellipsoid heights since the two will generally change together in time (Kaula 1986).

In consideration that standards of accuracies for vertical control by spirit leveling should be different from those by GPS relative positioning and other 3-D geometric techniques. In appendix E, elevation (orthometric height) difference accuracy standards for geometric relative positioning techniques are proposed. The minimum accuracies for the geoid height differences that are required to achieve the desired elevation difference accuracy standard are also given. This, in effect, separates the accuracy standard for allowable geometric relative positioning error from the accuracy standards for elevation differences.

Specifications for a survey might include only a geometric accuracy standard but not an elevation difference accuracy standard. For example, one might perform a purely geometric survey if primarily interested in changing geometry such as plate motion investigations, subsidence monitoring, dam deformation studies, etc. The geometric relative position measurements can be evaluated to meet these high-precision purposes independent of the geoid.

In summary, the heights produced from GPS surveys are with respect to a reference ellipsoid. To convert these ellipsoid (also known as geodetic) heights to orthometric heights or elevations, the survey must include adequate connections to network control points with orthometric heights established by differential leveling techniques and referenced to the National Geodetic Vertical Datum (NGVD). When reliable estimates for the geoid height differences between all stations of the project are available, orthometric heights derived from the GPS survey results can be computed. The accuracy of the GPS derived orthometric heights will depend on the accuracy of the GPS ellipsoidal height differences, the accuracy of the orthometric heights for the vertical control, and the accuracy of geoid height differences.

The maximum azimuth accuracy from GPS relative position determinations is based on a minimum spacing between a pair of stations that are intervisible. The azimuth between a station pair is determined after adjustment of the vector baseline in the satellite reference system to the local datum reference system. For a specified azimuth accuracy and expected accuracy for the GPS vector baseline determinations at the 95 percent confidence level, a minimum spacing between a pair of stations can be computed. More discussion on azimuth determinations from GPS relative positioning surveys is contained in the next section.

These accuracy standards were developed in consideration of a critical need for statistically-based positional accuracy standards that is appropriate for three-dimensional measurement techniques such as use of the GPS. There is also recognized that statistically-based positional accuracy standards need to be developed for property and cadastral surveys (Vonderohe 1986). In Vonderohe's paper, it was indicated that relative error ellipses may be viable as standards. There is also a critical need for positional accuracy standards when making deformation measurements (monitoring vertical and horizontal movement) or for other precise engineering surveying applications.

As noted by Vonderohe (1986), discussion of positional accuracy standards should consider the practicality of users implementing them. The use of these standards requires a fundamental understanding of statistics and adjustments. But these educational requirements are not unique for the implementation of these standards. If the surveyor wants to help ensure successful use of GPS surveying techniques in a variety of applications, it would be prudent to acquire appropriate knowledge in statistics, adjustments, and analysis of observations.

Research or studies into the appropriate definition of statistically-based positional accuracy standards is clearly needed. Thus, as such research or studies bear new information, modification and refinements of these geometric accuracy standards are expected during the next few years.

Monumentation

With the increasing use of space system measurement techniques, such as use of GPS, it is important that station markers have the properties of permanence and stability. The markers must be stable in all three dimensions.

Factors that may affect the stability of a monument include frost heave action, changes in groundwater level, and settlement (Sliwa 1987). When selecting sites for stations of a high-precision primary network or for monitoring deformation, it is recommended that soil and geotechnical specialists be consulted.

Markers for existing network control should show no historical evidence of significant movement. If an existing network control marker does not exhibit adequately the properties of permanence and stability, it may have to be replaced by a new marker. The decision to replace old markers will depend on there use and purpose in future surveys.

The type of marker best suited for a given type or condition of terrain will depend on such factors as local conditions, transportation, materials available, equipment available for setting marks, and cost. Sites for new markers will, whenever possible, be located on public property such as road right-of-ways, public building grounds, school yards, etc.

To meet the requirements for permanent and stable monumentation, the markers are usually corrosion-resistant metal disks that may be set in a rock outcrop or large masses of concrete such as bridge abutments and other structural foundations.

When bedrock or large, massive structures are not available, it is more difficult to ensure the marker has the properties of permanence and stability. Traditional concrete monuments, with or without an underground mark, are not recommended as a suitable choice for preserving the three-dimensional coordinates.

The recommended alternative is a three-dimensional rod mark (Beard 1986). The principle component of the mark is a 9/16-inch stainless steel rod driven into the ground until the driving rate with a gasoline powered reciprocating hammer slows to 60 seconds per foot or slower. When in position, the top of the rod is just below ground level. The top of the rod is rounded and centerpunched, to mark the exact point to be positioned.

A grease-filled, 1-inch PVC pipe (sleeve) surrounds the rod from just below its top to a depth of at least 3 feet. It is preferable that the sleeve depth is equal to the depth of maximum frost penetration. Extreme depths of frost penetration for the conterminous U.S. is shown in figure 2. A hole must be dug for the sleeve during installation. The 1-inch sleeve reduces vertical stress to the rod caused by frost heave or other soil movements. It also helps restrict horizontal movement to an insignificant amount. The grease used to fill the sleeve should be an insoluble, non-corrosive, cold-weather type such as that conforming to U.S. military specification G-10924D. The grease is contained within the sleeve with pipe caps center drilled to 9/16 inch + 0.005 inch, allowing the rod to penetrate.

A 5-inch PVC pipe and cap with access cover is placed in concrete around the top of the assembly for protection and to aid in locating the mark. It is installed at or slightly above ground level The space between the 1-inch and 5-inch PVC pipe is filled with fine grain sand. (See appendix H for detailed setting procedures.)

When the sites for new points are being selected, surveyors should attempt to locate the new points on existing bench marks tied to the National Geodetic Vertical Network. Besides being prudent and cost-saving, this procedure will help

Figure 2.--Extreme depth of frost penetration (in meters) for conterminous U.S.

meet the requirements for connecting the markers with unknown elevations to the existing vertical network control. Should the permanency and/or stability of the bench mark be questionable, an offset marker may need to be set.

Reference marks are optional except in special circumstances. These circumstances could include: stations established for the National Crustal Motion Network, the primary National Geodetic Reference System, or other precise geodetic applications where recovery of a primary station is important for historical or legal reasons.

Whenever it is not possible to occupy a station directly and an offset point must be established, the offset point will be monumented and connected to the control station by survey techniques consistent with the accuracy standard specified for the GPS survey.

When practical, new stations should be located at sites that are accessible by ground transportation.

SPECIFICATIONS

The specifications recommended in the following sections are based on considerable practical experience. Some of the parameters may still reflect conservation estimates and will require further studies before they can be refined.

Development of the specifications is an evolutionary process that is not expected to stabilize before 1992 or after the Block II constellation of GPS satellites are launched and fully operational. Appendix J summarize the proposed Launch Dates, for the Block II GPS Satellites as of February 1988.

Network Design, Geometry, and Connections

The location and relative disposition of the control points do not depend significantly on factors such as network shape or intervisibility (except when establishing azimuth reference points) but rather on optimum layout for carrying out the intent of the survey.

Table 2 summarizes the specifications for the network design and connection factors, including minimum station spacing, ties to existing horizontal and vertical network control points, and direct connection requirements.

Checks should be made to ensure that no existing network control points have been moved or disturbed. It may be necessary to occupy more than the minimum number of network control points to ensure the survey is tied into points with sufficient accuracy or internal consistency.

If bench marks are located in areas subjected to vertical motion, it may be necessary to perform a vertical survey by differential or precise trigonometric leveling methods to ensure all bench marks are connected to a common epoch.

It is stated in the present FGCC specifications that whenever the distance between two unconnected survey points is less than 20 percent of the distance between those points traced along existing or new connections, a direct connection should be made between those survey points (FGCC 1984). The enforcement of this rule is optional depending on circumstances for stations located within the area of the GPS survey project.

At least three factors should be considered when determining whether direct connections between adjacent stations is desirable: (1) if an existing station, can it be recovered, (2) is the station reasonably accessible (i.e., it is quite likely it may be occupied during future surveys), and (3) what is the distance between the adjacent stations? When direct connections are desirable, table 2 provides guidelines for corresponding accuracy standards. If enforcement of the 'adjacent-station' rule is not practical, appropriate statements about those stations affected must be included in the project report.

