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# Flexible Transit Services

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16. Abstract Public transit services in Massachusetts typically operate fixed routes with a published schedule. Fixed routes are not cost effective for serving low density, dispersed demand. The first part of this study is to conduct a theoretical analysis of the various forms of flexible transit service, ranging from deviating routes and schedules to fully demand-responsive service. The models are used to compare costs of different services or communities of varying density and size. The second part of the study involves analysis of data from flexible transit pilot programs implemented by Regional Transit Authorities (RTAs) in Massachusetts in order to synthesize needs and best practices related to data collection, management, and reporting. The experiences of RTAs are compared and combined with the results of the theoretical analysis to make recommendations for future flexible transit implementations.			
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# **Flexible Transit Services**

Final Report

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# **Disclaimer**

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# Executive Summary

Flexible transit services differ from conventional fixed-route service by adapting routes or schedules to customer demand. Historically, most transit services have operated on pre-planned routes, making stops at predefined locations according to a published schedule. This is a model that can work well in communities where there is sufficient demand to fill a transit vehicle, but in communities with low density of demand and dispersed origins and destinations, fixed routes are costly and inefficient. Although demand-responsive paratransit service has been operated for decades with advanced reservations in accordance with the Americans with Disabilities Act of 1990 (ADA), technologies like smartphones and global positioning systems now allow transit vehicles to respond to customer demands in real time.

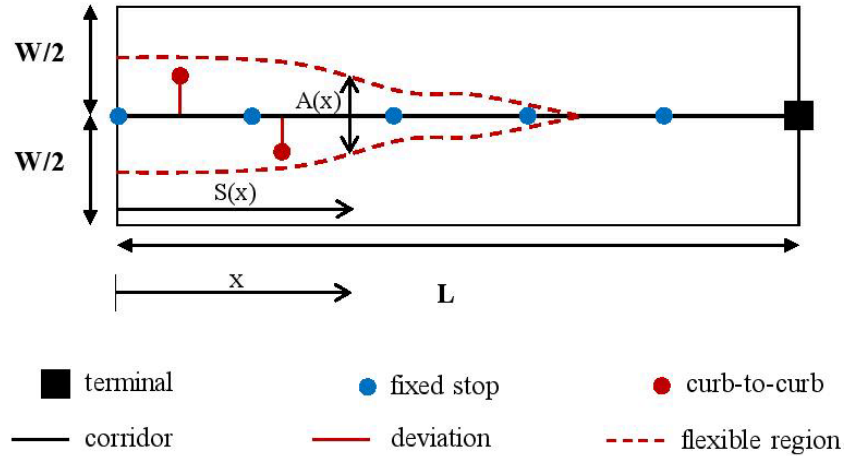
This report presents an evaluation of the potential for flexible transit to provide service in places that cannot be well served by fixed routes. The study has three main components: a modeling analysis to systematically compare the performance of flexible transit services with fixed route transit; an analysis of fully demand-responsive microtransit pilots implemented across Massachusetts in order to gain insights and synthesize the lessons learned; and an analysis of the data requirements for planning, operating, and monitoring flexible transit services. Altogether, this report serves as a resource for identifying markets where flexible transit has the potential to be effective transit service and for providing guidance based on existing experiences with microtransit in Massachusetts.

## Models of Flexible Transit

Transit services can be classified on a spectrum of flexibility, with conventional fixed-route services on one end (with routes, stops, and schedules all published in advance) and fully demand-responsive services on the other (with vehicles routed to carry customers from door to door, based on their requested location and time). Technologies for real-time communications between users and transit agencies, notably the ubiquity of GPS-equipped smartphones, now allow for many other types of flexible service in between, which allow vehicles to deviate from a defined route or within a defined corridor to serve passengers closer to their origins or destination. The trade-off is that low flexibility allows transit vehicles to operate more efficiently when there is high enough demand. A flexible service requires each vehicle to spend more time traveling greater distances to reach each customer, and this slows down service unless demand is low.

In order to conduct a systematic comparison between fixed-route and flexible transit for regions of different sizes and demand densities, a model was developed to predict the vehicle fleet size, hours of operations, and miles of operation that would be required to provide flexible service in a region. **Figure 1** shows an example of a corridor of width  $W$  and length  $L$  where passengers from distributed origins request service to a terminal, which may represent a town center or transit hub. A fixed-route transit service would stop only at fixed stops (blue dots), the spacing of which can be optimized at each location,  $S(x)$ . A flexible service can deviate from the route to pick up and drop off eligible customers who request trips within an area defined as the flexible region spanning a distance  $A(x)$  from the route at each location.





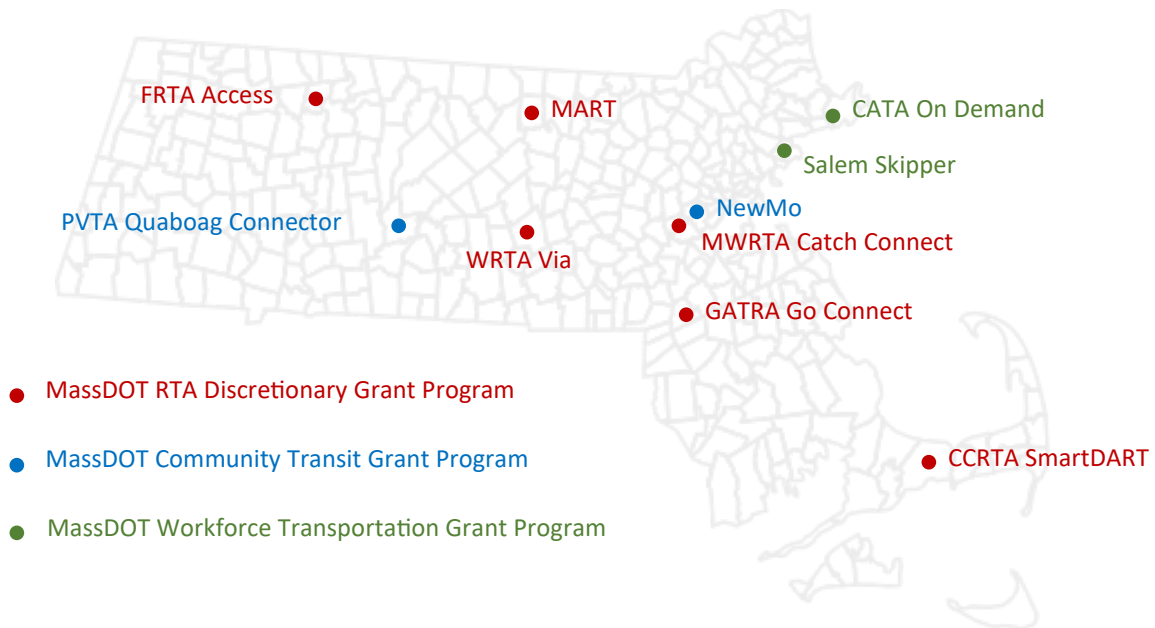
**Figure 1: System configuration for fixed-route service with route deviation**

The advantage of a model like this is that impact of region size and demand density on transit operations, costs, and the travel times experienced by users can all be quantified and optimized in terms of the design parameters: fixed stop spacing, width of flexible service region, and service headway. The results of a systematic quantitative analysis show that greater demand density leads to lower optimal flexibility, with the system converging toward a conventional fixed-route service. At very low demand density, it is most cost-effective to operate the entire area as a flexible demand-responsive service. The demand threshold to select flexible transit or microtransit depends on the size of the region, the distribution of demand in time and space, and the characteristics of the vehicles. Both the literature and modeling suggest that when demand is distributed and the density is less than 10 trips per hour per square mile, flexible transit is more efficient than fixed routes.

### Microtransit Pilots in Massachusetts

In an effort to provide more innovative and effective transit service across the Commonwealth of Massachusetts, the Massachusetts Dept. of Transportation (MassDOT) has funded a series of microtransit pilot projects with Regional Transit Authorities (RTAs). **Figure 2** shows the six pilots funded through the RTA Discretionary Grant Program, as well as pilots funded through the Community Transit Grant Program and Workforce Transportation Grant Program. The microtransit pilots span communities from very rural areas in Western Massachusetts to suburban communities in the Greater Boston Metropolitan Area. All of the microtransit pilots operated in areas with demand well below the 10 trips per hour per square mile threshold identified previously, so the operation of flexible transit rather than fixed-route service was clearly justified in all cases.

RTAs in Massachusetts have generally viewed microtransit pilots as successful. Demand-responsive microtransit either replaced under-performing fixed-route service or introduced service to areas with low demand density. Several RTAs have expanded or are planning to expand microtransit to more geographic areas or more hours of service.



**Figure 2: Microtransit pilot programs funded by MassDOT**

The following patterns and lessons learned emerged from analysis of the microtransit pilots.

1. ***Microtransit is used to serve rural and low-density suburban areas.*** Microtransit is used to serve dispersed demand in low-density areas where fixed-route transit does not perform well. RTAs used microtransit either to replace underperforming fixed routes or to provide new transit service where none had been provided before.
2. ***Microtransit is most efficient for short trips within small service areas.*** Most of the implementations were limited to small service regions that constitute only a part of the RTA's full service area. This scale makes trips easier to schedule, leads to fewer miles of empty vehicle repositioning, and supports the function of microtransit as a local complement to a larger fixed-route network.
3. ***Start small and expand.*** All RTAs advocated for starting with a small microtransit service that is limited in geographic coverage and hours of operations. This allows the agency to respond to the challenges of initial implementation and fix problem before expanding the service more broadly.
4. ***Microtransit fares are set above those for fixed route services.*** Fares for microtransit were generally 1.5 to 2 times the fixed-route fare, which is comparable to the fares for ADA paratransit. The rationale is to give users an incentive to keep using existing fixed-route services without burdening people who depend on transit.
5. ***Microtransit provided valuable service during the coronavirus pandemic.*** The low vehicle occupancies associated with microtransit are compatible with public health constraints of the pandemic. Several RTAs used the microtransit service to provide necessary transit service during the pandemic.

6. ***Marketing and outreach is a challenge.*** A common experience across RTAs was the challenge in communicating to the public how to use the new microtransit service. Successful outreach involved extensive effort to coordinate with community organizations, senior centers, and existing transit riders about the new opportunities associated with on-demand microtransit.

## **Data Requirements**

### ***Planning and System Design***

The data requirements for planning a flexible transit system are the same as the inputs for the quantitative model of system performance. Planning an effective service requires defining the extent of the intended service area, so that a comparison can be made between the cost and performance of a flexible service and a conventional fixed-route service. These data fall into three main categories:

1. Service Area Characteristics
  - Service Area [ $\text{mi}^2$ ]
  - Service Corridor Length [mi] and Width [mi]
  - Operational Hours [hrs]
2. Demand Characteristics
  - Density of Demand [trips/ $\text{mi}^2/\text{hr}$ ]
  - Distribution of Demand
  - Percentage Eligible for Curb-to-Curb Service [%]
  - Passenger Value of Time [\$ / hr]
3. Mode Characteristics
  - Access Mode Speed [mi/hr]
  - Transit Vehicle Speed [mi/hr]
  - Dwell Time at Stops [sec/stop]
  - Cost of Vehicles [\$ / veh]
  - Cost per Vehicle Hour [\$ / veh-hr] and per Vehicle Mile [\$ / veh-mi]

### ***Operations***

Unlike fixed-route transit systems, in which routes, stop locations, and schedules are planned in advance, flexible transit services require information about the locations and times that customers wish to travel. Real-time systems typically use an app-based platform for users to request service and to dispatch vehicles. Data requirements for flexible transit operations include:

- Trip Origin/Destination Location and Time
- Vehicle Fleet Size
- Vehicle Capacity
- Vehicle Location Tracking (Real Time)

Systems may also incorporate fare payment information and subscriptions for repeated trips.

There are commercial software products that provide app-based platforms to support flexible transit operations (e.g., Ecolane, Transloc, Via). These products have similar functionalities in that they include a smartphone app that facilitates communication between users and the agency. Commercial vendors also sell products to facilitate real-time scheduling and dispatch of transit vehicles. An advantage of commercial products is that they can be implemented relatively quickly, but there are two main downsides: 1) commercial products typically require ongoing payments for service; and 2) proprietary software restricts data access, compatibility with other products, and the types of operational policies that can be implemented.

An alternative to commercial software is to develop an app in-house, as is used by MWRTA CatchConnect and CCRTA SmartDART. Although in-house app development is more time-consuming and requires staff with relevant skills and knowledge, the end result is a platform that gives agencies full control of their data, communications, and operations. The CatchConnect and SmartDART apps include functionality for trip booking, vehicle tracking, automated dispatch, routing, and ongoing system monitoring. The rationale for making the investment in an in-house app were the following:

- Maintain Flexibility
- Integrate with Existing Services
- Maintain Control of Operations
- More Flexible Budgeting

### ***Ongoing Monitoring***

Finally, it is important to monitor flexible transit services in order to track the performance of each system relative to its own benchmarks. Comparisons between regions are of limited value, because the performance and cost is so dependent on specific characteristics of the region served. However, it can be valuable to compare flexible transit to a fixed route that it replaces or to track changes over time. Examples of relevant performance measures that reflect the supply side and the user experience include:

- Trips per Revenue Hour [trips/veh-hr] and Revenue Mile [trips/veh-mi]
- Average and Distribution of Wait Time [hrs]
- Average and Distribution of Access Distance or Travel Time [mi or hrs]
- Average and Distribution of In-Vehicle Travel Time
- Average Vehicle Occupancy [pax/veh]
- Average Fare Paid [\$ /trip]

### **Conclusion**

Flexible transit can take a variety of forms, but the principle of adjusting transit operations to serve demand provides opportunities to provide more efficient and higher-quality transit service. The modeling and analysis of microtransit pilots undertaken for this study show that flexible transit is a viable solution for providing transit service in parts of Massachusetts that are difficult to serve with conventional fixed routes. The coronavirus pandemic affected transit demand but also illustrated how flexible transit services are resilient to unforeseen

events, because the service adapts to demands as they change. With adequate preparation and data collection, there are many opportunities to improve transit service with flexible systems.

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# List of Acronyms

<b>Acronym</b>	<b>Expansion</b>
ACTA	Airport Corridor Transportation Association
ADA	Americans with Disabilities Act
AVL	Automatic Vehicle Locators
CARTA	Charleston Area Regional Transportation Authority
CAT	Capital Area Transit
CATA	Cape Ann Transit Authority
CCRTA	Cape Cod Regional Transit Authority
DAS	Demand Adaptive Transportation System
DRT	Demand-Responsive Transit
FMOD	Flexible Mobility on Demand
FRTA	Franklin Regional Transit Authority
GATRA	Greater Attleboro Taunton Regional Transit Authority
GPS	Global Positioning Systems
GTFS	General Transit Feed Specification
HRT	Hampton Roads Transit
IPC	Information Processing Center
JTA	Jacksonville Transportation Authority
MassDOT	Massachusetts Department of Transportation
MART	Montachusett Regional Transit Authority
MAST	Mobility Allowance Shuttle Systems
MBTA	Massachusetts Bay Transportation Authority
MDT	Mobile Data Terminals
MFB	Micro-Flex Bus
MTA	Mass Transportation Association
MTA	Metropolitan Transportation Authority
MWRTA	MetroWest Regional Transit Authority
OTS	Oahu Transit Service, Inc.
PRTC	Potomac and Rappahannock Transportation Commission
PSTA	Pinellas Suncoast Transit Authority
PVTA	Pioneer Valley Transit Authority
RTA	Regional Transit Authority
RTD	Regional Transportation District
SCAT	Sarasota County Area Transit
TCRP	Transit Cooperative Research Program
UTA	Utah Transit Authority
Votran	Volusia County Transit
WRTA	Worcester Regional Transit Authority

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# 1.0 Introduction

Public transportation service in Massachusetts is operated by the MBTA and 15 Regional Transit Authorities (RTAs). Outside of the larger cities, the density of demand for transit is low, which makes the provision of service costly. The research problem is to identify if there are flexible transit services that could be operated more cost-effectively in rural and low-density communities than conventional fixed routes to increase ridership. Flexible transit can take many forms, ranging from a fully flexible paratransit system to a more structured service that allows flag stops or route deviations. This research will synthesize insights from the pilot programs that are now being started in order to develop guidelines for best practices based on the experiences of local agencies.

## 1.1 Project Overview

---

Conventional public transit services operate vehicles on routes with predefined stops according to published schedules. In order to use a bus service, for example, passengers must adapt their travel routes and schedules to be at the stop when the bus arrives. By contrast, a person traveling by an individual mode (e.g., by driving a car) can choose to travel at any time and on any route they choose. In exchange for giving up some of this flexibility, a transit customer benefits from the efficiency of traveling on a vehicle with many other people, which uses fewer resources and may cost less money. The flexibility that the transit customers give up is transferred to the transit agency, which is responsible for planning routes, stop locations, and schedules to cost-effectively meet the mobility needs for the population served. In dense urban environments, conventional transit works well for transit users and agencies, because there is enough demand to justify many routes and stops with services operated at high frequencies. The result is that transit services in cities allow customers to travel without sacrificing much of the flexibility they would have using other transportation modes.

*Flex transit* services can adapt their routes or schedules to customer demand. By expending more resources on operations, these services can reduce the distance that customers must travel or the time that they must wait in order to board a transit vehicle. If viewed as a spectrum, one end is a conventional fixed-route service that does not adapt at all to customer demands, and the other is a fully *demand-responsive transit* (DRT) service or an on-demand service that is sometimes called *microtransit*. There are inherent trade-offs in determining how much flexibility to offer, because a system that adapts more to customer demands can provide better quality of service to riders in exchange for greater operating costs. Although conventional transit can be very efficient in urban environments, it is often not viable in suburban and rural areas, where customers would have to walk too far or wait too long to make fixed-route transit a practical mobility solution.

Technologies have now developed to allow for public transit services to be operated that are more flexible than conventional fixed routes with fixed schedules. Although paratransit

services have been operated for decades with call-in reservation systems, app- and web-based platforms now allow customers to communicate with operators in real time. Many models of flex transit are enabled by such systems, which allow customers to request service from an origin to a destination and operators to communicate where and when a vehicle will be able to provide service. It has long been a challenge to operate viable transit services in suburban and rural communities, because the quality of service is either too low for customers, the cost is too great for agencies, or both. The opportunity to use flex transit to offer cost-effective public mobility in these historically underserved communities is now emerging with support from technology and data standards, such as General Transit Feed Specification (GTFS) Flex.

The proposed research approach will pursue two lines of study in parallel. First, analytical methods will be used to model the operations and costs of various forms of flexible transit services, including demand-responsive door-to-door van services and more structured services allowing various degrees of route and schedule deviations. The cost-effectiveness of different flexible transit configurations will depend on demand density, size of service area, and the availability of competing modes. The literature suggests that there is a range of region characteristics for which flexible transit outperforms individual taxi-like services and fixed-route transit. Identifying the market configurations for which these service configurations are most cost-effective and competitive will provide guidance for identifying appropriate markets for flexible transit in Massachusetts.

Second, the data requirements for agencies to implement and monitor flex transit will be identified and defined.

- Information that is needed to identify which communities would benefit from the introduction of a flex transit service and what type of service to introduce.
- Information from passengers and operators that is needed to make real-time scheduling and routing decisions.
- Information that is needed to monitor and assess system performance, which is needed to evaluate the effectiveness of a flex transit implementation.

The analysis of the data requirements for flex transit will start from the literature review and theoretical analysis. Additional insights from pilot programs in Massachusetts to implement flex transit or microtransit in suburban and rural communities will link the general understanding of these systems to actual implementations in the Commonwealth. With this broad view that encompasses technical modeling of flex transit operations and data from implementations, this research will result in guidance for MassDOT and agencies to identify where flex transit systems should be operated, the data needs for implementation, and the appropriate metrics for monitoring performance.

## 1.2 Study Objectives

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This project has three main objectives:

1. To develop a method for identifying potential markets for flexible transit service and the type of flexible service that would most cost-effectively serve the demand.
2. To use data from flexible transit pilot programs in Massachusetts as the data become available, to compare theoretical analysis and pilot program data to identify lessons learned in practice and develop guidelines for future implementations.
3. To identify the data requirements for implementation of an automated reservation system for flexible services and to support operation of these services.

The first anticipated outcome of this project is a procedure for identifying appropriate markets for flexible transit service in Massachusetts. It is expected that this will depend on the density of demand, the size of the service area, and the availability of alternative transportation services. A second anticipated outcome is a set of guidelines or recommendations for best practices for the data requirements for new transit services. The guidelines should address the types of data that are most important for selecting markets, designing flexible services, implementing the desired systems for trip reservations, and monitoring ongoing system performance.



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## **2.0 Research Methodology**

The research approach for this study consists of four main components: a literature review to assess the current state of practice for flexible transit planning and operations, presented in this section; development of models to compare the performance and costs of different types of transit services in different contexts; analysis of data from microtransit pilot programs in Massachusetts; and analysis of data requirements for implementing flexible transit service. These sections culminate in a set of recommendations and lessons learned that are pertinent to transit operators across Massachusetts regarding the appropriate markets for implementation of flexible transit services.

### **2.1 Literature Review of Flexible Transit Services**

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Public transportation systems can be classified in fixed-route and non-fixed-route systems, according to their service mode. The first case includes systems that operate on a specific route and without the ability of deviating from their time schedules, if needed. These systems are characterized by low costs, which mostly arise from their pre-arranged schedule, the vehicles' high loading capacity, and other ride-sharing related benefits. Despite their low cost, however, they are often associated with high user dissatisfaction. Public transit can be considered as a spectrum in which demand-responsive transit systems are the opposite extreme point, consisting of systems with great flexibility in service but very high operating costs.

Current economic trends and growth patterns pose challenges regarding the operation of fixed-route systems, whereas the use of private automobiles increases pollution and congestion in urban centers. Flexible transit services (often called route deviation transit or mobility allowance shuttle transit) are an intermediate system between conventional fixed-route and demand-responsive transit services and could be a solution to the above-mentioned inefficiencies. Among the various motivations for moving toward greater use of flexible services are the improvement of transit systems, service of low-density areas, and reduction of operating costs.

Flexible-route systems are preferable in areas with low demand densities, where large fixed systems can't be accommodated. Also, areas where transit station accessibility is not assured for disabled or elder people are ideal candidates for implementing such systems. Flexible transit systems can be designed under different service configurations according to service area characteristics and demand needs. Since flexible services have the environmental and economic benefits of conventional fixed-route transit services, it is important to properly identify the service areas where such systems may be effective, as well as what type of flexible transit is the most appropriate.

Existing literature includes flexible transit–related surveys that aim at portraying the current conditions under which flexible transit services operate. Flexible service types, utilized technology systems, and best practices for increased performance are included in the information that such surveys reveal. Literature also includes many flexible transit modeling approaches, such as analytical methods, simulation, empirical, and stochastic processes. The research efforts are mostly put on planning and designing flexible operations, oftentimes through optimizing key design parameters such as headways and service area sizes. The two types of existing literature sources, surveys on implemented flexible transit services around the United States and relevant modeling approaches, are described as follows.

### 2.1.1 Surveys of Agencies Operating Flexible Transit Services

This part of the study is based on existing reports of surveys conducted to analyze flexible transit services across the United States. The reports that were considered here are the following.

**TCRP Synthesis 53—Operational Experiences with Flexible Transit Services.** Flexible transit services are implemented in more than 50 transit agencies throughout North America. A written survey by Koffman (*1*) was distributed among 81 transit systems throughout the United States. There were 24 responses, which were followed by interviews with the agency staff. **Table 2.1** shows the 26 identified flexible transit services that are operated by the 24 transit agencies that responded to the survey.

The agencies’ websites were an additional source of information regarding the operation of flexible services. The developed and distributed questionnaire was structured with the following sections:

1. Service Design
2. Service Coordination
3. Planning and Marketing
4. Performance Measurements and Standards
5. Operations
6. Barriers and Opportunities

**TCRP Synthesis 76—Integration of Paratransit and Fixed-Route Transit Services.** This study by Weiner (*2*) complements TCRP Synthesis 53 (*1*) by focusing on integrated flexible transit services that either were designed according to ADA (1990) or have proved beneficial for riders with disabilities. A web-based survey was distributed to more than 300 transit agencies and consultants throughout North America, 21 of whom responded that they indeed operate such integrated services. These agencies are presented in **Table 2.2**. The initial survey was followed by telephone interviews and site visits. The questionnaires were structured to account for each type of flexible system separately, as follows:

1. Feeder Service for People with Disabilities
2. Demand-Responsive Connector Service
3. Circulator and Community Bus Services
4. Point and Route Deviation Service
5. Other Integrated Services

For each type, the structure was similar to the questionnaires discussed in TCRP Synthesis 53 (1). The report also presents and discusses cases where integrated services were discontinued, such as Sarasota County Area Transit (SCAT), Calgary Transit, Access-A-Ride in New York, Whatcom Transportation Authority (Bellingham, WA), and Ridesource (Eugene, OR).

**Table 2.1: Service names and transit agencies for survey participants for TCRP Synthesis 53**

<b>Transit Agency</b>	<b>City, State</b>	<b>Flexible Service Name</b>
<b>ARC Transit</b>	Palatka, FL	Ride Solution
<b>Capital Area Transit</b>	Raleigh, NC	CAT Connector
<b>Central Oklahoma Transit and Parking Authority</b>	Oklahoma City, OK	METRO Link
<b>Corpus Christi Regional Transport Authority</b>	Corpus Christi, TX	Route 67 Bishop Driscoll
<b>Decatur Public Transit System</b>	Decatur, IL	Decatur Public Transit System
<b>Fort Worth Transport. Authority</b>	Fort Worth, TX	Rider Request
<b>Greater Richmond Transit Company</b>	Richmond, VA	Chesterfield LINK
<b>Hampton Roads Transit</b>	Hampton, VA	HRT On Call
<b>Lane Transit District</b>	Eugene, OR	Diamond Express
<b>Madison County Transit</b>	Granite City, IL	Madison County Transit
<b>Mason County Transportation Authority</b>	Shelton, WA	Mason Transit
<b>METRO Regional Transit Authority</b>	Akron, OH	Night zones
<b>METRO Regional Transit Authority</b>	Akron, OH	Town Center Routes
<b>Metropolitan Transit Development Board</b>	San Diego, CA	Routes 961–964
<b>Minnesota Valley Transit Authority</b>	Burnsville, MN	Local Route 440
<b>Minnesota Valley Transit Authority</b>	Burnsville, MN	Flex Routes 420 and 421
<b>Napa County Transport Planning Agency</b>	Napa, CA	St. Helena and Yountville Shuttles
<b>Ottumwa Transit Authority</b>	Ottumwa, IA	Ottumwa Transit Authority
<b>Pierce Transit</b>	Tacoma, WA	Key Loop, Orting Loop
<b>Potomac and Rappahannock Transport Commission</b>	Woodbridge, VA	OmniLink
<b>River Valley Metro Mass Transit District</b>	Kankakee, IL	Bourbonnais Flex
<b>Sarasota County Area Transit</b>	Sarasota, FL	SCAT About
<b>St. Joseph Transit</b>	St. Joseph, MO	St. Joseph Transit
<b>Tillamook County Transport. District</b>	Tillamook, OR	Deviated Fixed Route
<b>Tri-Met</b>	Portland, OR	Cedar Mill Shuttle
<b>Winnipeg Transit System</b>	Winnipeg, Manitoba	DART

**Table 2.2: Service names and transit agencies for survey participants for TCRP Synthesis 76**

<b>Transit Agency</b>	<b>City, State</b>	<b>Flexible Service Name(s)</b>
<b>Access Services, Inc. (ASI)</b>	Los Angeles, CA	Fare Free Program
<b>ACCESS Transportation Systems</b>	Pittsburgh, PA	ACCESS Transportation Systems; ACTA employer shuttles; ACTA "just in time" rides; Work Link; Ship of Zion; Elder Express
<b>Madison County Transit</b>	Granite City, IL	Agency for Community Transit
<b>Amador Regional Transit System</b>	Jackson, CA	Amador Regional Transit System
<b>Broward County Paratransit Services</b>	Palm Beach, FL	Community Bus Service
<b>Island Transit</b>	Coupeville, WA	Island Transit
<b>Laketran</b>	Painesville, OH	Dial-a-Ride
<b>RideSource</b>	Eugene, OR	RideSource; Diamond Express; Rhody Express
<b>Mason Transit</b>	Shelton, WA	Dial-a-Ride
<b>Mass Transportation Authority (MTA)</b>	Flint, MI	Your Ride
<b>Mountain Mobility</b>	Asheville, NC	Black Mountain and Enka-Candler Trailblazers
<b>Oahu Transit Service, Inc. (OTS)</b>	Honolulu, HI	Community Access Service
<b>Pierce Transit</b>	Tacoma, WA	Bus Plus Mid-County; Bus Plus
<b>Denver Regional Transportation District (RTD)</b>	Denver, CO	Call-n-Ride
<b>Sacramento Regional Transit District (RTD)</b>	Sacramento, CA	Paratransit; Neighborhood Ride
<b>San Mateo County Transit District (SamTrans)</b>	San Carlos, CA	RediCoast & RediWheels; Bayshore/Brisbane Shuttle
<b>San Joaquin Regional Transit District</b>	Stockton, CA	Hopper
<b>Sarasota County Area Transit (SCAT)</b>	Sarasota, FL	SCAT-About
<b>South Coast British Columbia Transportation Authority</b>	Vancouver, BC	HandyDART
<b>Utah Transit Authority (UTA)</b>	Salt Lake City, UT	Paratransit; UTA Route F94; Brigham Lift
<b>Whatcom Transport. Authority</b>	Bellingham, WA	Safety Net; FLEX

### **TCRP Report 140—A Guide for Planning and Operating Flexible Public**

**Transportation Services.** This study by Potts et al. (3) aims at providing a practical guide regarding the implementation of flexible transit services through studying both the agencies that operate flexible transit services and those that do not. The web-based conducted survey involved nearly 1,100 transit representatives. The 500 responses revealed that 39% of them operated some type of flexible public transportation service. The questions included in this survey are categorized as follows:

- Information about survey respondents
- Types of flexible public transportation service
- Flexible public transportation service users and productivity of the service

- Operation of flexible public transportation service
- Communication strategies used for flexible public transportation service
- Other considerations for flexible public transportation service:
  - Technologies
  - Motivation
  - Promotion
  - Advice

The results of the initial survey indicated that the following factors were significant in identifying flexible transit service agencies for further study (i.e., best practices):

- Geography
- Agency Type and Size
- Density
- Area Served by Flexible Service
- Type of Flexible Public Transportation Service Operated.
- Participation in TCRP Synthesis 53: Operational Experiences with Flexible Transit Services.