If azimuth marks are required, the azimuth reference can be established by GPS surveys. There are at least four factors to consider when establishing azimuth references by GPS relative positioning techniques rather than using conventional

Table 2. -- Guidelines for network design, geometry and connections

Geometric accuracy standards — Group / Order / ppm / base(cm)	AA / AA / 0.01 / 0.3	A / A / 0.1 / 0.5	B / B / 1.0 / 0.8	C / 1,2-I&II,3 / 10,20,50,100 / 1 2 3 5
<u>Horizontal network control of NGRS[a], minimum number of stations</u>:				
When connections are to orders AA, A or B....	4	3	3	2
When connections are to order 1..............	na[b]	na[b]	na[b]	3
When connections are to orders 2 or 3........	na[b]	na[b]	na[b]	4
<u>Vertical network control of NGRS[a], minimum number of stations[c][d]</u>..........	5	5	5	4
<u>Continuous tracking stations (master or fiducials), minimum number of stations</u>....	4	3	2	op
<u>Station Spacing (km)</u>:				
Between "existing network control" and CENTER of project:				
Not <u>more</u> than............................	100d	10d	7d	5d
50 percent not <u>less</u> than................	$\sqrt{5d}$	$\sqrt{5d}$	$\sqrt{5d}$	d/5
Between "existing network control" located <u>outside</u> of project's outer boundary and the edge of the boundary, not <u>more</u> than.	3000	300	100	50
<u>Location of network control</u> (relative to center of project); number of "quadrants", not less than..............	4	4	3	3
<u>Direct connections should be performed, if practical, between</u>: ANY adjacent stations (new or old, GPS or non-GPS) located near or within project area, when spacing is <u>less</u> than (km)..........	30	30	10	5

Legend: d - is the maximum distance in (km) between the center of the project area and any station of the project.
NGRS - National Geodetic Reference System
CL - Confidence level; na - not applicable; op - optional

4-17-89

Table 2. -- Guidelines for network design, geometry and connections (continued)

NOTE: If it is not practical to plan a survey that is within the criteria, minor adjustments may be made provided that it is authorized by the agency requesting the survey.

Remarks: (a) Consult National Geodetic Survey officials whenever it is necessary to consider exceptions to these criteria, particularly, when the GPS survey project data are to be submitted to NGS for incorporation in the NGRS.

(b) If a survey with an accuracy standard of AA, A, or B is specified and one objective in the survey is to upgrade the existing network, then connections to a minimum of four stations are required or at least one station in each one-degree block with a minimum of four stations.

(c) First choice is vertical network control established and/or maintained by the National Geodetic Survey. When it is not possible to occupy the minimum number of NGRS points, non-NGRS control points may be used. This should be documented in the project report.

(d) If it is expected that the constrained adjustment for determination of the elevations within the project area will be based on more than one "bias group" (see discussion under section on Office procedures, Analysis and Adjustments) then the minimum number of stations specified is that which is required within the area for each "bias group." For example, if there two bias groups and ties required to four bench marks, then four bench marks will be incorporated within each area of the "bias group" for a total of 8 bench marks.

5-09-88

astronomical methods. They are: (1) cost, (2) a pair of stations will be located close to each other with coordinates established at the same order of accuracy, (3) repeat observations between the azimuth and main station can be used to verify the relative stability of the two marks in all three dimensions, and (4) check observations or redundancy is not possible when azimuth reference is determined from only a single set of astronomic observations.

Table 3 summarizes minimum spacings between station-pairs for corresponding relative position accuracies possibly achieved from a GPS survey and for a range of azimuth accuracy standards.

Table 3. -- Guidelines for minimum spacings for establishing pairs of intervisible stations to meet azimuth reference requirements.

Spacing between a "pair" of stations, not less than (meters)	Azimuth accuracy required in seconds of arc (95 percent confidence level) 1	2	4	6	10
	GPS relative position precision (mm) (95 percent confidence level)				
100	-	-	2	3	5
200	-	2	4	6	10
300	-	3	6	9	14
400	2	4	8	12	19
500	3	5	10	14	24
600	3	6	12	18	29

1-01-86

Example: If the expected relative position precision from a GPS survey between two marks spaced less than 1000 meters apart is 2 mm at the 95 percent confidence level, then to achieve an azimuth accuracy of 2 seconds at the 95 percent confidence level, the minimum spacing between the pair of stations is 200 meters.

Instrumentation

GPS geodetic receivers may receive one or both carrier frequencies transmitted by the GPS satellites. Two frequency receivers are required for the most precise surveys to correct for the effects of ionospheric refraction where the magnitude of the error may range from 1 to 10 ppm. The receivers must record the phase of the satellite signals, the receiver clock times, and the signal strength. Data collected with different receivers may be combined in the processing, however, observations need to be taken approximately simultaneously.

Generally, GPS satellite geodetic surveying equipment will consist of three major components: the antenna, receiver/processor, and recording unit. Depending on type of cable used, the lengths will usually range from 10 to 60 meters. The maximum length and type of cable may depend on the manufacturer's specifications. The receiver should have the capability to track a minimum of four GPS satellites.

Some receivers may have multiple data ports for handling printer output, data input from automatic weather instruments, and remote control operations. It should be possible to operate the receiver in the unattended mode. However, when commanded, information should be available for display to ensure that the receiver is functioning normally and the data quality meets acceptable standards.

The receivers may be codeless or have the capability to receive and decode the P and/or CA coded data. If it is codeless, the receiver must have the appropriate output and input ports for synchronizing the clocks among instruments and with respect to UTC (Universal Coordinated Time).

The required stability of the reference frequency of the GPS receiver is dependent on the receiver design. The amount of the initial time offset between receivers and the relative drift which can be tolerated is highly dependent on the sophistication of the processing software (i.e., the physical model). All GPS receivers should have a signal input port for an external frequency standard.

For high precision results, while allowing the widest choice of processing software, it is recommended that codeless receivers be initially synchronized and the relative drift rates be maintained to less than 10 microseconds per hour. (This is equivalent to approximately 4.4 Hz difference in the GPS receiver's L1 frequency.) It is generally recommended that codeless receivers be compared again at the end of the surveying day. This is not strictly required; it is possible to perform the clock check the following day prior to synchronization.

For codeless sets it is recommended that a high quality wrist watch be standard equipment. In rare cases, the receiver clock may experience a time problem on the way to or at the survey site. In such a case a synchronization of the receiver clock to the wrist watch will likely result in a successful survey. The final processed results may be somewhat degraded, however.

The height of the "phase" center (L_1) or centers (L_1 and L_2) above a defined reference point on the antenna or an adaptor connected to the antenna is usually predetermined by the manufacturer. This will be a constant for a particular antenna model. Combining this height constant with the height of the defined reference point above the station mark will give the total height used to reduce the baseline measurements from phase-center to phase-center down to mark-to-mark. The location of the phase center may not be marked on the antenna.

Using the appropriate constant for a particular antenna model is very important when different antennas are used during the same project. If the bias in height between different antenna models is not well known, it is recommended that test surveys be conducted between nearby marks which have accurately known height differences. Then the constant for one of the antennas will be adjusted for any significant height bias between different antenna models.

Calibration

Field calibration is necessary to control systematic errors that may be critical to GPS satellite surveys. This will verify the adequacy of the GPS survey equipment, observation procedures, the processing software, and steps implemented in the data analysis. The field calibration consists of testing the GPS equipment performance and the associated base line processing software on a three-dimensional test network.

The three-dimensional test network should be composed of four or more stations spaced approximately 50 m to 10 km apart. The location of the stations should permit base lines to be measured which are nearly at right angles to each other.

Three-dimensional relative position measurements will be established to be accurate in any component to within $\pm\sqrt{(3mm)^2+(0.1d1ppm)^2}$ at the 95 percent confidence level.

The field procedures found in table 4 for order B will be used to establish the test network. The data will be reduced in the fixed orbit mode using precise ephemerides available from the National Geodetic Survey (Remondi 1986). Single base line, multiple base line (session) processing software, or other software that will give results with comparable precision shall be used. The network shall be established with a minimum of four receivers collecting three observing days.

A special three-dimensional geodetic test network established by the FGCC has been used to test GPS survey systems since 1983. This network is located in the vicinity of Washington, D.C. (Hothem and Fronczek 1983).

If different receivers and/or different model antennas are used in a survey, it will be necessary to conduct calibration tests to determine whether significant biases exist. For example, if the markings for the location of the phase center are not at the true location for different antennas, this will cause a bias in the height component of the GPS base line measurements. Other tests may be needed to determine procedures to ensure optimum orientation of the antenna and to determine the error contribution due to multipath.

Field Procedures

The precision of the GPS vector base line results depends on the number of satellites visible simultaneously from each station during an observing session, their geometric relationships, duration of the period when the desired number of satellites can be observed simultaneously, the uncorrected effects of ionospheric and tropospheric refraction, and the length of line. The number of possible observing sessions per observing day is a function of the required survey accuracy, satellite availability, and project logistical considerations such as travel and set up time required between observing sessions.

The specifications for field procedures will be common for all surveys with "intended" accuracies specified as 1:100,000 or lower. This is because vector base lines can be measured routinely with uncertainties of better than 10 ppm (1:100,000) using data sets from collection periods of 30 to 60 minutes. Even data collection periods of a few minutes can also produce good results during optimal satellite visibility conditions.

Although there are no differences in the field procedures for 1:100,000 and lower order surveys, there will be different criteria for each standard in the section on office procedures. The criteria for establishing the "final" classification will differ significantly to take into account factors which affected the results and either were not known at the time the observations were being collected or they could not be controlled by altering the field procedures.

Factors possibly affecting the results include: unexpected degraded accuracy for the orbital coordinates, satellite transmission problems, significant atmospheric disturbances, and receiver problems that went undetected before the survey team departed from the project area. It will be possible from the office procedures to evaluate surveys affected by unexpected problems and determine a final classification that, although maybe lower than the "intended" accuracy, may still meet minimum criteria for a project.