The transit agencies that were identified as best practices for further research were 26, out of which the following 10 agreed to participate:

1. South Central Adult Services Council, Inc. (South Central Adult Services)
2. Mountain Rides Transportation Authority (Mountain Rides)
3. Mason County Transportation Authority
4. Charleston Area Regional Transportation Authority (CARTA)
5. Jacksonville Transportation Authority (JTA)
6. Potomac and Rappahannock Transportation Commission (PRTC)/Omniride
7. City of St. Joseph
8. Omnitrans
9. Pierce Transit
10. Denver Regional Transportation District (RTD)

**Flexible Public Transportation Services in Florida.** This study by Goodwill and Staes (4) provides an overview of flexible transportation services in Florida. The required information was collected through a survey and the subsequent identification and examination of case studies. The questionnaire included an open-ended question in which the agencies that operated some type of flexible transportation service were asked to describe this type. More specifically, six Florida transit agencies were identified and asked to provide information about their flexible service routes. The six case-study agencies are:

1. Hillsborough Area Transit Authority (HART), Tampa, FL
2. Jacksonville Transportation Authority (JTA), Jacksonville, FL
3. LYNX, Orlando, FL
4. Lakeland Area Mass Transit District, Citrus Connection, Lakeland, FL
5. Pinellas Suncoast Transit Authority (PSTA), St. Petersburg/Clearwater, FL

## 6. Volusia County Transit (Votran), Daytona, FL

### 2.1.2 Types of Flexible Transit Systems

There are many variations of flexible-route services, and it is very likely to meet the same type of service called by different names, since individual transit agencies tend to not follow any standard naming practice. According to Koffman (1), there are four elements of service design that could assist in defining the type of flexible service:

1. Where vehicles operate
2. Boarding and alighting locations
3. Schedule
4. Advance notice requirements

Potts et al. (3) and Goodwill and Staes (4) adopt these definitions of flexible transit types as well. According to (3), the most common type of flexible-route system (60% of the sample case studies considered there) is route deviation, followed by demand-responsive connector (30% of sample case studies), often called “demand-responsive feeder service.”

One way to think about flexible or demand-responsive transit systems is in terms of how customers access the vehicles:

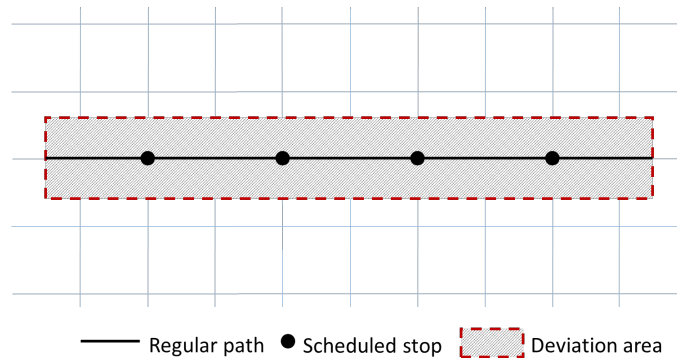
- Curb-to-Curb. A service that picks up passengers from the curb at the address of their choice and drops them at the curb at the address of their choice. This is a little different from door-to-door service in that drivers do not help transfer passengers between a building and the vehicle, although the two terms are sometimes used interchangeably.
- Stop-to-Stop. A service that picks up and drops off passengers only at specific stop locations that are designated by the transit agency. These stops may not be at the address of a passenger’s origin or destination, thereby requiring users to access the transit service by walking or some other means. Typically, the goal of consolidating pickups and drop-offs into stops would be to reduce the number of vehicle stops and/or the vehicle distance traveled.

Another way to classify services is by the way that vehicles are routed and scheduled to serve demand. Conventional fixed-route transit is a special case of stop-to-stop service in which the stop locations, routes, and operating schedules are all planned in advanced and operated regardless of the realized demand. There are several other ways that transit demand can be served more flexibly. A synthesis of the typologies introduced in (1) and (2) is presented in the following subsections.

#### ***Route Deviation***

Vehicles operate on a regular schedule along a well-defined path, with or without marked bus stops, and deviate to serve demand-responsive requests within a zone around the path (see **Figure 2.1**). The width or extent of the zone may be precisely established or flexible. The deviations of the route occur within a specified corridor and during specified times of the

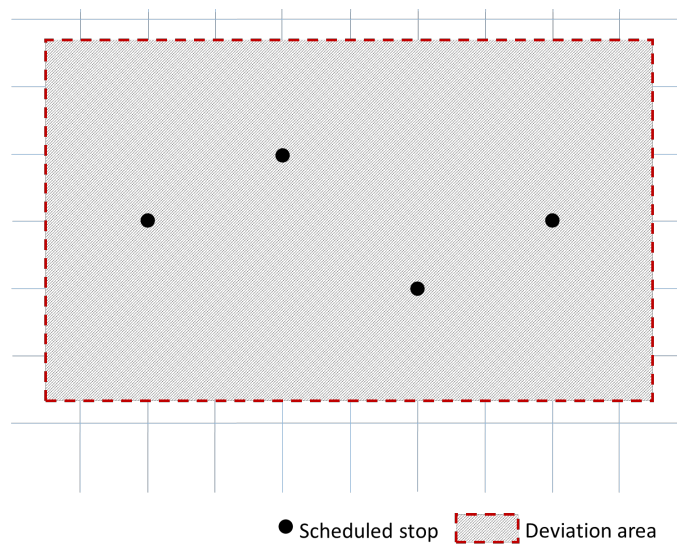
day. The fixed-route service may deviate only for people with disabilities or older adults, or it may deviate for any customer.



**Figure 2.1: Route Deviation**

### ***Point Deviation***

Vehicles serve demand-responsive requests within a zone and also serve a limited number of stops within the zone without any regular path between the stops. The deviations of the route occur within a specified corridor and during specified times of the day (see **Figure 2.2**). The service may be restricted to people with disabilities and older adults, or it may be available to all customers.



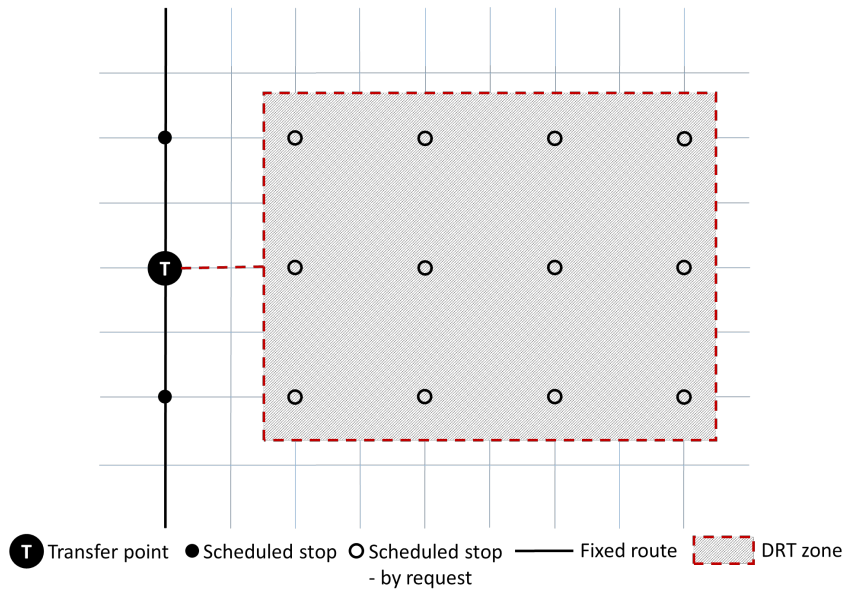
**Figure 2.2: Point Deviation**

### ***Demand-Response Corridor***

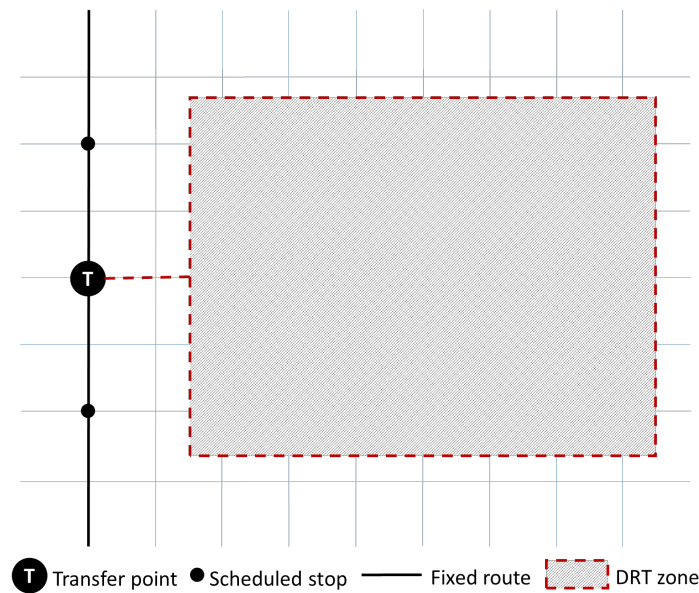
Vehicles operate in demand-responsive mode within a zone, with one or more scheduled transfer points that connect with a fixed-route network (see **Figure 2.3**). A high percentage of ridership consists of trips to or from the transfer points. Demand-responsive feeder services



like this may be exclusively for people with disabilities (e.g., paratransit service) or available to carry the general public to/from fixed-route service at bus stops, park-and-rides, and rail stations.



(a) with demand-responsive stop locations

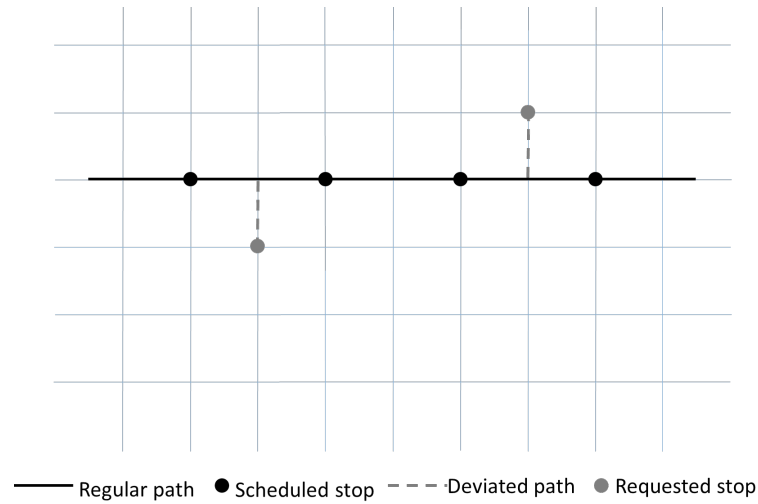


(b) without demand-responsive stop locations

**Figure 2.3 Demand responsive connector**

### ***Request Stops***

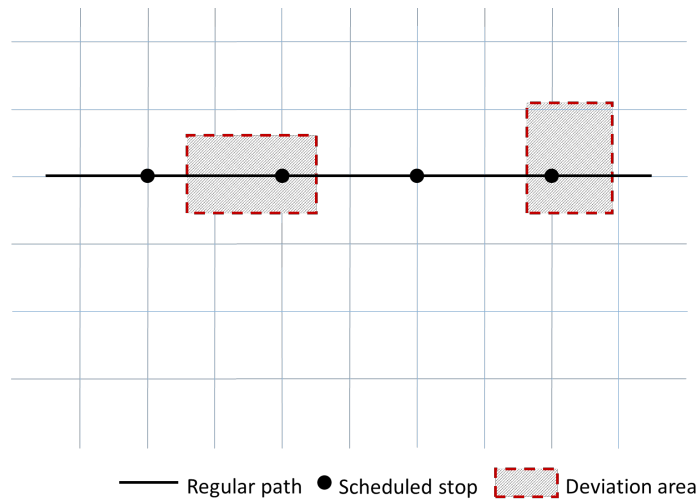
Vehicles operate in conventional fixed-route, fixed-schedule mode and also serve a limited number of defined stops near the route in response to passenger requests. Request stops differ from “flag stops” in that they are not located directly on the route (see **Figure 2.4**).



**Figure 2.4: Request stops**

### ***Flexible-route Segments***

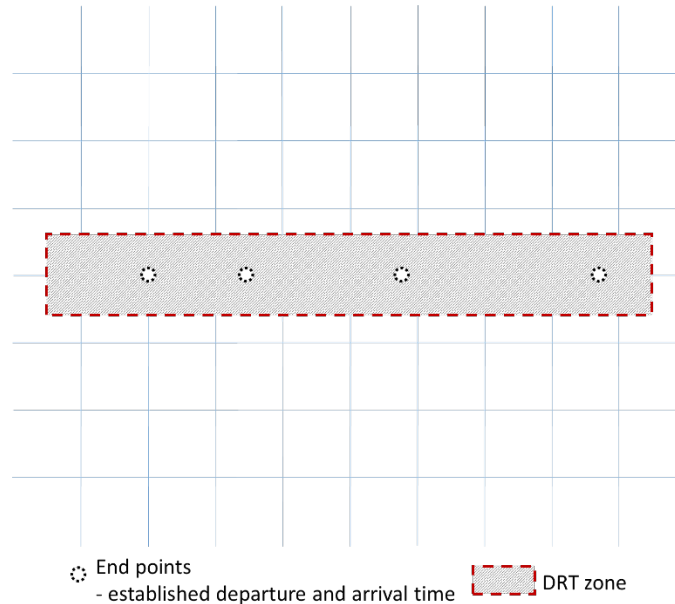
Vehicles operate in conventional fixed-route, fixed-schedule mode but switch to demand-responsive operation for a limited portion of the route (see **Figure 2.5**). The number, size, and shape of the flexible segments are all design variables that can be adapted to suit the specific needs of the corridor being served.



**Figure 2.5: Flexible-route segments**

### ***Zone Route***

Vehicles operate in demand-responsive mode along a corridor with established departure and arrival times at one or more end points (**Figure 2.6**).



**Figure 2.6: Zone route**

### ***Other Integrated Services***

There are assisted travel programs that facilitate transfers between paratransit and fixed-route or two fixed-route modes at transit centers, for those riders who would not be able to transfer unassisted. These include fare-free programs for paratransit registrants riding fixed-route service and shopping shuttles geared toward seniors and people with disabilities.

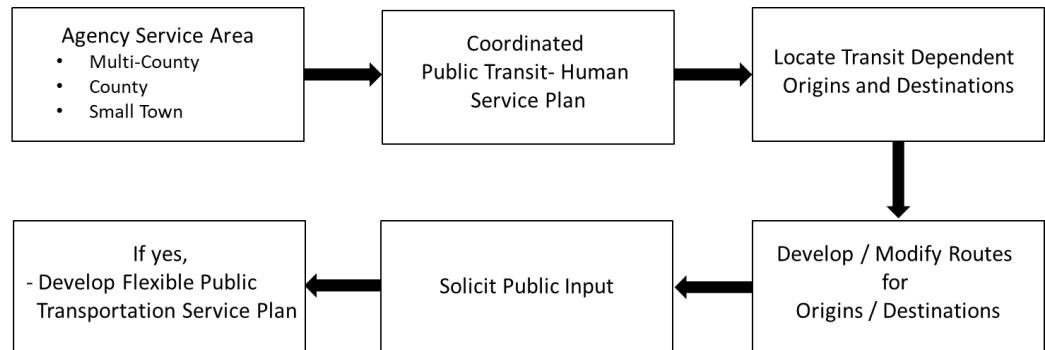
#### **2.1.3 Implementation of Flexible Transit Services**

Chapter 3 of Potts et al. (3) includes guidelines for implementing new flexible transit services for different types of service areas (rural, small urban areas, and large urban and suburban areas). The authors also present flowcharts for guiding the decision making of transit agencies in these three different types of areas. A brief summary of the information that a transit agency should acquire before studying the implementation of flexible transit systems is as follows.

**Rural areas** (up to 500 persons per square mile; see **Figure 2.7**):

- Population density
- Senior citizen density
- Youth density
- Low-income and/or subsidized housing
- Senior citizen housing

- Trip destination locations (e.g., discount stores, hospitals, senior citizen centers, and activity centers for persons with disabilities)
- Trip purpose



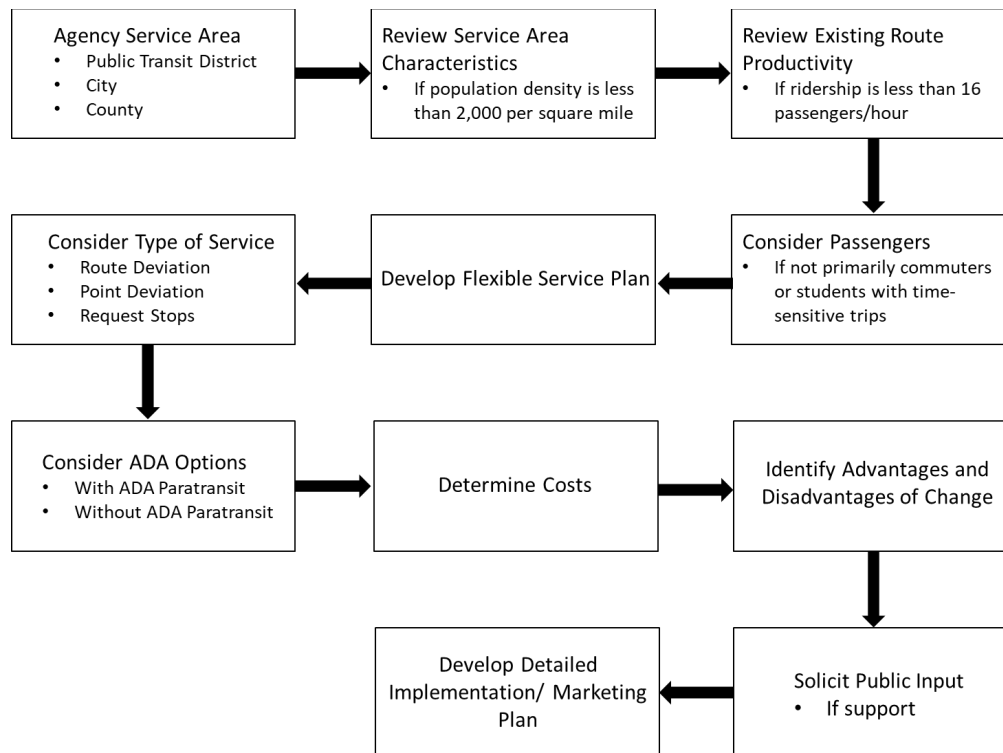
**Figure 2.7: Decision guide for rural areas**

**Small Urban Areas** (50,000 to 200,000 in population; see **Figure 2.8**):

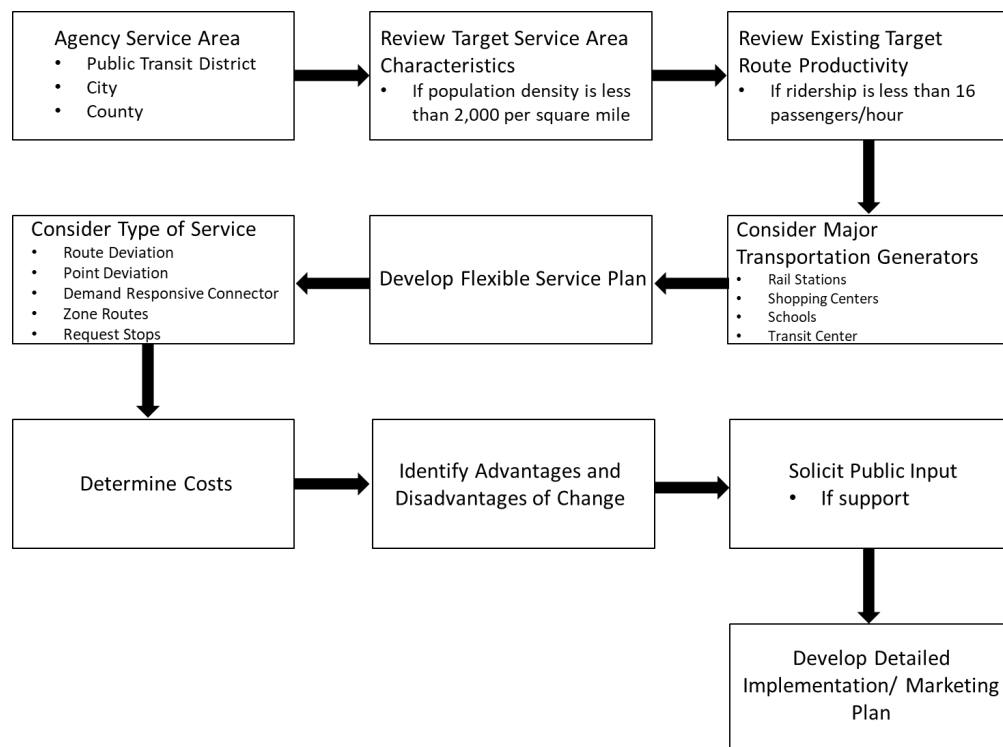
- Current route productivity
- Population density
- Senior citizen density
- Youth density
- Income levels
- Trip purpose

**Large Urban Areas and Suburban Areas** (population over 200,000; see **Figure 2.9**):

- Population density
- Size of area to be served
- Travel time to connector or time point
- Employment density
- Household density
- Auto ownership
- Senior citizen density
- Youth density
- Median income
- Productivity of existing routes, if any



**Figure 2.8: Decision guide for small urban areas**



**Figure 2.9: Decision guide for large urban and suburban areas**

Best practices for planning different types of public transit successfully are included in Mistretta et al. (5). For flexible systems particularly, some examples are:

- Service should be provided within  $\frac{3}{4}$  mile (as designated by ADA).
- On-time performance standard of “not early and no more than three minutes late” should be achieved 90% of the time.
- Minimum ridership productivity is that systems should average 6 passengers per hour.

## 2.2 Modeling Flexible Transit Services

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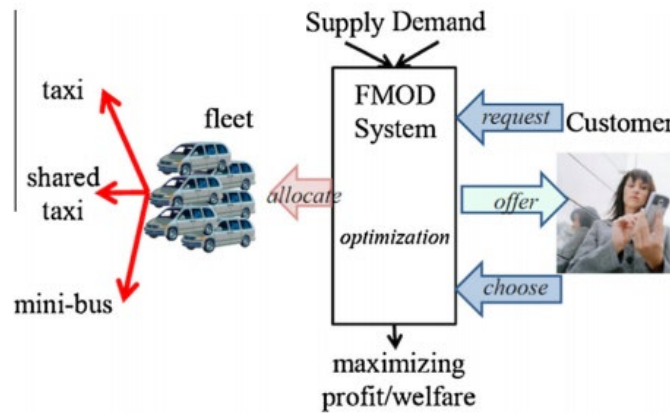
Recognizing that there are many different types of flexible transit systems, the research method is to develop models for direct comparison of performance across fixed-route, route-deviation, flexible transit, and on-demand (route-less) services. This starts with a review of the literature on models for flexible transit systems. The remaining task is to adapt these existing models so that they use the same inputs and provide estimates for the same performance measures so the analysis of model can be used to provide useful results about where and when each type of flexible transit service is most appropriate for implementation.

The main goals of flexible systems are twofold: to improve the convenience of public transportation and to maintain a comparable price to existing public transit systems (Mulley and Nelson (6)). A survey by Koffman (1) reveals that the majority of flexible transit services are planned and designed without established guidelines. Errico et al. (7) classified existing studies on flexible transit into two categories. The first group includes studies that describe practical experiences, whereas the second refers to methodological contributions to assist planning processes. There are only a few cases of implementing optimization techniques for flexible systems (3,7,8). Many approaches are based on analytical modeling, considering rectilinear distances, because rectilinear movement of vehicles is a good approximation of the reality, according to Dessouky et al. (9).

Most studies on public transit user preferences focus on the competition between fixed-route and demand-responsive services (10,11). Few studies have focused, in general, on flex-route transit so far (12). Broome et al. (13) completed a study showing the public’s positive perception of flexible transit systems. Chavis and Gayah (14) performed a stated preference survey to develop a mode choice model that can be used to describe how transit users select among competitive transit options. Their study covered the entire public transit spectrum, from traditional fixed-route to flexible and pure on-demand services (including e-hailing such as Uber and Lyft). Although there are passengers that always choose the same mode, the results also indicated that there are statistically significant predictors of the flexible service type selected. More specifically, these predictors are monetary cost, expected in-vehicle waiting time, expected waiting time, and walking time.

Atasoy et al. (15) present the concept and perform the impact analysis of Flexible Mobility on Demand (FMOD), which allows for personalized services to public transit users as shown in **Figure 2.10**. The latter have the flexibility to choose among taxi, shared-taxi, and fixed-route mini-bus services, and a fleet of vehicles is dynamically allocated among the three

modes. All three alternatives are associated with different benefits and drawbacks for both operators and users. The FMOD system's adaptation to various demand patterns is expected to bring profits to operators, whereas the consumer surplus, and consequently the user satisfaction, will be increased. These are indeed confirmed by the study results. Broome et al. (13) evaluated the performance of a flexible route in Hervey Bay, Queensland, Australia, in terms of satisfying the needs of older people. Ticket sale data analysis proved that the replacement of the conventional fixed route by a flexible one led to approximately doubled use of the service by older people. The authors conclude that flexible-route bus transport is a promising technology for the transport needs of older people.



**Figure 2.10: Flexible vehicle allocation based on passenger request**

Among the many types of flexible transit services, deviated fixed-route services are the most widely used, per Qiu et al. (16). Such services are often met with different names. Durvasula et al. (17) and Lu et al. (18) refer to them as route deviation transit. Quadrifoglio et al. (19) and Zhao and Dessouky (20) call them Mobility Allowance Shuttle Transit (MAST). Crainic et al. (21) use the term Demand Adaptive Transit System (DAS). Fu et al. (22) refer to deviated fixed-route services as flex-route transit. A study by Pei et al. (23) summarizes valuable findings from the existing literature on modeling approaches for flexible transit systems, reproduced in **Table 2.3**. The authors highlight that the most commonly minimized objective function for operating flexible systems is the system's overall cost, including both agency and user costs in most cases. Almost all the papers that the authors reviewed included time windows of service, while many of them also accounted for slack in the schedule. Dynamic approaches seem to be fewer than the static, most likely due to technological limitations.

**Table 2.3: Summaries of previous studies on flexible transit systems, from Pei et al.**

Reference	Study Area	System Type	Study Purpose	Objective	Solution Method	Case Study
Diana et al. (2006)	Square Area	Demand Responsive Connector (Fleet)	Fleet Size with Time Windows	Min. System Cost	Continuous Approximation; Regret Insertion Algorithm	Computational Experiments
Quadrifoglio et al. (2006)	Square Area	Deviated Fixed Route (Single Line)	Max. Longitudinal Velocity	Min. System Cost	Continuous Approximation	Computational Experiments
Quadrifoglio et al. (2007)	Slim Service Area	Deviated Fixed Route (Single Line)	Scheduling	Min. System Cost	Mixed-Integer Program	Line 646, Los Angeles
Quadrifoglio et al. (2008a)	Loop Between Terminals	Deviated Fixed Route (Single Vehicle)	Scheduling	Min. Weighted Sum of Factors	Mixed-Integer Program	DRT Service, Los Angeles
Quadrifoglio et al. (2008b)	Zone District	Deviated Fixed Route (Fleet)	Scheduling	Min. Additional Distance	Simulation Model	Los Angeles County
Quadrifoglio & Dessouky (2008)	Slim Service Area	Deviated Fixed Route	Shape of Service Area	Min. System Cost	Simulation Model	Line 646, Los Angeles
Li & Quadrifoglio (2009)	Square Area	Demand Responsive Connector (Single Line)	Number of Zones	Min. System Cost	Simulation	Computational Experiments
Nourbakhsh & Ouyang (2012)	Area Around Terminals	Demand Responsive Connector (Single Line)	Scheduling	Min. System Cost	Simulation	City of Oakville (Toronto)
Asfalalfah & Shalaby (2012)	Hybrid Network	Zone Route (Fleet)	Identify Decision Variables	Min. System Cost	Constrained Nonlinear Programming	Numerical Examples
Lu et al. (2016)	Irregular Network	Deviated Fixed Route (Fleet)	Routing	Min. Total Time	Traveling Salesman Problem	Numerical Examples
Pei et al. (2019)	Slim Service Area	Demand Responsive (Fleet)	Routing and Scheduling	Min. Total Time	Binary Integer Programming	Line 15, Guangzhou



The following subsections present models of different types of flexible public transportation services.

### 2.2.1 Review of Models for Route and Point Deviation Services

In Pei et al. (23), the authors integrate the ideas of flexible routes, U-turns, and uninterrupted operation to design a “flexible-line-length system.” This system is composed of a bus line where several buses operate at the same time. A bus stop is served only when at least one passenger requests it (either pickup or drop-off), and routing and scheduling are updated after a new request. An Information Processing Center (IPC) is responsible for identifying the farthest U-turn at the end of the line (both directions), and the optimal U-turn point is determined.

An analytical function with proper constraints is utilized to describe the system’s operating performance. A real-time control process is followed for simulation purposes. The proposed methodology can be briefly summarized as follows.