Currently, the Block I GPS satellite constellation includes only seven usable satellites. Depending on the location of the project, this limits the observing period when four or more satellites are available to approximately 5 hours each day. When the Block II 21- to 24-satellite constellation becomes operational in the 1990's (See appendix I), in general at least six satellites will be available for simultaneous observations from anywhere on Earth 24 hours a day.

Table 4 summarizes the field procedures that should be followed to achieve the desired accuracy standards. These field procedures are valid only for relative positioning surveys and are subject to change as more satellites become available and processing techniques are refined.

Although there has not been any report of interference affecting quality of data, it advisable that the antenna be located where potential radio interference is minimal for the 1227.6 and 1575.42 MHz frequencies (GPS L1 and L2 signals). The distance between the potential radio interference and the GPS survey system may be an important consideration. For example, stations located adjacent to high-powered radio and high frequency, high-powered radar and transmission antennas should be avoided.

If one or more of the stations in a project network is continuously reoccupied during each session, these stations are generally called "master" or "fiducial" stations. In this observing scheme, the observations for the "master" station(s) are in common to most or all the other observing sessions for a project. The data for observing sessions linked by a master station can be processed simultaneously either in the fixed orbit or adjusted orbit mode. This is usually called a network base line solution.

Other procedures for processing the simultaneous observations include processing single or session base line solutions. In a session base line solution, all data collected simultaneously during an observing session are combined for simultaneous multiple-base line determinations.

Depending on the number of receivers available, project observing schemes that include one or more "master" stations may result in less efficient operations compared with the so-called "leapfrog" approach to planning the observing schemes. For example, efficiency is improved 20 to 35 percent when four receivers are operated in the "leapfrog" observing scheme rather than if one of the four receivers was used for continuous deployment at a "master" station.

On the other hand, the "master" station approach (also referred to as fiducial stations) might be highly desirable if the highest accuracy is required. For example, GPS observations might be collected continuously at the "master" stations located at sites of other space systems such as Very Long Baseline Interferometry (VLBI) or satellite laser ranging. These data can be processed while holding fixed the "master" station coordinates determined from the other space systems.

Table 4. -- Guidelines for GPS field survey procedures

Geometric relative positioning standards — Group / Order / ppm	AA / AA / 0.01	A / A / 0.1	B / B / 1.0	C / 1,2-I&II,3 / 10,20,50,100
Two frequency observations (1 and L2) required[a]: Daylight observations[b].....	Y	Y	Y	op
Recommended number of receivers observing simultaneously, not less than:·.·.........	5	5	4	3
Satellite Observations: RDOP values during observing session (meters/cycle)[d]....... [TO BE ADDED IN FUTURE VERSION]				
Period of observing session (observing span), not less than (min):				
[4 or more simultaneous satellite observations][e]				
Triple difference processing[f]...........	na	na	240	60-120
Other processing techniques[g]: General requirements[h][i]..............	240	240	120	30-60
Continuous and simultaneous between all receivers, period not less than[i][j].	180	120	60	20-30
Data sampling rate - maximum time interval between observations (sec)...............	15	30	30	15-30
Minimum number of quadrants from which satellite signals are observed...........	4	4	3	3 or 2[k]
Maximum angle above horizon for obstructions[v] (degrees).................	10	15	20	20-40
Independent occupations per station(l):				
Three or more (percent of all stations, not less than).................................	80	40	20	10
Two or more (percent of stations, not less than):				
New stations.............................	100	80	50	30
Vertical control stations................	100	100	100	100
Horizontal control stations..............	100	75	50	25
Two or more for each station of "station-pairs"(m)........................	Y	Y	Y	Y

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Table 4. -- Guidelines for GPS field survey procedures (continued)

Geometric relative positioning standards	Group / Order / ppm	AA / AA / 0.01	A / A / 0.1	B / B / 1.0	C / 1,2-I&II,3 / 10,20,50,100
Master or fiducial stations(n):					
Required, yes or no(o)		Y	Y	Y	op
If yes, minimum number		4	3	2	-
Repeat base line measurements, about equal number in N-S and E-W directions, minimum not less than (percent of total independently [nontrivial] determined base lines)		25	15	5	5
Loop closure, requirements when forming loops for post-analyses:					
Base lines from independent observing sessions, not less than		3	3	2	2
Base lines in each loop, total not more than.		6	8	10	10
Loop length, generally not more than (Km)		2000	300	100	100
[NOTE: Also, see table 5]					
Loop closure (Continued):					
Base lines not meeting criteria for inclusion in any loop, not more than [percent of all independent nontrivial lines(p)]		0	5	20	30
Stations not meeting criteria for inclusion in any loop, not more than (percent of all stations)		0	5	10	15
Direct connections are required: Between ANY adjacent (NGRS and/or new GPS) stations (new or old, GPS or non-GPS) located near or within project area, when spacing is less than (Km)		30	10	5	3
Antenna setup:					
Number of antenna phase center height measurements per session, not less than		3(q)	3(q)	2	2
Independent plumb point check required(r)		Y	Y	Y	op

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Table 4. -- Guidelines for GPS field survey procedures (Continued)

Geometric relative positioning standards	Group / Order / ppm	AA / AA / 0.01	A / A / 0.1	B / B / 1.0	C / 1,2-I&II,3 / 10,20,50,100
Photograph (closeup) and/or pencil rubbing required for each mark occupied		Y	Y	Y	Y
Meteorological observations:					
Per observing session, not less than		3(s)	3(s)	2(t)	2(t) or op
Sampling rate (measurement interval), not more than (min)		30	30	60	60
Water vapor radiometer measurements required at selected stations?		op	op	N	N
Frequency standard warm-up time (hr)(u):					
Crystal		12	12	(u)	(u)
Atomic		1	1	(t)	(t)

LEGEND: nr - not required, na - not applicable, op - optional

REMARKS:

(a) If two-frequency observations can not be obtained, it is possible that an alternate method for estimating the ionospheric refraction correction would be acceptable, such as modeling the ionosphere using two-frequency data obtained from other sources.

Or, if observations are during darkness, single frequency observations may may be acceptable depending on the expected magnitude of the ionospheric refraction error.

(b) When spacing between any two stations occupied during an observing session is more than 50 km, two frequency observations may need to be considered for Accuracy Standards of Order 2 or higher.

(c) Multiple baseline processing techniques.

(d) Studies are underway to investigate the relationship of Geometric Dilution of Precision (GDOP) values to the accuracy of the base line determinations. Initial results of these studies indicate there is a possible correlation. It appears the best results may be achieved when the GDOP values are changing in value during the observing session.

(e) The number of satellites that are observed simultaneously cannot be less than the number specified for more than 25 percent of the specified period for each observing session.

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Table 4. -- Guidelines for GPS field survey procedures (Continued)

(f) Absolute minimum criteria is 100 percent of specified period.

(g) "Other" includes processing carrier phase data using single, double, nondifferencing, or other comparable precise relative positioning processing techniques.

(h) The times for the observing span are conservative estimates to ensure the data quantity and quality will give results that will meet the desired accuracy standard.

(i)

(j) Absolute minimum criteria for the data collection observing span is that period specified for an observing session that includes continuous and simultaneous observations. Continuous observations are data collected that do not have any breaks involving <u>all</u> satellites; occasional breaks for individual satellites caused by obstructions are acceptable, however, these must be minimized. A set of observations for each measurement epoch is considered simultaneous when it includes data from at least 75 percent of the receivers participating in the observing session.

(k) Satellites should pass through quadrants diagonally opposite of each other

(l) Two or more independent occupations for the stations of a network are specified to help detect instrument and operator errors. Operator errors include those caused by antenna centering and height offset blunders.

When a station is occupied during two or more sessions, back to back, the antenna/tripod will be reset and replumbed between sessions to meet the criteria for an independent occupation. To separate biases caused by receiver and/or antenna equipment problems from operator induced blunders, a calibration test may need to be performed.

(m) Redundant occupations are required when pairs of intervisible stations are established to meet azimuth requirements, when the distance between the station pair is less than 2 km, and when the order is 2 or higher.

(n) Master or fiducial stations are those that are continuously monitored during a sequence of sessions, perhaps for the complete project. These could be sites with permanently tracking equipment in operation where the data are available for use in processing with data collected with the mobile units.

(o) If simultaneous observations are to be processed in the session or network for base line determinations while adjusting one or more components of the orbit, then two or more master stations shall be established.

Table 4. -- Guidelines for GPS field survey procedures (Continued)

(p) For each observing session there are r-1 independent base lines where r is the number of receivers collecting data simultaneously during a session, e.g. if there were 10 sessions and 4 receivers used in each session, 30 independent base lines would be observed. (See appendix F and I.)

(q) A measurement will be made both in meters and feet, at the beginning, mid-point, and end of each station occupation.

(r) To ensure the antenna was centered accurately with the optical plummet over the reference point on the marker, when specified, a heavy weight plumb bob will be used to check that the plumb point is within specifications.