- Step 1: Find the real-time location of each bus.
- Step 2: Search the waiting list of each bus and determine the U-turn points for each bus.
- Step 3: Optimize real-time routes and add new requests to the waiting list of the bus.
- Step 4: Output the new route and schedule.

**Figure 2.11** presents the proposed flexible system. According to the authors, such limited-stop services can benefit both operators and users. A numerical example considers the No. 14 bus line in Nansha District, Guangzhou, China. It is a low-density bus line in an industrial park, with approximately 8.0 km length. Results confirm lower travel times compared to conventional fixed-route services.

Zheng et al. (12) propose a methodology to assist planners’ decision-making process when choosing between route deviation policy and point deviation policy. According to the authors, these are the two promising types of flexible transit services. The quality of service provided by the two systems is measured through a user cost function. Analytical models are developed to compare the system performance, considering both expected and unexpected demand levels. A residential community is modeled as a rectangle service area of width  $W$  and length  $L$ , as shown in **Figure 2.12** for route deviation and **Figure 2.13** for point deviation policy. Trip origins and destinations are considered the passengers’ houses and connection centers, from where they transfer to the major transit network.

The study is based on various modeling assumptions. A modeling assumption is that the trip origins and destinations outside checkpoints are uniformly and independently distributed in the service area and follow a homogeneous spatial Poisson process. The variation of demand locations is eliminated by implementing the bus cruising speed control strategy. The performance of the two flexible transit systems is implemented assuming the same size  $M$  in feet and passenger distribution for the two services. The following three types of passengers are considered here:

- Type I: both pickup and drop-off at checkpoints.
- Type II: pickup at checkpoints, and drop-off not at checkpoints.
- Type III: pickup not at checkpoints, and drop-off at checkpoints.

The results' analysis indicates that point deviation policy is more efficient at low-demand levels, while route deviation policy is a better choice at low-to-moderate demand levels. When demand levels are unexpectedly high, route deviation policy can accommodate rejected passengers better than point deviation policy.

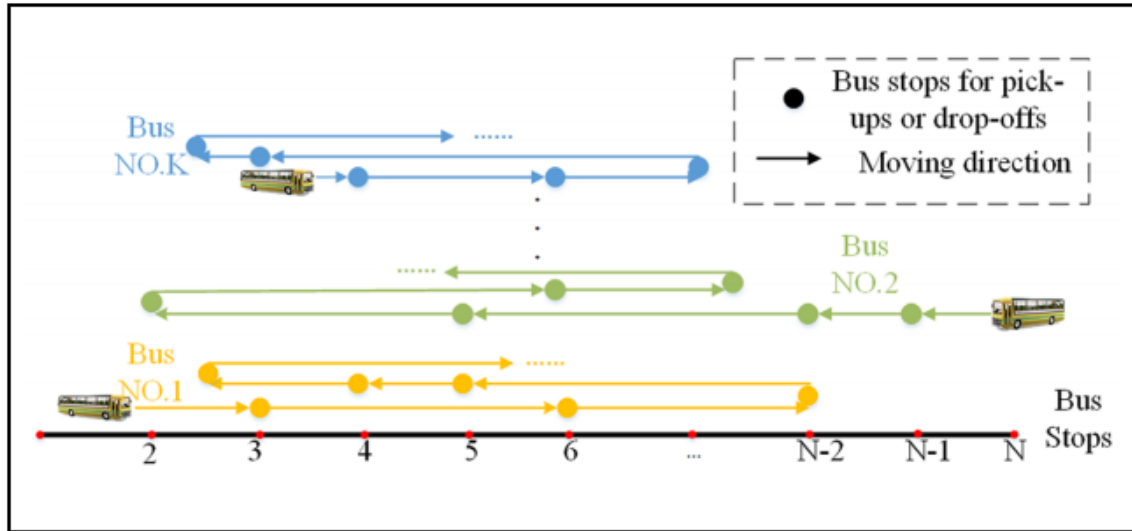


Figure 2.11: General scheme of structured flexible transit system

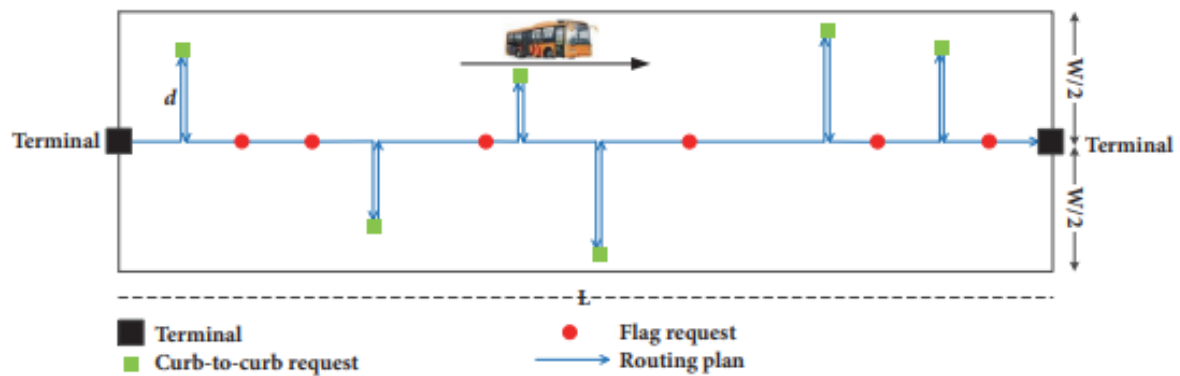
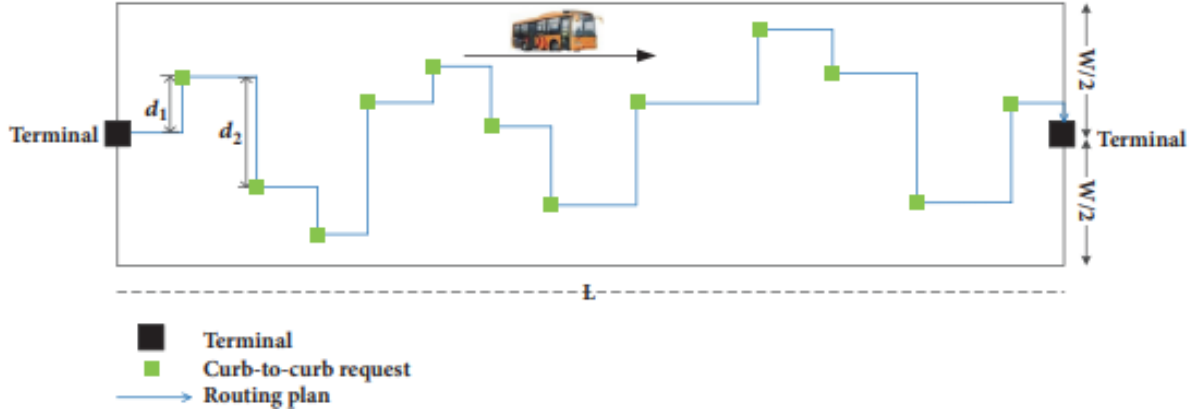


Figure 2.12: Route deviation operating policy



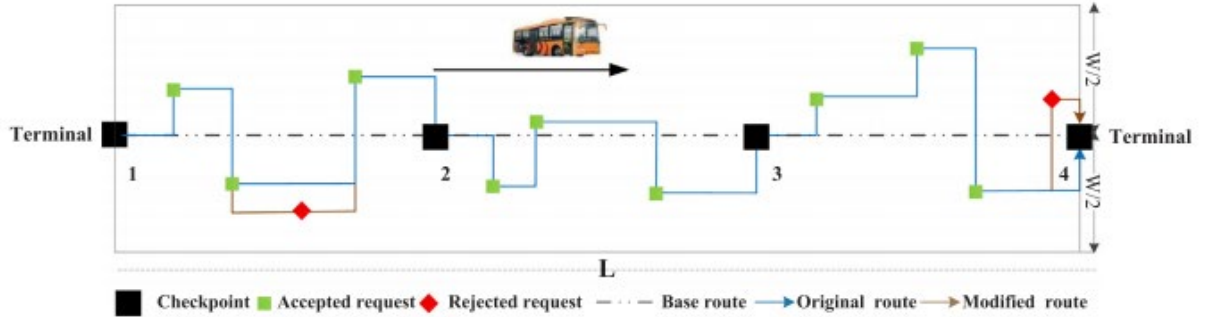
**Figure 2.13: Point deviation operating policy**

Qiu et al. (16) propose a methodology to assist transit planners to decide between conventional fixed-route and flexible-route systems, considering varying passenger demand. Demand densities were used to define critical points for switching between the two competing services. The performance of the two competing systems is measured through a service quality function composed of expected walk time, expected waiting time, and expected riding time for the users. The transit system in this study is based on the MTA Line 646 flex-route service in Los Angeles County. A single service vehicle and a rectangular service area of length  $L$  and width  $W$  are considered. The demand is assumed uniformly distributed (both at the checkpoints and outside of them).

The fixed-route case is studied through analytical models, whereas the flexible-route service is simulated. The latter is justified, due to the limitations of slack time, which poses challenges in developing theoretical models for service quality function in flexible-route services. An insertion heuristic algorithm to reproduce vehicle movement is the basis of the simulation method. Since reservations in this flexible service are made in advance, the heuristic algorithm implemented in this study is not real time. Heuristic algorithms are recognized by the authors as “a classical and efficient approach to operating flexible transit systems” (16).

A study by Zheng et al. (12) proposes a slack arrival strategy to improve the operations of a flexible-route system, using the same case study (MTA line 646). The proposed slack arrival strategy for a flexible-route system is shown in **Figure 2.14**. The performance measure considered here is the system cost, composed of the sum of the vehicle operation cost per customer and average customer cost. The cost components’ formulation is investigated in the following demand scenarios:

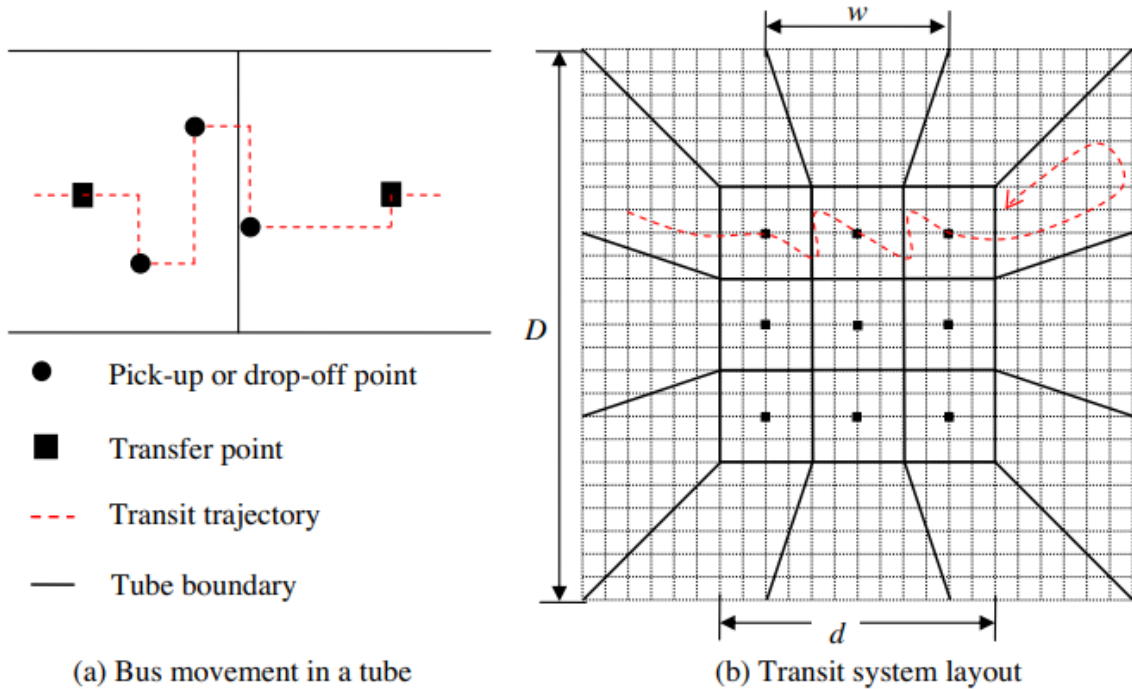
1. Expected demand level
2. Unexpectedly low demand level
3. Unexpectedly high demand levels



**Figure 2.14: Diagram of a slack arrival policy**

The theoretical models are evaluated using simulation based on a first-come, first-serve heuristic algorithm. The study results indicate that the proposed slack arrival strategy could effectively reduce both the rejection rate and the idle time at checkpoints. Users' riding and waiting times, however, are expected to be slightly increased.

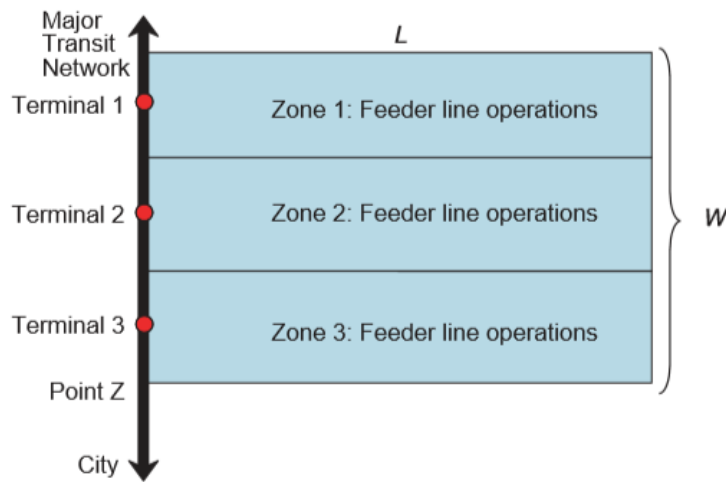
Nourbakhsh et al. (24) analyze the agency and user cost components of a flexible transit system considering idealized square cities. Their goal is to minimize the total system cost through determining a) the optimum network layout; b) service area (of each bus); and c) bus headway. Analytical formulas are obtained and validated using simulations. The general scheme for a proposed flexible system is shown in **Figure 2.15**. This system is compared with conventional fixed-route systems and taxis, and the passenger demand levels for its proper implementation are determined. The proposed flexible transit system is found to have the lowest cost for low-to-moderate demand levels.



**Figure 2.15: General scheme of structured flexible transit system**

### 2.2.2 Review of Models for Demand-responsive Connectors

Li et al. (25) developed an analytical model to determine the optimal number of transit service zones in a residential service area. Transit vehicles can operate either as conventional fixed-route or as demand-responsive in each zone. A representation of zones with demand-responsive service is shown in **Figure 2.16**. The optimization process is focused on service quality and vehicle operating costs. Customers are assumed to request a trip through phone or Internet booking service. The analytical models are validated using an insertion heuristic algorithm to schedule the requests, with no real-time scheduling. Idle time between trips is not considered. Three cases of areas were investigated: a) a large service area ( $L = 2$  mi,  $W = 6$  mi) with demand of 80 customers/h; b) a large service area ( $L = 2$  mi,  $W = 6$  mi), with high demand of 200 customers/h; and c) a small service area ( $L = 2$  mi,  $W = 2$  mi), with low demand of 10 customers/h.

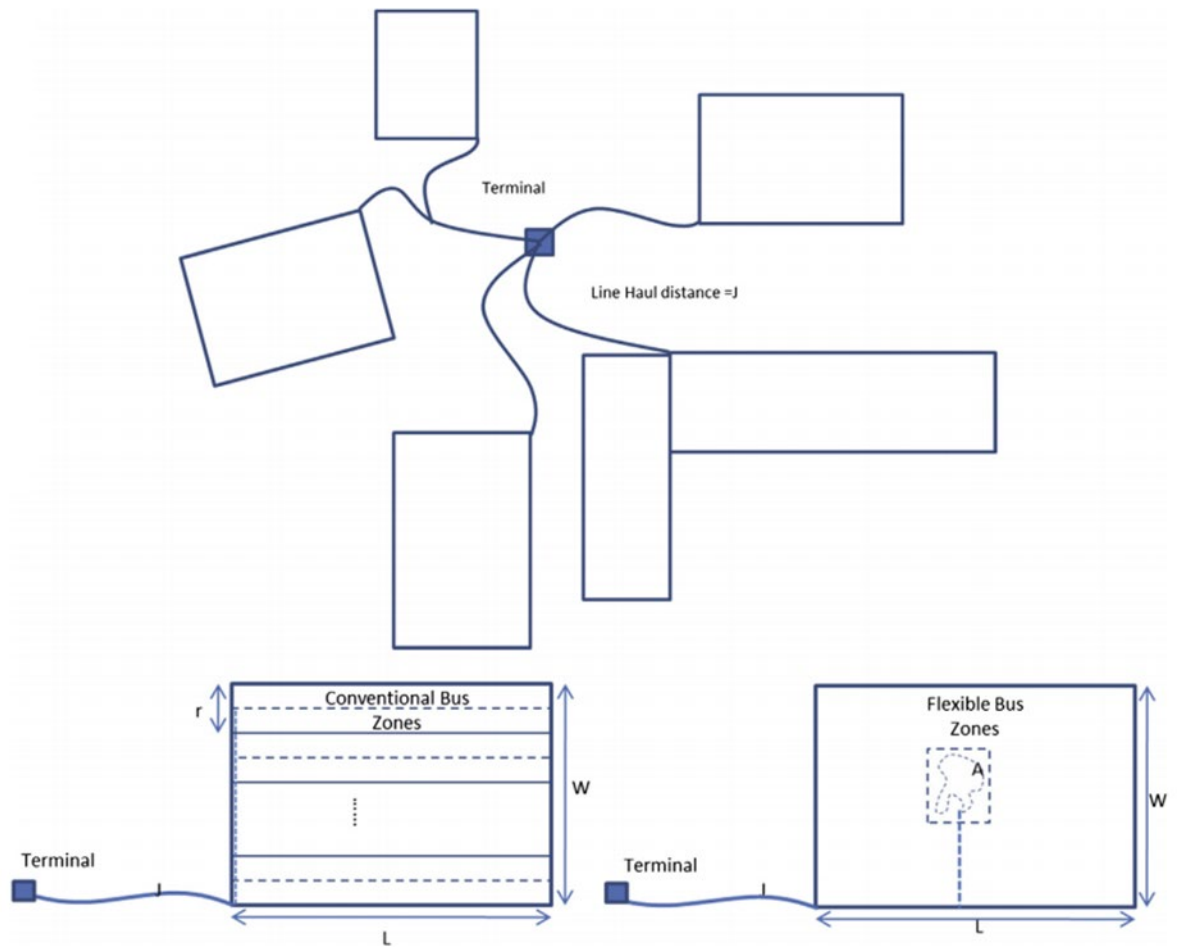


**Figure 2.16: Service area with three zones**

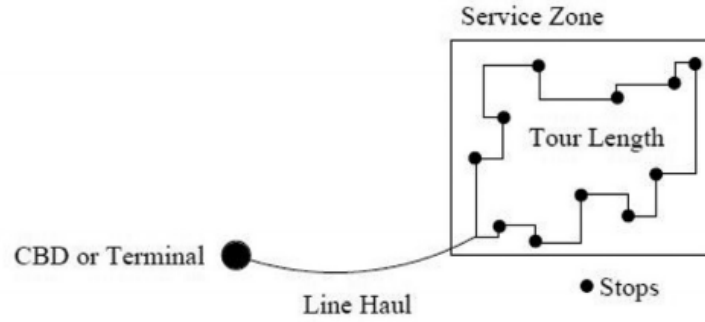
Kim et al., 2015 (26), after studying variable type bus services in 2012 (27), developed formulas to estimate the elasticities of demand (for both conventional and flexible transit services) with respect to factors such as fares, travel times, waiting times, and access times, in order to maximize system welfare. The Real Coded Genetic Algorithm is used to solve the resulting mixed integer nonlinear welfare maximization problem. According to the authors, the reasons that exact solutions cannot be achieved here are the multiple dissimilar regions and time-dependent demand included in the formulations.

For modeling purposes, the service region is assumed to be rectangular and is divided into zones for fixed routes or flexible routes, as shown in **Figure 2.17**. The proposed models are based on various assumptions for both the fixed and the flexible service that are explicitly described by the authors. Model inputs include potential demand, service time, line-haul distance, and sizes of regions. This study's (26) contributions include optimized variables of service type, zone sizes, headways, and fares. The authors also propose a maximum welfare threshold between optimized conventional and flexible services.

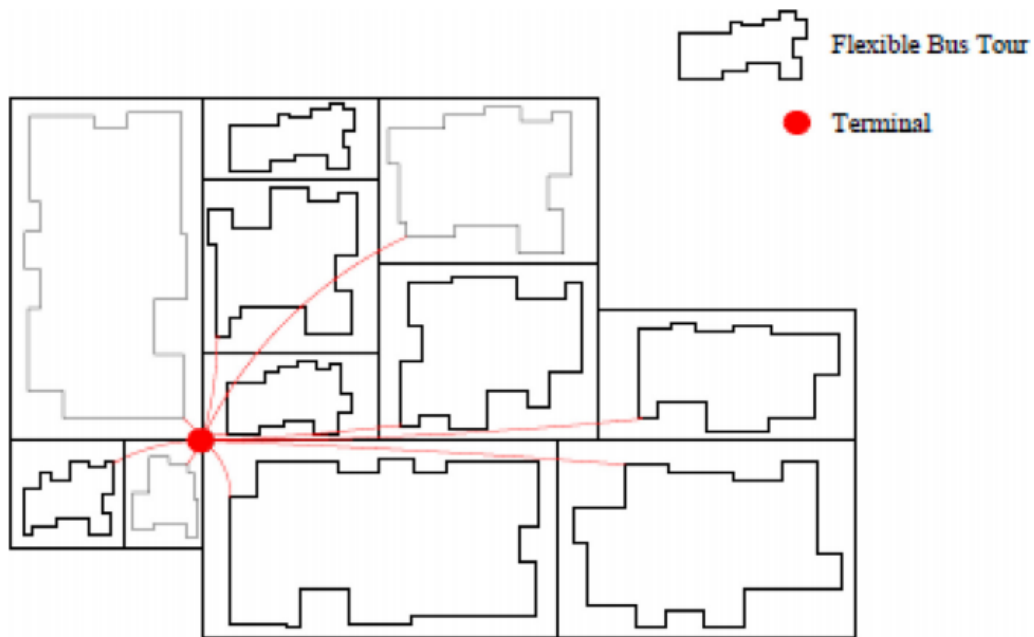
Kim et al., 2019 (28), present a planning model for optimizing a flexible system serving many-to-one and one-to-many demand patterns. This study aimed to fill the gap in existing literature regarding relations among optimal zone sizes, headways, and relevant exogenous factors for flexible-route services (e.g., demand density, distance from the major terminal, applicable unit costs, bus speeds). An analytic relation between optimal headway and optimal zone size is proposed to minimize the average cost per passenger trip. Flexible-route modules with one bus route connecting a local service zone to a major terminal through an express segment are considered in this study. Each module includes a route that serves a many-to-one (M-to-1) or one-to-many (1-to-M) demand pattern, and each module is optimized individually in this study. **Figure 2.18** illustrates the configuration of this service, and **Figure 2.19** presents the extension of such a service to a multizone flexible-route system (M-to-M).



**Figure 2.17: Local regions and bus operations**



**Figure 2.18: Configuration of flexible route bus service**



**Figure 2.19: Potential extension to multizone flexible-route system**

For each module considered here, trip origins and destinations are assumed to be randomly and uniformly distributed over space and time (e.g., suburban neighborhood outside a large city). The total cost for flexible bus services is estimated using the sum of approximations for operating costs and user costs expressed as both in-vehicle and waiting costs (no access time considered). The required approximations for the components of the proposed model (e.g., fleet size, number of passengers boarding per stop, tour length within a zone) are calculated based on some fundamental assumptions that the authors explain thoroughly. For instance, dwell and stopping times within a zone are taken into consideration through the average speed.

### **2.2.3 Methodology for Modeling Flexible Transit**

In this study, modeling of flexible transit systems is conducted in three stages. First, an analytical model is developed using the continuous approximation technique. The model describes the physical operation of transit vehicles to quantify the experience of system users and the cost to the agency. The models take the form of mathematical expressions, so that performance measures are expressed as functions of the input variables, including characteristics of the service area, demand, and vehicles.

The analytical model is then used to conduct an optimization analysis to identify configurations of flexible transit service that minimize the total cost of transit service, accounting the experience of users as well as costs to the agency. Since the analytical model is for a hybrid flexible transit system that combines curb-to-curb services with fixed stops, the decision variables are the size of the flexible service region and the locations of the fixed stops. This structure has the flexibility to represent a wide range of service types, depending on the characteristics of the region served. For example, if the flexible service region is very small, the system will converge toward a conventional fixed-route service in which all users board and alight vehicles at fixed stops. If the flexible region is very large and all of the demand is eligible for curb-to-curb service, the system will converge toward a fully demand-responsive service in which the vehicle travels to serve customers at their preferred origin/destination. Solutions in between represent a hybrid solution of a structure transit service that allows some flexibility in operations.

Finally, a simulation model was constructed to represent more realistic operations with the continuous approximations, which treat indivisible values (e.g., number of stops and number of vehicles) as continuous variables. The simulation model generates random realizations of demand within a service area, and then the transit vehicle is routed to serve the demand according to the optimized service design from the analytical analysis. The simulated costs experienced by users and incurred by the agency are then compared with the analytical approximation to evaluate the accuracy of the developed models.

## **2.3 Flexible Transit/Microtransit Pilots in Massachusetts**

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In an effort to provide more innovative and effective transit service across the Commonwealth of Massachusetts, MassDOT has funded a series of microtransit pilot projects with RTAs. These pilots are intended as demonstration projects to test whether flexible microtransit services are viable in different types of communities around Massachusetts. The idea is that microtransit, which can take any of the forms of flexible transit service presented in this report, can provide a customizable solution to fill the gaps in traditional fixed route transit networks. In some communities, this means replacing underperforming fixed-route services with a microtransit service area. In other communities, this means introducing transit service where none had been previously available. MassDOT has supported six microtransit pilots through the RTA Discretionary Grant Program, which uses state funds. Two pilots were supported through the competitive Community Transit Grant



Program using federal Section 5310 and state funds to support transportation options for seniors and persons with disabilities. Three other pilots were supported through the Workforce Transportation Program, with the CATA and GATRA pilots using federal funds and the Salem Skipper supported with funds from the state and the federal Congestion Mitigation and Air Quality Improvement Program. These are listed in **Table 2.4**.

The analysis of the microtransit pilots in Massachusetts has two main parts. First, to the extent that quantitative data is available about the operating regions and demand characteristics, this real-world data provides a basis for comparison with the analytical and simulation models that are developed from theory.

The second part of the microtransit study is to provide a qualitative analysis of the pilots to identify patterns and lessons learned related to the rationale for implementing microtransit, the design and policy decisions associated with the implementations, and the challenges that were identified in the real-world implementations. The coronavirus pandemic had a major impact on all the pilot projects, because the lockdown measures that were widely implemented in March 2020 occurred soon after or before several planned project launches. Since the pandemic has had a significant impact on transit demand (encouraging people to avoid crowded transit vehicles and reducing total demand for travel), the demand data is not considered representative of normal operating conditions. Nevertheless, the pilots have provided RTAs with a wealth of experience to guide future implementations.

**Table 2.4: Microtransit pilot programs in Massachusetts**

Funding Program	RTA	Microtransit Program
RTA Discretionary Grant Program	CCRTA	SmartDART
RTA Discretionary Grant Program	FRTA	Access
RTA Discretionary Grant Program	GATRA	Go Coastline/Go Connect
RTA Discretionary Grant Program	MART	Subscription Service
RTA Discretionary Grant Program	MWRTA	CatchConnect
RTA Discretionary Grant Program	WRTA	VIA Partnership
Community Transit Grant Program		NewMo (Newton)
Community Transit Grant Program	PVTA	Quaboag Region Microtransit
Workforce Transportation Grant Program	CATA	VIA Partnership
Workforce Transportation Grant Program	GATRA	Go Connect
Workforce Transportation Grant Program		Salem Skipper

## 2.4 Data Requirements for Implementation

The analysis of data requirements for implementing flexible transit services, including microtransit systems, includes three components. First, there are data requirements for planning and designing a flexible transit service. These are the data that are needed in order to make an informed decisions about what type of service (e.g., fixed-route, route-deviation, hybrid, or fully demand-responsive) would be most appropriate. This is also the data that

would be needed to specify details of the design, such as the number of vehicles and frequency of dispatch (if relevant). In the planning stage, it would likely be useful for agencies to have estimates of the performance of the system in terms of costs to users and to the agency itself. The analytical models provide the basis for this planning-level analysis, because the models explicitly identify the data inputs that will affect the design and performance of the system.

Second, there are data that are required to operate the flexible transit system in real time. These include data from users about the specific locations and time of their requested trip as well as vehicle operations data, including the locations of vehicles, number of passengers onboard, and road network conditions. This part of the analysis follows closely a detailed look at the development of an app use by MWRTA and CCRTA for their microtransit operation. This is related to a comparison of the benefits and drawbacks of developing an in-house app for vehicle dispatch and routing versus using an off-the-shelf product.

Finally, there are data requirements for ongoing monitoring of the system. In addition to the data reporting requirements for the Federal Transit Administration, there are many performance measures that reflect the customer experience or the cost-effectiveness of a flexible transit service that would be useful for an agency to track, either to compare a flexible service with an alternative, such as fixed route, or to track progress over time.



## 3.0 Results: Modeling Flexible Transit Systems

Flexible transit systems are modeled in order to provide a systematic comparison between different types of services. First, a set of analytical models are developed to relate demand and vehicle characteristics to performance metrics that reflect the cost to agencies and users. Second, these models are used to analyze the range of performance outcomes for regions with different sizes and demand densities. Finally, the different types of service are compared to provide insights about the conditions that are best suited to fixed-route transit, hybrid flexible services, and demand-responsive transit.