(s) Measurements of station pressure (in millibars), relative humidity, and air temperature (in°C) will be recorded at the beginning, midpoint, and end depending on the period of the observing session.

(t) Report only unusual weather conditions, such as major storm fronts passing over the sites during the data collection period. This report will include station pressure, relative humidity, and air temperature.

(u) The amount of warm-up time required is very instrument dependent. It is very important to follow the manufacturer's specifications.

(v) An obstruction is any object that would effectively block the signal arriving from the satellite. These include buildings, trees, fences, humans, vehicles, etc.

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One or more of the orbital parameters may be free in the adjustment while simultaneously solving for the base line vectors.

If a network solution is desired for ultimate accuracy, the observing scheme must include two or more "master" stations. The "master" stations should be located on opposite sides from the center of the project. All other criteria for the field procedures for either the "master" or other observing schemes are given in table 4. Which observing scheme is best, "leapfrog" or the "master" approach, will depend on the accuracy standard for the survey, the accuracy of the orbit coordinate data, and number of receivers available for the project. These and other factors will dictate the final observing strategy for the project.

For all surveys, the antenna must be stably located over the station mark for the duration of the observations within the allowable antenna setup error specified in appendix D. The height differences will be measured in feet and metric units and all will be recorded. Experience has demonstrated that blunders can be minimized by making this double measurement before and after each survey session.

The antenna phase center will be plumbed over the survey point using an optical plummet, collimator or similar instrument for control surveys. The each station occupation. The adjustment of optical plummets should be checked frequently, at least once per week or whenever there is an indication the plumb error exceeds the tolerance specified in appendix D. This check is for the purpose of determining gross plumb errors of 1 cm or more.

If an antenna is moved during an observing session, the set of observations for that session may not be acceptable. This will depend on such factors as the total data collection span before or after the antenna was moved, the quality of the data, and the quality and completeness of the data collected at the other observing stations.

The power source for the survey equipment should be stable and continuous especially for the high-accuracy surveys to minimize unnecessary breaks in the observations or damage to the equipment that would affect the quality of the data.

When observations of temperature and relative humidity are specified, these data shall be collected near the location of the antenna and at approximately the same height above the ground. Observations of wet-bulb and dry-bulb temperature readings should be recorded to the nearest 1.0°C. The relative humidity should be determined to the nearest 5 percent. Barometric readings at the station site should be recorded to the nearest millibar and corrected for any significant difference in height between the antenna phase center and location of the barometer. The meteorological instruments should be brought together and compared at least once per week and compared against a standard at least once per month. The logs shall include the name of manufacturer, model, and serial numbers of instruments used.

Office Procedures

Data Processing

Software to process the raw tracking data has been developed to handle either single or multiple base line input. The software incorporates a variety of models and differences in capabilities. Software adopted for processing the raw data must be certified as capable of producing results that meet the accuracy standards specified for a survey. Software can be certified by processing test data sets collected on FGCC 3-D test networks.

Numerous groups are investigating improvements to processing software. Major areas of work underway include: (a) orbit refinement modeling, (b) difference (single, double or triple) versus nondifference processing of carrier phase observations, (c) improved techniques for resolving carrier phase ambiguity and cycle-slips, and (d) improved atmospheric refraction modeling (ionosphere and troposphere).

All software must be able to produce from the raw data relative position coordinates and corresponding variance-covariance statistics which in turn can be used as input to three-dimensional network adjustment programs.

Criteria for processing and determining the quality of GPS relative positioning results are as follows (Remondi 1984 and Beutler et al. 1987):

1. The cutoff angle for data points should be no greater than 20.

2. The point position (absolute) coordinates for the station held fixed in each single, session, or network base line solution must be referenced to the datum for the satellite orbital coordinates (ephemerides). This datum is now called the World Geodetic System 1984 (WGS-84) (DMA 1987).

   The accuracy required for these coordinates will depend on the order of the survey. The order and corresponding accuracies are:

Order AA:	± 0.5 meter
Order A:	± 0.5 meter
Order B:	± 2.5 meter
Order 1 and lower:	± 25 meter

   In order of descending accuracies, the following are acceptable methods for estimating the fixed coordinates:

   a. Point position reduction of the GPS observations using Doppler smoothed pseudorange (code phase) measurements.

   b. Point position coordinates determined from unsmoothed GPS pseudorange measurements.

   c. Point position reduction of Transit Doppler observations using the precise ephemerides and transformed to WGS-84.

   d. Use of NAD 1983 published coordinates.

   e. Transformation of coordinates in a non-geocentric datum (e.g. NAD 1927) to the WGS-84 datum. In this method, the surveyor must be careful in obtaining transformation values that reflect with sufficient accuracy the differences between the non-geocentric local datum and the WGS-84 system.

3. Processing must account for the offset of antenna phase center relative to the station mark in both horizontal and vertical components.

4. As a rule of thumb, the number of simultaneous phase observations rejected (excluding those affected by cutoff angle and nonsimultaneous observations) for a solution should be less than 5 percent for accuracy standards AA, A and B, and 10 percent for the remaining standards.

5. Depending on the number of observations, quality of data, method of reduction, and length of base lines, the standard deviation of the range residuals in the base line solution should be between 0.1 and 2 cm for orders A, B, and 1; 1 to 4 cm for order 2; and, 1 to 8 cm for order 3.

6. The maximum allowable formal standard errors for the base line components may

depend on the particular software. With proper weighting in a fixed orbit solution, the values should be less than the expected accuracy for the orbit data. Typically, these range within 2 cm for base lines with lengths of less than 50 km.

Analysis and Adjustments

In practice, there will be two classifications for a GPS relative positioning survey. One would be based on the internal consistency of the GPS network adjusted independently of the local network control. This would be called the "geometric" classification. The second classification, if required, would be based on the results of a constrained adjustment where stations of the GPS survey network connected to the local network control are held fixed to vertical and horizontal coordinates in the National Geodetic Reference System (NGVD 1929 and NAD 1983). This is referred to as the "NGRS" classification.

Table 5 summarizes the specifications to aid in classifying the results for a GPS survey project.

Loop closures and differences in repeat base line measurements will be computed to check for blunders and to obtain initial estimates for the internal consistency of the GPS network.

Error of closure is the ratio of the length of the line representing the equivalent of the resultant errors in the base line vector components to the length of the perimeter of the figure constituting the survey loop analyzed. The error of closure is valid for orders A and B surveys only when there are three or more independently determined base lines (from three or more observing sessions) included in the loop closure analysis. For orders 1 and lower, independently determined base lines from a minimum of two observing sessions are required for a valid analysis. Loop closures incorporating only base lines determined from a common observing session (simultaneous observations) are not valid for analyzing the internal consistency of the GPS survey network.

After adjusting for any blunders, a minimally constrained (sometimes called a "free") least squares adjustment should be performed and the normalized residuals examined. The normalized residual is the residual multiplied by the square root of its weight, i.e. the ratio of the residual to the *a priori* standard error. Examining the normalized residuals helps to detect bad baseline vectors. In the "free" adjustment, one arbitrary station is held fixed in all three coordinates and the four bias unknowns (3 rotations and one scale parameter) are set to zero values (Vincenty 1987). The observation weights should be verified as realistic by inspecting the estimate of the variance of unit weight, which should be close to 1. However, in practice, it may be higher, perhaps in the range of 3 to 5 because for a particular GPS baseline solution software, the formal errors from the base line solutions may be too optimistic.

Vector component (relative position) standard errors computed by error propagation between points in a correctly weighted minimally constrained least squares adjustment will indicate the maximum achievable precision for the "geometric" classification.

Table 5. -- Office procedures for classifying GPS relative positioning networks independent of connections to existing control

Geometric relative positioning standards — Order: / ppm :	AA 0.01	A 0.1	B 1.0	1 10	2-I 20	2-II 50	3 100
Ephemerides:							
Orbit accuracy, minimum (ppm)............	0.008	0.05	0.5	5	10	25	50
Precise ephemerides required?............	Y[a]	Y[a]	Y	op	op	N	N
Loop closure analyses(b) - When forming loops, the following are minimum criteria:							
Base lines in loop from independent observations not less than............	4	3	2	2	2	2	2
Base lines in each loop, total not more than................................	6	8	10	10	10	15	15
Loop length, not more than (Km)..........	2000	300	100	100	100	100	100
Base lines not meeting criteria for inclusion in any loop, not more than (percent of all independent lines)....	0	0	5	20	30	30	30
In any component (X,Y,Z), "maximum" misclosure not to exceed (cm).........	10	10	15	25	30	50	100
In any component (X,Y,Z), "maximum" misclosure, in terms of loop length, not to exceed (ppm)..................	0.2	0.2	1.25	12.5	25	60	125
In any component (X,Y,Z), "average" misclosure, in terms of loop length, not to exceed (ppm)..................	0.09	0.09	0.9	8	16	40	80
Repeat base line differences:							
Base line length, not more than (Km).....	2000	2000	500	250	250	100	50
In any component (X,Y,Z), "maximum" not to exceed (ppm)..................	0.01	0.1	1.0	10	20	50	100
Minimally constrained adjustment analyses:							
(Criteria is being developed and will appear in an updated version of this document)							

Table 5. -- Office procedures for classifying GPS relative positioning networks independent of connections to existing control (continued)

REMARKS:
(a) The precise ephemerides is presently limited to an accuracy of about 1 ppm. By late 1989, it is expected the accuracy will improve to about 0.1 ppm. It is unlikely orbital coordinate accuracies of 0.01 ppm will be achieved in the near future. Thus to achieve precisions approaching 0.01 ppm, it will be necessary to collect data simultaneously with continuous trackers or fiducial stations. (see criteria for field procedures, table 5.) Then the all data is processed in a session or network solution mode where the initial orbital coordinates are adjusted while solvinmg for the base lines. In this method of processing the carrier phase data, the coordinates at the continuous trackers are held fixed.
(b) Between any combination of stations, it must be possible to form a loop through three or more stations which never passes through the same station more than once.

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The constrained least squares adjustment will use models which account for: the reference ellipsoid for the network control, the orientation and scale differences between the satellite and network control datums, geoid-ellipsoid relationships, the distortions and/or reliability in the network control, and instability in the control network due to horizontal and/or vertical deformation. A survey variance factor ratio will be computed to aid in determining the "NGRS" classification of the adjustment. The classification for the adjustment into the NGRS should not exceed the order for the combined control network.

The constrained adjustment determines the appropriate orientation and scale corrections to the GPS Baseline vectors so it with conform to the local network control. Because of possible significant inconsistencies in the network control between sections of the project area, it may be necessary to compute several sets of orientation and scale corrections. This is done by dividing the project area into smaller "bias groups", provided that in each such group there is sufficient existing control with adequate distribution that is tied to the GPS network (Vincenty 1987).

If reliable geoid height data are available, the adjustment to determine elevations should be done in terms of heights above the ellipsoid. However, useful estimates for elevations above mean sea level can be determined if geoidal height data are not available by fixing in an adjustment at least three stations with elevations. The stations with elevations must be well-distributed to permit fitting a plane through the three heights. The effect of ignoring the slope means that the geoidal slope is absorbed by two rotation angles (around the north and east axes in a horizon system) and geoidal heights are absorbed by the scale correction in a constarined 3-D adjustment (Vincenty). If there is one or more significant changes in the geoidal slope within the project area, the project can be divided into smaller "bias groups", provided there is at least three vertical control stations appropriately distributed within the "bias group" area.

The discussion related to "bias groups" points out the importance in the planning for a GPS survey project to insure there is included in the survey adequate connections to the horizontal and vertical control network.

See appendix G for examples of a network of points surveyed by GPS, each designed to meet different classification criteria. The field survey statistics are also summarized.

SUMMARY

Geometric relative positioning accuracy standards have been developed to meet classification requirements for control surveys and high-precision engineering surveys performed by GPS relative positioning techniques and other three-dimensional measurement systems such as VLBI. Relative positioning accuracies at the 1.0 cm + 1-2 ppm level can be achieved routinely from GPS carrier phase observations. The proposed standards augments the FGCC horizontal distance accuracy standards.

The specifications for geodetic surveys performed by GPS relative positioning techniques are based on extensive field and office experience gained at NGS, from special test surveys, and from reports prepared by numerous researchers within and outside of the United States. Much of the criteria reflects conservative estimates and will require further research and studies before they can be refined.

Development of the geometric accuracy standards and GPS relative positioning specifications is an evolutionary process that will continue for the foreseeable future or at least until after the Block II constellation of GPS satellites are deployed and fully operational in the early 1990's.

This document is presently undergoing a review by the U.S. Federal Geodetic Control Committee and will be considered for formal adoption. This process is expected to reach a conclusion by late summer 1988.

Until this document is formally adopted and published by the FGCC, users are cautioned to use this as only a guideline for the planning and execution of GPS relative positioning surveys.

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APPENDIX A.--FEDERAL GEODETIC CONTROL COMMITTEE MEMBERSHIP

The Federal Geodetic Control Committee (FGCC), chartered in 1968, assists and advises the Federal Coordinator for Geodetic Control and Related Surveys. The Federal Coordinator for Geodetic Control is responsible for coordinating, planning, and executing national geodetic control surveys and related survey activities of Federal agencies.

The Methodology Subcommittee of FGCC is responsible for revising and updating the Standards and Specifications for Geodetic Control Networks.

MEMBER ORGANIZATIONS

Department of Commerce
Department of Agriculture
Department of Defense
Corps of Engineers, U.S. Army
Department of Energy
Department of Housing and Urban Development
Department of Interior
Department of Transportation
National Aeronautics and Space Administration
Bureau of Land Management
International Boundary Commission

APPENDIX B.--ONE-DIMENSIONAL AND THREE-DIMENSIONAL (ELLIPSOIDAL AND SPHERICAL) ERRORS

Suppose the value m quantifies one of the components of the relative position between two marks, which may be, for example, relative height or the east-west base line component. Then the term "relative accuracy" for m will be defined as the ratio, ε/d, where the interval m- to m+ corresponds to the 95% confidence region for m while d equals the distance between two marks and ε equals the component error.

For a network of stations surveyed by GPS relative positioning techniques the three components of the relative position can be determined. The term "relative position accuracy" denotes the relative accuracy of the various components for a representative pair of network marks.

Consequently, a GPS network is said to have a relative positioning accuracy of 1 ppm (1:1,000,000) when each component of a representative base line has a relative accuracy of at least 1 ppm. The concept of relative position accuracy can be applied to networks where relative positions have been determined either by single-dimensional measurements or by three-dimensional space-based measurements (R. Snay, NGS, 1986 personal communications).

Accuracy standards for geometric relative positioning are based on the assumption that errors can be assumed to follow a normal distribution. Normal distribution applys only to independent random errors, assuming that systematic errors and blunders have been eliminated or reduced sufficiently to permit treatment as random errors.

Although, truly normal error distribution seldom occurs in a sample of observations, it is desirable to assume a normal distribution for ease of computation and understanding.

A three-dimensional error is the error in a quantity defined by three random variables. The components of a vector base line can be expressed in terms of dX, dY, and dZ. It is assumed that the spherical standard error (σ_s) is equal to the linear standard error for the components or $\sigma_s = \sigma_x = \sigma_y = \sigma_z$.

A one-sigma spherical standard error (σ_s) represents 19.9 percent probability. This compares to a one-sigma linear standard error (σ_x) which represents 68.3 percent probability.

At the 95 percent probability or confidence level, the spherical accuracy standard is $2.79\sigma_s$ compared to $1.96\sigma_x$ for a linear accuracy standard (Greenwalt and Shultz 1962).

The probability level of 95 percent is consistent with the Standards and Specifications for Geodetic Control Networks (FGCC 1984). On page 1-2 of this document, it is stated "... a safety factor of two ..." is "... incorporated in the standards and specifications." Since those accuracy standards were based on one-dimensional errors that exist in such positional data as elevation differences and observed lengths of lines, the factor of two, a $2\sigma_x$ linear accuracy standard, is a probability or confidence level of about 95 percent.

APPENDIX C.--CONVERSION OF MINIMUM GEOMETRIC ACCURACIES AT THE 95 PERCENT CONFIDENCE LEVEL FROM TABLE 1 TO MINIMUM "ONE-SIGMA" STANDARD ERRORS

The "one-sigma" three- and one-dimensional standard errors are computed by:

$\sigma_s = p/2.79$ and, $\sigma_x = p/1.96$

where, p = minimum geometric relative accuracies in (ppm) at the 95 percent confidence level

σ_s = "one-sigma" three-dimensional minimum error (ppm)

σ_x = "one-sigma" one-dimensional minimum error (ppm)

Tabulation of "one-sigma" errors for corresponding minimum geometric accuracies at the 95 percent confidence level.

Order	Class	Relative accuracies (95 percent) confidence level		Minimum geometric "One-sigma" standard errors			
				Three-dimensional (σ_s)		One-dimensional (σ_x)	
		p (ppm)	a (1:a)	(ppm)	(1:T)	(ppm)	(1:L)
AA	-	0.01	1:100,000,000	0.0036	1:279,000,000	0.005	1:200,000,000
A	-	0.1	1:10,000,000	0.036	1:27,900,000	0.05	1:20,000,000
B	-	1	1:1,000,000	0.36	1:2,790,000	0.5	1:2,000,000
1	-	10	1:100,000	3.58	1:279,000	5	1:200,000
2	I	20	1:50,000	7.17	1:140,000	10	1:100,000
2	II	50	1:20,000	17.9	1:56,000	25	1:40,000
3	I	100	1:10,000	35.8	1:28,000	50	1:20,000

APPENDIX D.--EXPECTED MINIMUM/MAXIMUM ANTENNA SETUP ERRORS

k = the repeatable setup error in (cm) for any component (horizontal and vertical) at the 95 percent confidence level

$k = 0.1pd(\beta)$, where, k_{min} = 0.3 cm and k_{max} = 10 cm

NOTE: The value for k_{min} is based on current estimates for expected setup errors when the antenna is set on a tripod at a total height of less than 5 m. When the antenna is set on a mast or twoer where the height is greater than 5 m, the esimated minimum value for k may be greater than 0.3 cm. On the other hand, if the antenna is mounted on a fixed or permanently installed stand, then K_{min} should be less than 0.1 cm.