### 3.1 System Description for Flexible Transit Model

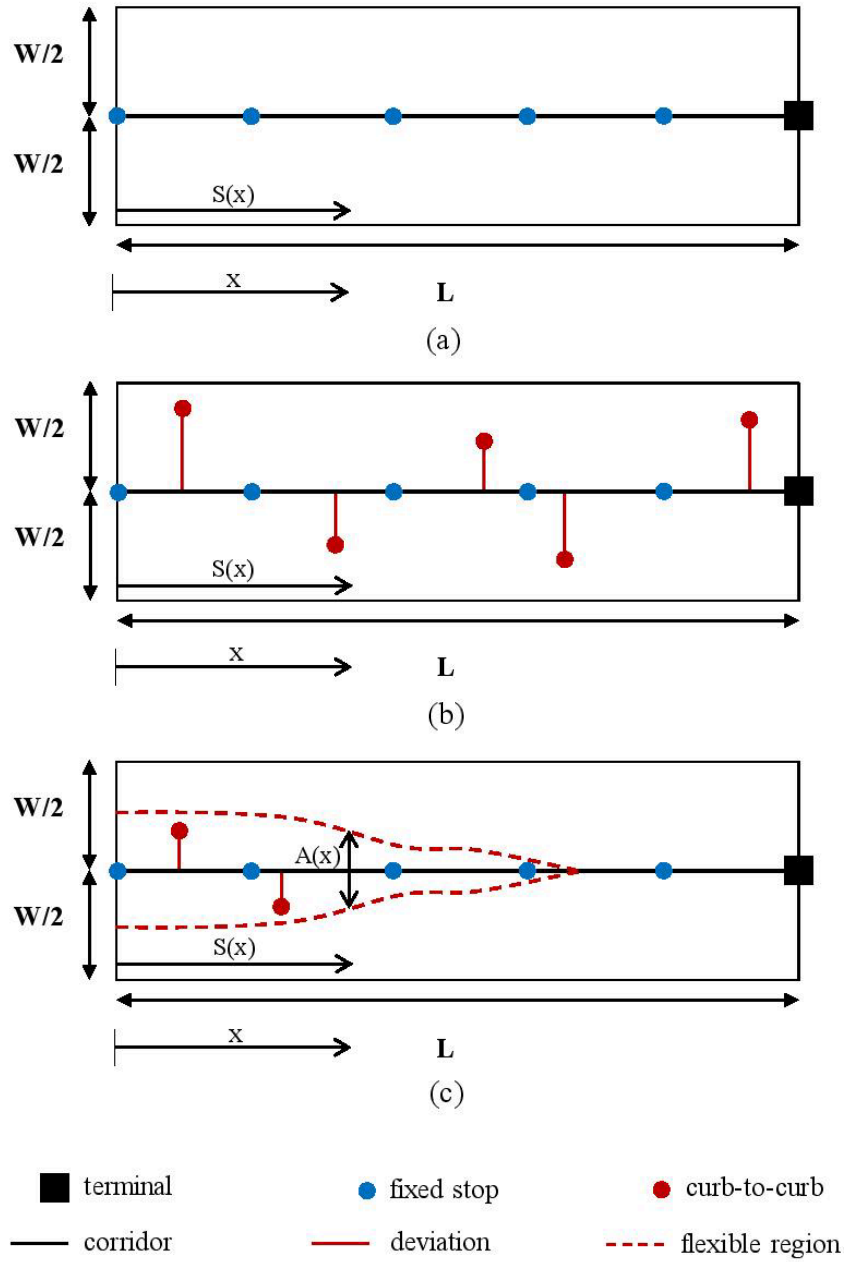
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The service area considered in this study is rectangular, with size  $W (mi) \times L (mi)$ , and the tours are routed on a rectilinear street network. A straight-line corridor for fixed-route services is assumed to operate in the middle of the service area, with one end being a major terminal station. A typical configuration for this network is given in **Figure 3.1a**. The demand,  $Q (pax/mi^2/hr)$  is uniformly distributed over space and time. The vehicle average speed,  $V (mph)$ , is considered to account for stopping times and delays. Vehicle headways,  $H (hr/veh)$ , are uniform, and no passenger capacity is considered. The stop spacing across the fixed-route corridor,  $S(x) (mi)$ , is a continuous function of  $x$ .

Users are assumed to travel from a location within the service area to a terminal station, or vice-versa. The terminal station is assumed to connect the service area with a big city center or other major destination, and one end of the trip is always the terminal station. Thus, it is considered that passengers only board the vehicle as it moves toward the terminal station, and they only alight in the opposite direction. Two types of users are analyzed:

- **Curb-to-curb users**—system users who request curb-to-curb service either for their pickup or drop-off.
- **Fixed-route users**—system users who use only the fixed-route service provided by the flexible system.

Different types of flexible services might involve only one or both the types of users presented above. An example of curb-to-curb requests includes users who are under the ADA and are thus eligible for such service. Another example refers to passengers who want to avoid the efforts associated with accessing a fixed stop and probably the inconvenience of waiting at a transit stop rather than their own private space. Such phenomena are expected to increase substantially during the ongoing coronavirus pandemic, since public transit users aim to reduce their infection risks to the maximum possible extent. Alternatively, curb-to-curb requests might simply be the first  $\alpha(\%)$  of trips requested, considering a capacity on how many users can be served curb-to-curb during a single trip time.



**Figure 3.1: Examples of system configuration for a) conventional fixed route; b) flexible with route deviation; c) hybrid fixed with route deviation**

The modeling approach presented in this study assumes that all users are served as they desire, either curb-to-curb or at fixed stops. Thus, the factors that could lead to reject service, as, for example, vehicle seating capacity, are considered negligible. Both types of demand are perfectly inelastic, which means that they are not affected by the quality of service. The flexible service considered in this study is the route deviation, as described in the following section.

## 3.2 Modeling Route Deviation Service

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A vehicle starts its trip from the terminal station and serves customers in a given corridor at fixed stops or by deviating to serve the curb-to-curb demand, which makes up a fraction  $a \in [0,1]$  of the total demand. The locations of fixed stops are defined in terms of the stop spacing at location  $x$ ,  $S(x)$ . The curb-to-curb users are assumed to request their pickups or drop-offs with sufficient advanced notice that the vehicle routing can be scheduled and determined prior to dispatch. The route has a longitudinal length,  $L$ , which is the length of the corridor. For each requested stop, the vehicle travels a lateral distance,  $d$ , to pick up or drop off the curb-to-curb requests and then the same distance,  $d$ , to return to the main route. The expected distance of a uniformly distributed requested stop from the main route is  $W/4$ . The vehicle does not backtrack to serve curb-to-curb demand. The remaining  $(1 - a)$  portion of total demand is associated with passengers who walk to the nearest fixed stop and wait at that location for service. A typical configuration of a flexible system with route deviation is shown in **Figure 3.1b**.

The focus of this study is to optimize the operation of a transit system in order to identify when and where flexible service will be more beneficial for both agency and users. The resulting system is a hybrid system between a conventional fixed-route and a flexible-route deviation system. An example of such a system's configuration is given in **Figure 3.1c**. The dashed line indicates the flexible region where the vehicles may deviate from the fixed corridor to serve the curb-to-curb requested demand. The width of the flexible area around a point  $x$  along the fixed corridor is  $A(x)$ , where  $A(x) \in [0, W]$ . The expected deviation is  $A(x)/4$ .

The following are the research team's calculations for distributed demand and vehicle operations in a corridor heading toward the terminal. The reverse direction, with distributed destinations for passengers heading away from the terminal, is symmetric. The number of passengers boarding each vehicle per unit distance traveled in the corridor is the product of the demand rate, the headway since the last vehicle, and the corridor width,  $QHW$ . Of this total demand, the number of passengers with request stop service is  $aQHA(x)$ , where the width of the flexible service area can vary as a function of the location in the corridor,  $x$ .

Vehicle distance and travel time can be calculated by integrating across the incremental vehicle distance and time required for the transit vehicle to traverse a distance  $dx$  at any location  $x$ . The total distance and time required to traverse the corridor is obtained by integrating the incremental values over the length  $L$ . The one-directional value is then doubled to obtain the distance and travel time associated with a cycle of travel from the terminal back to the terminal.

The vehicle distance is the sum of longitudinal distance traveled along the corridor and the lateral distance traveled to serve each requested stop, as shown in equation 1,

$$VMT = 2 \int_0^L \left[ 1 + aQHA(x) \frac{A(x)}{2} \right] dx \quad (1)$$

The first term is the longitudinal distance traveled per unit length of the corridor; the total longitudinal distance is  $2L$  per cycle. The second term is the product of the expected number of passengers with request stop service per unit length of the corridor and the expected lateral distance per request stop, which is twice  $A(x)/4$ .

The cycle time,  $C$ , includes the travel times for the longitudinal and lateral travel at speed  $V$ . It also includes dwell time for three kinds of stops: the dwell time at fixed stops,  $\tau^f$ ; the dwell time at requested stops,  $\tau^r$ ; and the dwell time at the terminal station,  $\tau^t$ . Fixed stops have spacing  $S(x)$ , as defined above, so the expected number of fixed stops per unit length of corridor is  $1/S(x)$ . The number of requested stops per unit length of the corridor is the same as the expected number of passengers with request stop service, because each request trip is served individually. The vehicle stops once at the terminal. As a result, the cycle time is given by equation 2, as follows,

$$C = 2 \int_0^L \left[ \frac{1}{V} + aQHA(x) \frac{A(x)}{2V} + \tau^f \frac{1}{S(x)} + aQHA(x) \tau^r \right] dx + \tau^t \quad (2)$$

### 3.3 Modeling Costs of Flexible Transit Systems

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The continuous approximation approach is adopted here to determine the optimal width of the flexible service area,  $A(x)$ , as well as the optimal spacing between fixed stops,  $S(x)$ . Both characteristics are treated as continuous functions of the distance,  $x$ , from the edge of the corridor and serve as decision variables in the optimization process presented in this report. Specifically,  $A(x)$  can be implemented as a continuous function, and  $S(x)$  is approximated by a continuous function. Like the formulation for  $VMT$  and  $C$ , the analysis is focused on the costs associated with the cycle of vehicle traversing the corridor from the terminal to the end and back.

#### 3.3.1 Agency Costs

The agency cost per vehicle cycle,  $AC$ , consists of three parts: costs attributed to vehicle distance traveled, costs attributed to vehicle hours of operation, and costs associated with the fleet size. Each of these costs is calculated by multiplying a cost coefficient by the corresponding value, as shown in equation 3,

$$AC = a_{VMT}VMT + a_{VHT}C + a_M M \frac{H}{O} \quad (3)$$

where  $a_{VMT}$  is the cost per vehicle distance traveled,  $a_{VHT}$  is the cost per vehicle time operated,  $a_M$  is the daily capital cost per vehicle, and  $M$  is the number of vehicles in the fleet. The vehicle distance traveled per cycle,  $VMT$ , and the cycle time,  $C$ , are given by equations (1) and (2). The fleet size is considered to be constant for this analysis, so its cost must be spread over the number of vehicle cycle operated within a daily period of operations. Details on properly selecting  $M$  are given in the following section. If the daily operating hours are denoted by  $O$  and the service headway is  $H$ , then there are  $O/H$  vehicle cycles operated per day.

### 3.3.2 User Costs

User costs include costs associated with walking, waiting, and riding as experienced by the users. Like the analysis of vehicle operations and agency costs, the user costs can be calculated by integrating the incremental user cost associated with each unit length across the corridor. As a result, the total daily user cost is the sum of these components, weighted by corresponding user cost coefficients:  $a_{WK}$  for time spent walking per vehicle cycle,  $WK$ ;  $a_{WT}$  for time spent waiting per vehicle cycle,  $WT$ ; and  $a_R$  for time spent riding per vehicle cycle,  $R$ , as shown in equation 4,

$$UC = a_{WK}WK + a_{WT}WT + a_R R \quad (4)$$

The models for each of these components of time spent by users are presented in the following subsections.

#### ***Walking***

Passengers who receive request stop service do not experience walking time, so the remaining demand  $QH(W - aA(x))$  per unit length of the corridor must walk to the nearest fixed transit stop. On average, this is  $W/4$  in the direction perpendicular to the main corridor and  $S(x)/4$  along the corridor. The walking speed is assumed to be  $V_{WK}$ . The total walking time for all users served in a vehicle cycle is thus given by equation 5,

$$WK = 2 \int_0^L QH(W - aA(x)) \frac{W + S(x)}{4V_{WK}} dx \quad (5)$$



### ***Waiting***

All transit users, either served curb-to-curb or at fixed stops, are expected to experience waiting time equal to half the headway. User choices, such as planning trips around the timetable, are not considered. The total waiting time for all passengers served in a vehicle cycle is simply the product of the demand,  $2QHW$ , and the average waiting time,  $H/2$ , as shown in equation 6,

$$WT = QH^2W \quad (6)$$

### ***Riding***

The expected riding time is calculated based on the incremental riding time experienced by all passengers onboard a vehicle as it traverses a unit length of the corridor at location  $x$ . The number of passengers onboard the vehicles is the cumulative number of passengers who have boarded since the beginning of the line. It is useful to think of this in terms of a vehicle trip from the edge of the corridor that starts empty and picks up passengers enroute to the terminal. By the time the vehicle reaches location  $x$ , there are  $QHWx$  passengers onboard. Each of these passengers experiences travel time associated with longitudinal and lateral vehicle distance as well as loss time per fixed and requested stop. The incremental travel time per unit length of the corridor for all passengers is the product of  $QHWx$  and the incremental vehicle travel time, which is the integrand of equation (2). Therefore, total riding time for a vehicle cycle,  $R$ , has a similar structure to the expression for cycle time,  $C$ , as expressed in equation 7,

$$R = 2 \int_0^L QHWx \left[ \frac{1}{V} + aQHA(x) \frac{A(x)}{2V} + \tau^f \frac{1}{S(x)} + aQHA(x)\tau^r \right] dx + QHWL\tau^t \quad (7)$$

In order to estimate the total riding costs, the dwell time at the terminal should also be considered. For a fixed corridor of length  $L$ , there are  $2QHWL$  passengers who each experience half of the dwell time at the terminal,  $\tau^t/2$ .

### **3.3.3 Total Weighted Generalized Costs**

The generalized cost for a day of flexible transit operations,  $GC$ , is the sum of agency costs,  $AC$ , and user costs,  $UC$ , weighted by  $w_{AC}$  and  $w_{UC}$ , respectively, as expressed in equation 8,

$$GC = w_{AC}AC + w_{UC}UC \quad (8)$$

This cost depends on the size of the flexible region,  $A(x)$ , and the fixed stop spacing,  $S(x)$ , which can be designed as functions of  $x$ .

The total daily generalized cost,  $TGC$ , is then calculated by multiplying the cost per cycle by the number of vehicle cycles that are operated in a day,  $O/H$ , as expressed in equation 9,

$$TGC = GC \frac{O}{H} \quad (9)$$

The objective in this study is to minimize  $TGC$  with respect to  $A(x)$  and  $S(x)$  for given  $O$  and  $H$  in order to achieve the optimal performance for the hybrid transit system studied here. The respective analysis is presented in the following section.