The value for k_{max} is the expected largest value for the setup error; in practice, it should be much smaller than 10 cm, typically less than 1 cm.

p = minimum geometric accuracy standard in parts-per-million (ppm) (See table 1.)

d = distance between any two stations of a survey (km)

β = 0.05 = critical region factor for the 95 percent confidence level (1.00 - 0.95 = 0.05)

To convert setup error at the 95 percent confidence level to standard error (one-sigma), divide k by: 1.96 for 'linear' standard error, or 2.79 for 'spherical' standard error.

Tabulation of setup errors (k) in centimeters at 95 percent confidence level

Class	ppm	d = Distance between stations (km)										
		0.01	0.05	0.1	0.5	1	5	10	50	100	500	1000
AA	0.01	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3
A	0.1	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3
B	1	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.5	2.5	5
1	10	0.3	0.3	0.3	0.3	0.3	0.3	0.5	2.5	5	(10)	(10)
2-I	20	0.3	0.3	0.3	0.3	0.3	0.5	1.0	5	10	(10)	(10)
2-II	50	0.3	0.3	0.3	0.3	0.3	1.2	2.5	(10)	(10)	(10)	(10)
3-I	100	0.3	0.3	0.3	0.3	0.5	2.5	5	(10)	(10)	(10)	(10)

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APPENDIX E.--ELEVATION DIFFERENCE ACCURACY STANDARDS FOR GEOMETRIC RELATIVE POSITIONING TECHNIQUES

An elevation difference accuracy is the minimum allowable error at the 95 percent confidence level. For simplicity and ease of computations, elevation differences (dH) are assumed to be equal to orthometric height differences.

The height differences determined from space survey systems, such as GPS satellite surveying techniques, are with respect to a reference ellipsoid. These ellipsoid (geodetic) height differences (dh) can be converted to elevation differences (dh) by the relationship:

$$(dh) = (dH) - (dN)$$

where (dN) is the geoid height difference.

With accurate estimates for (dN) and adequate connections by GPS relative positioning techniques to network control points tied to National Geodetic Vertical Datum, elevations can be determine for stations with unknown or poorly known values.

> NOTE: If GPS ellipsoid height differences are being measured for the purpose of monitoring the change in height between stations, then it is not necessary to have any accurate information on the shape of the geoid. Thus, the accuracy of the height differences depends only on the accuracy of the GPS ellipsoid height differences.

The accuracy of the GPS derived elevations for points in a survey will depend on three factors: (1) accuracy of the GPS ellipsoid height differences, (2) accuracy of the elevations for the network control, and (3) accuracy of the geoid height difference estimates.

In the following table, elevation difference accuracy standards at the 95 percent confidence level are proposed. The order/class correspond to the proposed geometric relative position accuracy standards. At the high orders, the error is dominated by the accuracy for the (dN) values, whereas, for the lower orders, the major source of error is in the ellipsoid height differences.

> NOTE: In developing these standards, it is assumed that errors or inconsistencies in the vertical network control are negligible. Of course, this may not be true in many cases.

Elevation difference accuracy standards for geometric relative positioning techniques.

(95 percent confidence level)							
Order	Class	Minimum elevation difference accuracy standard		(From table 1) Minimum geometric relative position accuracy standard	Minimum geoid height difference accuracy standard	
		p_E (ppm)	1:e	p (ppm)	p_N (ppm)	1:n
AA	-	2	1:500,000	0.1	2	1:500,000
A	-	2	1:500,000	0.1	2	1:500,000
B	-	5	1:200,000	1	5	1:200,000
1	-	15	1: 67,000	10	10	1:100,000
2	I	20	1: 50,000	20	10	1:100,000
2	II	50	1: 20,000	50	20	1: 50,000
3	I	100	1: 10,000	100	40	1: 25,000

NOTE: THESE ELEVATION DIFFERENCE ACCURACY STANDARDS ARE TO BE USED ONLY FOR ELEVATION DIFFERENCES DETERMINED INDIRECTLY FROM ELLIPSOID HEIGHT DIFFERENCE MEASUREMENTS.

FOR DIRECT VERTICAL MEASUREMENT TECHNIQUES SUCH AS DIFFERENTIAL OR TRIGONOMETRIC LEVELING, USE ONLY THE ACCURACY STANDARDS GIVEN IN THE FGCC 1984 DOCUMENT, SECTION 2.2, PAGES 2-2 and 2-3.

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r = The number of GPS receivers used for each observing session

n = Minimum number of independent occupations per each station of a project

- If n = 1, (no check, no redundancy)
- If n = 1.5, (50 percent or more stations with 2 or more occupations)
- If n = 1.75, (75 percent or more stations with 2 or more occupations)
- If n = 2, (100 percent check, adequate redundancy)
- If n = 3, (excellent check, highest confidence)

NOTE: when, r = 2, n will always be 2 or greater.
when, r>2, then n = 1, 2, 3, or more occupations.

m = Total stations for the project (existing and new)

s = Number of observing sessions scheduled for the project

d = Average number of observing sessions scheduled per observing day (e.g. 1 per day, 2 per day, 2.5 per day, etc.)

NOTE: Depends on required observing span, satellite availability, and transportation requirements.

x = Number of observing days, where $x = s/d$

y = Number of observing days scheduled per week, generally 5 to 7.

w = Number of workweeks, where $w = x/y = s/(d \cdot y)$

p = Production factor (based on historical evidence of reliability; ratio of proposed observing sessions for a project versus final number of observed sessions)

$$p = f/i,$$

where: f = final number of observing sessions required to complete the project

i = Proposed (initial) number of observing sessions scheduled for the project, where:

$$i = (m \cdot n)/r$$

FORMULAS:

$$s = (m \cdot n)/r + (m \cdot n)(p-1)/r + k \cdot m$$

where, k is a safety factor: $k = 0.1$ for local projects; within 100 km radius.
$k = 0.2$ for all other

x = estimated number of observing days for a project: $x = s/d$

w = estimated number of work-weeks for a project: $w = x/y$

v = estimated total vectors for a project: $v = r \cdot s(r-1)/2$

b = estimated independent vectors for a project: $b = (r-1)s$

EXAMPLE:

If	$n =$	1.75	independent occupations per station
	$m =$	50	total stations for project
	$y =$	5	observing days per week
	$k =$	0.2	safety factor
	$r =$	4	number of GPS receivers per observing session
	$d =$	2.5	average observing sessions per day
	$p =$	1.1	production factor
Then	$s =$	22 + 3 + 10 = 35 observing sessions	
	$x =$	14 observing days	
	$w =$	2.8 workweeks	
	$b =$	105 independent vectors	

COMMENTS:

In the equation to compute the number of observing sessions (s), if there were no sessions lost due to receiver malfunctions, and no additional sessions required to cover such factors as human error and irregular network configuration, then

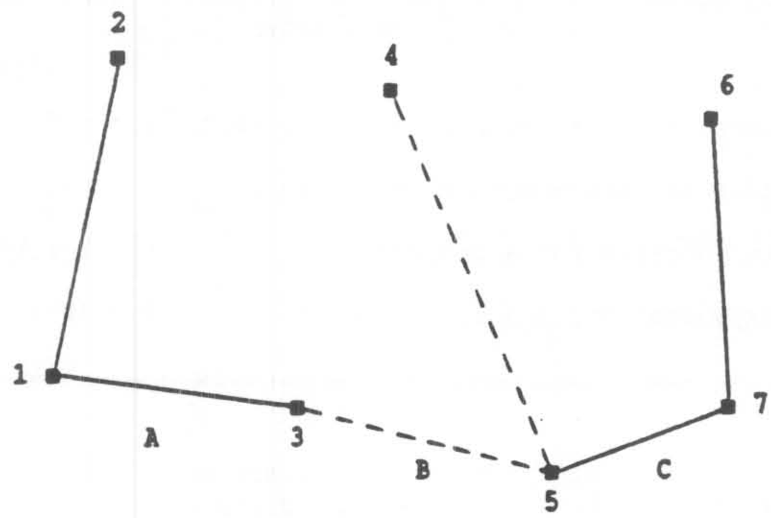
$$s = (m \cdot n)/r$$

However, the second part of the equation for computing "s" is to allow for additional sessions to offset scheduled sessions that may be lost due to equipment breakdown.

The third part of the equation, k(m), allows for additional sessions that may be required due to human error, irregular network configuration, etc.

APPENDIX G.--EXAMPLES OF GPS SURVEYS WITH SUMMARY OF STATISTICS USED TO CLASSIFY THE ORDER OF SURVEY BASED ON THE OBSERVING SCHEME AND DATA COLLECTION PROCEDURES

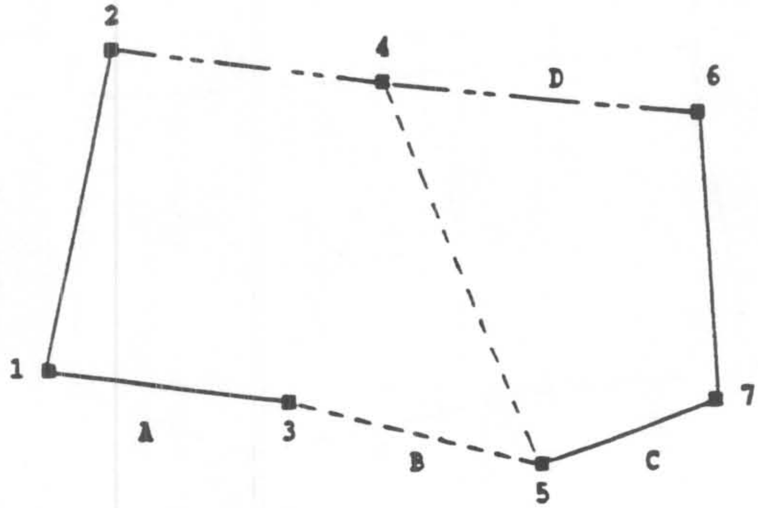
Example 1:



NOTE: Only nontrivial (independent) base lines are shown.

Observing sessions, total number (A,B, and C)......................	3
Receivers observing simultaneously................................	3
Stations, total number..	7
Station occupations:	
Single occupations (no redundancy)................................	5
Two or more occupations, number/percent of all stations..........	2/29
Three or more occupations, number/percent of all stations........	0/0
Base lines determined:	
All (trivial and nontrivial)......................................	9
Independent (nontrivial)..	6
Repeat base lines (N-S/E-W/percent of nontrivial)..................	0/0/0
Loop closure analyses:	
Valid loops formed?/Number of stations that can't be included....	No/7
Loops containing base lines from (2 or)/(3 or more) sessions?....	0/0
Geometric relative position classification (based on table 4)......	None

Example 2:

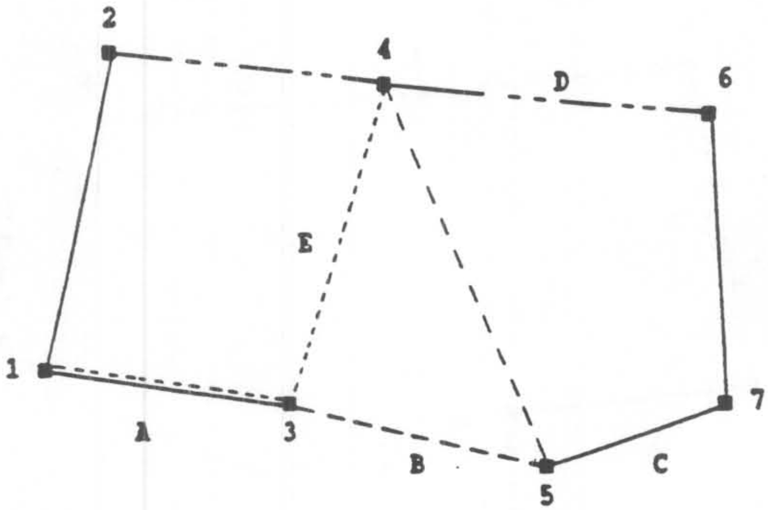


NOTE: Only nontrivial (independent) base lines are shown.

Observing sessions, total number (A,B,C, and D)	4
Receivers observing simultaneously..................................	3
Stations, total number..	7
Station occupations:	
Single occupations (no redundancy)..................................	2
Two or more occupations, number/percent of all stations..........	5/71
Three or more occupations, number/percent of all stations........	0/0
Base lines determined:	
All (trivial and nontrivial)......................................	12
Independent (nontrivial)...	8
Repeat base lines (N-S/E-W/percent of nontrivial)..................	0/0/0
Loop closure analyses:	
Valid loops formed?/Number of stations that can't be included....	Yes[a]/0
Loops containing base lines from (2 or)/(3 or more) sessions?....	0/2
Geometric relative position classification (based on table 4)......	Order "2-II"

(a) Loops formed:
1- 1(A)3 + 3(B)5 + 5(B)4 + 4(D)2 + 2(D)1 Includes 3 sessions
2- 5(C)7 + 7(C)6 + 6(D)4 + 4(B)5 Includes 3 sessions

Example 3:

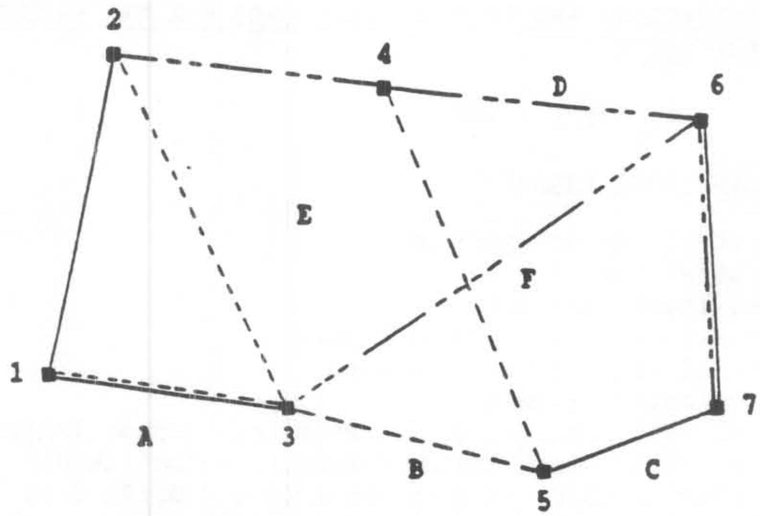


NOTE: Only nontrivial (independent) base lines are shown.

Observing sessions, total number (A,B,C,D, and E)	5
Receivers observing simultaneously	3
Stations, total number	7
Station occupations:	
Single occupations (no redundancy)	1
Two or more occupations, number/percent of all stations	6/86
Three or more occupations, number/percent of all stations	2/29
Base lines determined:	
All (trivial and nontrivial)	15
Independent (nontrivial)	10
Repeat base lines (N-S/E-W/percent of nontrivial)	0/1/10
Loop closure analyses:	
Valid loops formed?/Number of stations that can't be included	Yes[a]/0
Loops containing base lines from (2 or)/(3 or more) sessions?	1/2
Geometric relative position classification (based on table 4)	Order "1"

(a) Loops formed:
1- 1(A)3 + 3(E)4 + 4(D)2 + 4(A)1 — Includes 3 sessions
2- 3(B)5 + 5(B)4 + 4(E)3 — Includes 2 sessions
3- 5(C)7 + 7(C)6 + 6(D)4 + 4(B)5 — Includes 3 sessions

Example 4:



NOTE: Only nontrivial (independent) base lines are shown.

Observing sessions, total number (A,B,C,D,E, and F)	6
Receivers observing simultaneously....................................	3
Stations, total number..	7
Station occupations:	
Single occupations (no redundancy)................................	0
Two or more occupations, number/percent of all stations..........	7/100
Three or more occupations, number/percent of all stations........	3/43
Base lines determined:	
All (trivial and nontrivial)..	18
Independent (nontrivial)..	12
Repeat base lines (N-S/E-W/percent of nontrivial).................	2/1/25
Loop closure analyses:	
Valid loops formed?/Number of stations that can't be included....	Yes[a]/0
Loops containing base lines from 2 or / 3 or more sessions?......	0/4
Geometric relative position classification (based on table 4)......	Order "B"

NOTE: If one additional session was observed where session G would include stations 1,2 and 5 (or 7), then the survey would be classified with an Order of "A".

(a) Loops formed:	1-	1(A)3 + 3(E)4 + 4(D)2 + 2(A)1	Includes 3 sessions
	2-	3(B)5 + 5(C)7 + 7(C)6 + 6(F)3	Includes 3 sessions
	3-	6(D)4 + 4(B)5 + 5(C)7 + 7(F)6	Includes 3 sessions
	4-	1(E)3 + 3(B)5 + 5(B)4 + 4(E)2 + 2(A)1	Includes 4 sessions

APPENDIX H.--SPECIFICATIONS AND SETTING PROCEDURES FOR THREE-DIMENSIONAL MONUMENTATION

May 11, 1988

A. Materials required for each marker:

1. Rod, stainless steel, 4-foot sections
2. Rod, stainless steel, one 4-5 inch
2. Studs, stainless steel, 3/8 inch
3. Datum point, stainless steel, 3/8 inch bolt
4. Spiral (fluted) rod entry point, standard
5. NGS logo caps, standard, aluminum
6. Pipe, schedule 40 PVC, 5 inches inside diameter, 2-foot length
7. Pipe, schedule 40 PVC, 1 inch inside diameter, 3-foot length
8. Caps, schedule 40 PVC, (Slip-on caps centered and drilled to 0.567 inch ±0.002)
9. Cement for making concrete
10. Cement, PVC solvent
11. Loctite (2 oz. bottle)
12. Grease
13. Sand (washed or play)

B. Setting procedures:

1. The time required to set an average mark using the following procedures is 1 to 2 hours.

2. Using the solvent cement formulated specifically for PVC, glue the aluminum logo cap to a 2-foot section of 5-inch PVC pipe. This will allow the glue to set while continuing with the following setting procedures.

3. Glue the PVC cap with a drill hole on one end of a 3-foot section of schedule 40 PVC pipe 1-inch inside diameter. Pump the PVC pipe full of grease. Thoroughly clean the open end of the pipe with a solvent which will remove the grease. Then glue another cap with drill hole on the remaining open end. Set aside while continuing with the next step.

4. Using a power auger or post hole digger, drill or dig a hole in the ground 12-14 inches in diameter and 3 1/2 feet deep.

5. Attach a standard spiral (fluted) rod entry point to one end of a 4-foot section of stainless steel rod with the standard 3/8 inch stud. On the opposite end screw on a short 4 to 5 inch piece of rod which will be used as the impact point for driving the rod. Drive this section of rod with a reciprocating driver such as Whacker model BHB 25, Pionjar model 120, or another machine with an equivalent driving force.

6. Remove the short piece of rod used for driving and screw in a new stud. Attach another 4-foot section of rod. Tighten securely. Reattach the short piece of rod and drive the new section into the ground.

B. Setting procedures (continued):

7. Repeat step 6 until the rod refuses to drive further or until a driving rate of 60 seconds per foot is achieved. The top of the rod should terminate about 3 inches below the ground surface.

8. When the desired depth of the rod is reached, cut off the top removing the tapped and threaded portion of the rod leaving the top about three inches below ground surface. The top of the rod then must be shaped to a smooth rounded (hemispherical) top, using a portable grinding machine to produce a datum point. The datum point must then be center punched to provide a plumbing (centering) point.

   NOTE: For personnel that may not have the proper cutting or grinding equipment to produce the datum point, the following alternative procedure should be used if absolutely necessary. When the desired depth of the rod is obtained (an even 4-foot section), thoroughly clean the thread with a solvent to remove any possible remains of grease or oil that may have been used when the rod was tapped. Coat the threads of the datum point with Loctite and screw the datum point into the rod. Tighten the point firmly with vise grips to make sure it is secure. The datum point is a stainless steel 3/8 inch bolt with the head precisely machined to 9/16 inch.

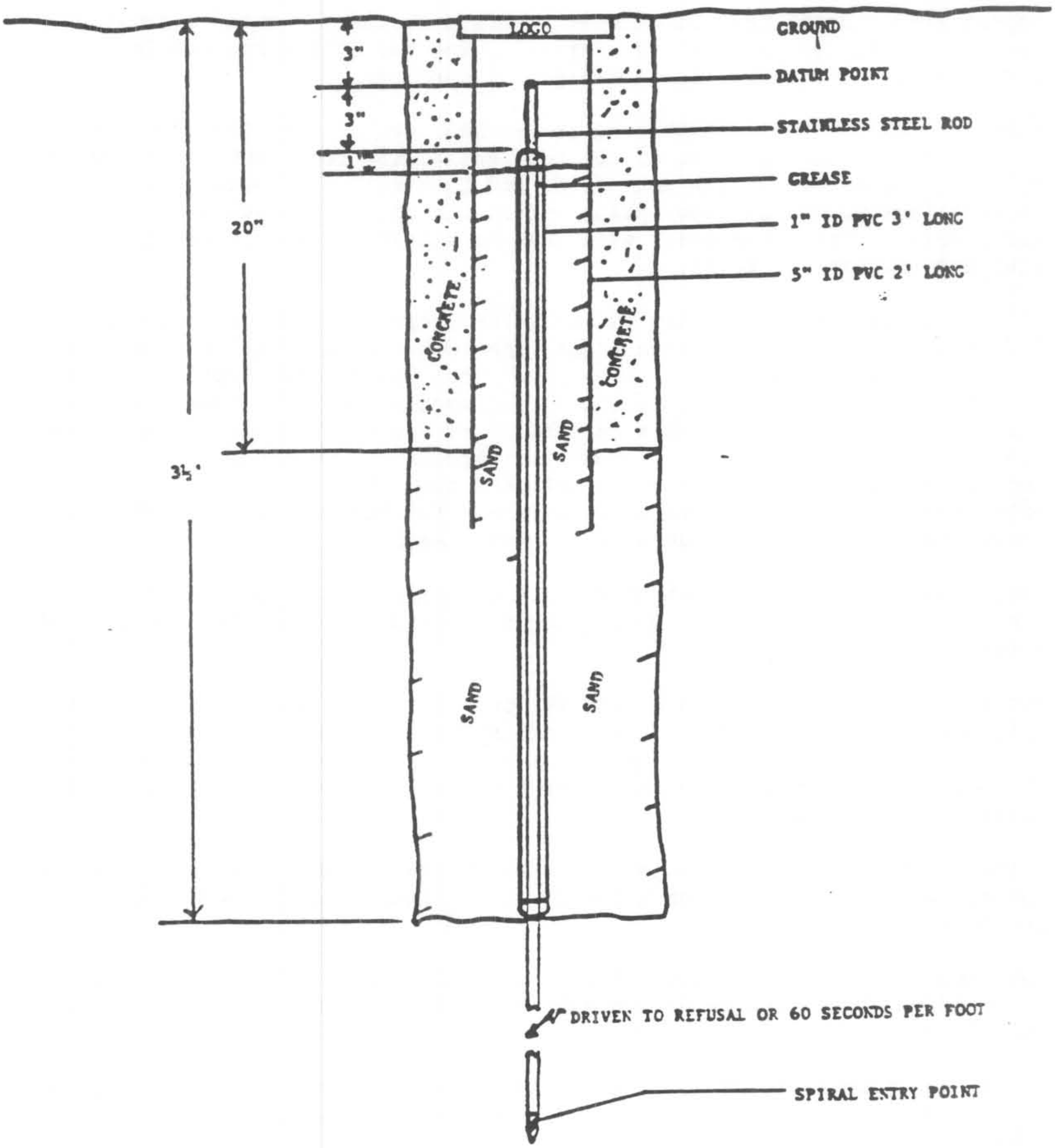
9. Insert the grease filled 3-foot section of 1-inch PVC pipe (sleeve) over the rod. The rod and datum point should protrude through the sleeve about 3 inches.

10. Backfill and pack with sand around the outside of the sleeve to 20 inches below ground surface. Place the 5-inch PVC and logo cap over and around the 1-inch sleeve and rod. The access cover on the logo cap should be flush with the ground. The datum point should be about 3 inches below the cover of the logo cap.

11. Place concrete around the outside of the 5-inch PVC and logo cap, up to the top of the logo cover. Trowel the concrete until a smooth neat finish is produced.

12. Continue to backfill and pack with sand inside the 5-inch PVC and around the outside of the 1-inch sleeve and rod to about 1 inch below the top of the sleeve.

13. Remove all debris and excess dirt to leave the area in the condition it was found. Make sure all excess grease is removed and the datum point is clean.



Schematic of the NGS 3-D marker

APPENDIX A-13

* RECOVERY REPORT

MASSACHUSETTS GEODETIC SURVEY
REPORT ON CONDITION OF SURVEY MARKER

DESIGNATION OF STATION ______________________ CITY OR TOWN ______________ USGS QUADRANGLE ______________

TYPE OF MARK ______________________ PRESENT CONDITION OF MARK ______________________

LETTERS OR NUMBERS FOUND STAMPED (NOT CAST IN) THE MARK ______________________

USE THE FOLLOWING SPACE FOR REPORTING UPON THE THOROUGHNESS OF THE SEARCH IN CASE MARK WAS NOT RECOVERED, OR FOR SUGGESTING NEEDED CHANGES IN THE PUBLISHED DESCRIPTION, TOGETHER WITH COMPLETE DESCRIPTION OF MARK AND ITS PLACEMENT, IF THE MARK WAS RECOVERED. IF MARK IS DISK, GIVE NAME OF AGENCY FOUND CAST IN DISK. (USE BACK OF CARD FOR SKETCH)

DATE ______________ (SIGNED) ______________________ TITLE ______________________

APPENDIX A-14

* NATIONAL GEODETIC SURVEY ADDRESSES AND TELEPHONE NUMBERS

NATIONAL GEODETIC SURVEY DIVISION, N/CGI
1315 EAST-WEST HIGHWAY, ROOM 9535
SILVER SPRING, MD 20910-3282

NATIONAL GEODETIC INFRORMATION CENTER (VOICE) (301) 713-3242

STATE ADVISORY

- NH
  - CURTIS CROWE (603) 271-1600 CONCORD, NH
    - clcrow@aol.com

- MA
  - CURTIS CROWE (617) 973-8466 BOSTON, MA
    - clcrow@aol.com

- VT
  - MILO ROBINSON (802) 828-2813 MONTPELIER, VT
    - mrobinson@state.vt.us