### 3.4 Optimal Stop Spacing and Size of Flexible Service Region

Given that the duration and daily operations,  $O$ , and the service headway,  $H$ , are treated as exogenous values in this analysis, the minimization of  $TGC$  is equivalent to minimizing  $GC$ . Thus, the optimization problem that this study addresses is represented by equations 10a through c,

$$\min_{A(x), S(x)} GC \quad (10a)$$

$$\text{s.t. } 0 \leq A(x) \leq W \quad \forall x \in (0, L) \quad (10b)$$

$$0 \leq S(x) \leq 2L \quad \forall x \in (0, L) \quad (10c)$$

The constraints on  $A(x)$  ensure that the flexible region is always a subset of the corridor. The stop spacing is constrained to  $2L$ , which would be the extreme case with one stop at the terminal, thereby forcing any customers who do not receive request stop service to walk all the way to their destination.

To facilitate the optimization, it is useful to note that in equations (1), (2), (5), (6), and (7), which are the inputs to equation (8), the decision variables,  $A(x)$  and  $S(x)$ , only appear within the integrand. This integrand containing the terms with decision variables can be rewritten as expression 11,

$$\begin{aligned} w_{AC} \left[ a_{VMT} \left( \frac{aQH}{2} A(x)^2 \right) + a_{VHT} \left( \frac{aQH}{2V} A(x)^2 + \frac{\tau^f}{S(x)} + aQH\tau^r A(x) \right) \right] \\ + w_{UC} \left[ a_{WK} \left( \frac{QH}{4V_{WK}} (W - aA(x))(W + S(x)) \right) \right. \\ \left. + a_R QHWx \left( \frac{aQH}{2V} A(x)^2 + \frac{\tau^f}{S(x)} + aQH\tau^r A(x) \right) \right] \quad (11) \end{aligned}$$

The value of the continuous approximation formulation is that the analysis can now focus on identifying the values of  $A(x)$  and  $S(x)$  that minimize the integrand at any  $x$ , and the results are functions that minimize the integral and thus the generalized cost.

Expression (11) is not quite separable with respect to  $S(x)$  and  $A(x)$ , because the term associated with walking cost includes  $(W - aA(x))(W + S(x))$ , which combines both decision variables. This combined term prevents the derivation of a closed form analytical solution for the optimal values for  $S(x)$  and  $A(x)$  at any location  $x$ ,  $S^*(x)$  and  $A^*(x)$ . Expression (11) is convex in  $S(x)$  if  $A(x)$  is treated as given, and it is convex in  $A(x)$  if  $S(x)$  is treated as given. Therefore, a closed form for the optimal stop spacing at each location,  $S^*(x)$ , can be expressed in terms of  $A(x)$  by solving the first order conditions for expression (11) with respect to  $S(x)$ ; i.e., setting the first derivative equal to zero and solving for  $S(x)$ , as shown in equation 12,

$$S^*(x) = 2 \left[ \frac{v_{WK} \tau^f (w_{AC} a_{VHT} + w_{UC} a_R QHWx)}{w_{UC} a_{WK} QH(W - aA(x))} \right]^{0.5} \quad (12)$$

Likewise, a closed form for the optimal size of the flexible service area at each location,  $A^*(x)$ , can be expressed in terms of  $S(x)$  by solving the first order conditions with respect to  $A(x)$ , as shown in equation 13,

$$A^*(x) = \frac{V}{4V_{WK}} \cdot \frac{w_{UC} a_{WK} (W + S(x)) - 4w_{UC} a_R QHWx \tau^r V_{WK} - 4w_{AC} a_{VHT} \tau^r V_{WK}}{w_{UC} a_R QHWx + w_{AC} a_{VMT} V + w_{AC} a_{VHT}} \quad (13)$$

Equations (12) and (13) are directly applicable to cases where one of the two decision variables is exogenous. For example, equation (12) provides the optimal fixed stop spacing for a system in which an agency has already decided how big the flexible service area should be (e.g.,  $A(x) = 1.5$  miles to satisfy minimum ADA requirements). Similarly, equation (13) defines the optimal size of the flexible service area for a transit agency that may not want to move the stop locations of a fixed-route service that has already been designed.

The more complex case is to optimize both decision variables,  $S(x)$  and  $A(x)$ , simultaneously, because each depends on the other. A computational approach can be implemented to identify the fixed-point solution satisfying equations (12) and (13). This can be solved substituting the expression for  $S^*(x)$  in equation (14) into equation (13) to obtain an expression with only  $A(x)$  terms. The optimal value,  $A^*(x)$ , that satisfies the equation can be identified by iterating through potential values of  $A(x) \in (0, W)$  for increments of  $x$ . A numerical solution can be obtained quickly with a computer. Once  $A^*(x)$  has been identified,  $S^*(x)$  is given by equation (12).

Finally, it is necessary to confirm that the available fleet size,  $M$ , is sufficient for the designed service operation. Although it is theoretically possible to make  $M$  a variable that depends on design variables, the reality is that flexible transit service in low-density corridors typically operates at such long headways that only a small number of vehicles are ever needed.

Therefore,  $M$  is treated as an input parameter. The fleet size must be at least large enough to sustain the headway,  $H$ , with the cycle time,  $C$ , as shown in equation 14,

$$M \geq \frac{C}{H} \quad (14)$$

### 3.5 Numerical Analysis of Flexible Transit Systems

A numerical analysis will illustrate application of the model to realistic corridors. Optimal values of  $A^*(x)$  and  $S^*(x)$  are calculated every 0.001 miles to provide a high-resolution representation of optimized functions. The input values for the numerical examples presented here are summarized in **Table 3.1**.

**Table 3.1: Model input values**

Parameter	Value	Units
Percent of Curb-to-Curb Demand, $\alpha$	0.50	unitless
Fleet Cost Coefficient, $a_M$	100	\$/veh
Riding Cost Coefficient, $a_R$	10	\$/hr
VHT Cost Coefficient, $a_{VHT}$	20	\$/veh. hr
VMT Cost Coefficient, $a_{VMT}$	0.5	\$/veh. mi
Walking Cost Coefficient, $a_{WK}$	20	\$/hr
Waiting Cost Coefficient, $a_{WT}$	10	\$/hr
Vehicle Headway, $H$	1	hr/veh
Operational Hours, $O$	18	hr/day
Cruising Speed, $V$	25	mph
Walking Speed, $V_{wk}$	3	mph
Weighting Factor for $AC$ , $w_{AC}$	1	unitless
Weighting Factor for $UC$ , $w_{UC}$	1	unitless
Dwell Time at Fixed Stops, $\tau^f$	0.008	hr/stop
Dwell Time at Curb-to-Curb Stops, $\tau^r$	0.005	hr/stop
Dwell Time at Terminal Stop, $\tau^t$	0.010	hr/stop

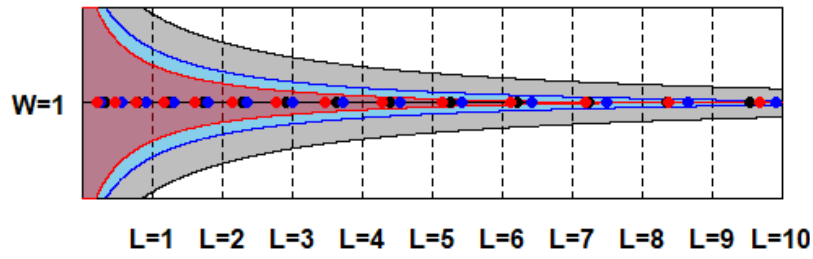
The fundamental assumption for user costs is that walking should have higher cost coefficients than waiting and riding, and the two latter are considered equal. Insights on the transit user cost coefficients can be found in Wardman (29). The magnitudes considered here for agency costs are derived from existing literature for the paratransit services in New Jersey and the Greater Boston area, which are considered the worst-case scenario, since demand-responsive operations in large cities tend to be made more expensive by the high costs of labor. For more details on the agency cost coefficients, readers are referred to Rahimi et al. (30) and Turmo et al. (31).

Real-world flexible service areas, where vehicles deviate their route to serve customers as needed, can be identified in existing literature. In Zheng et al. (12), Route 289 in a suburban area of Zhengzhou City, China, is evaluated for an implementation of point and route deviation services. A single service vehicle is considered for a service area of  $W = 1$  mile and  $L = 3$  miles. The demand density ranges from 4 to 17 ( $pax/mi^2/hr$ ). The MTA Line 646 in Los Angeles County is used in several case studies of flexible system (16,32). The service area has a width of  $W = 1$  mile and length of  $L = 10$  miles, with one operating service vehicle. In Zheng et al. (32), demand ranging from 0.8 to 2.8 ( $pax/mi^2/hr$ ) is considered. In Qiu et al. (16), slightly higher demand levels are considered for the same service area and a corridor of size  $W = 2$  and  $L = 5$  miles is evaluated. A third real-world case study for flexible systems is the Plymouth Area Link in the Greater Attleboro Taunton Regional Transit Authority in Massachusetts, which operates the Manomet/Cedarville Deviated Link, where two vehicles operate on a fixed corridor of  $L \approx 8$  miles, with a headway  $H = 1$  hour, which is a common headway for such services. The vehicles are allowed to deviate to serve passengers within  $3/4$  mile of the fixed route, indicating a service area of width  $W = 1.5$  miles.

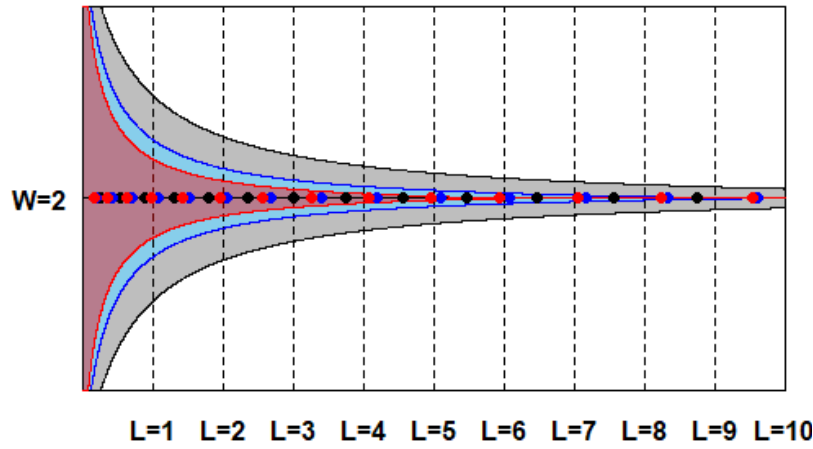
In the remaining analyses, the magnitudes of  $W$ ,  $L$ , and  $Q$  are based on values in existing literature to investigate the implementation of the proposed method under different service area scenarios. For input values with no clear indications from existing literature, a sensitivity analysis is performed to investigate their impact on the proposed flexible transit service.

### 3.5.1 Optimal Decision Variables

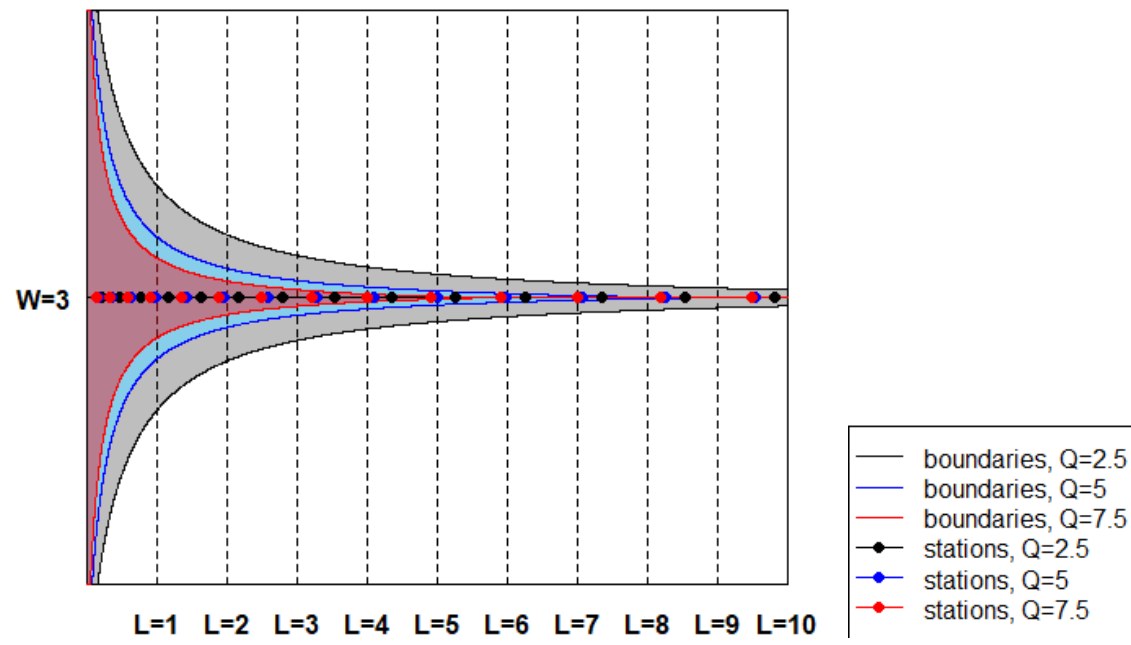
**Figure 3.2** shows the flexible region boundaries for  $W \in \{1,2,3\}$  miles for a service area of length  $L = 10$  miles. Since  $A^*(x)$  and  $S^*(x)$  depend only on cumulative demand up to  $x$ , shorter corridors are represented by the same figures, just truncated to  $L < 10$ . The horizontal line in the middle of each service area represents the fixed-route corridor. Three cases of demand density per direction are investigated in this figure,  $Q \in \{2.5, 5, 7.5\}$  ( $pax/mi^2/hr$ ). The three shaded areas represent the flexible regions in each case, colored gray, blue, and red, respectively. For all  $W$ , the lower value of  $Q$  leads to a greater flexible service region, and the flexible region gets smaller as  $W$  increases. Station locations are also shown for each demand density by black, blue, and red dots, respectively. The station spacing increases with  $x$ , because greater vehicle occupancy increases the generalized cost of stopping. For more details on determining the station location from a continuous function of spacing between stations, readers are referred to Wirasinghe et al. (33).



(a)



(b)



(c)

Figure 3.2: Service area configuration for a)  $W=1$ ; b)  $W=2$ ; c)  $W=3$

**Figure 3.3** shows that increasing  $W$ ,  $L$ , and  $Q$  increases all of the cost components. The costs associated with walking have the greatest impact, and the costs associated with  $VMT$  have the least impact. Fleet size costs shown in **Figure 3.3d** present a step increase of one vehicle after  $x = 4$ . The maximum fleet size for all scenarios investigated here is equal to two vehicles. Although fleet size costs and waiting costs are independent of the optimization process, their values offer insights to the relative magnitudes of the components of the generalized costs. In the case of  $VHT$  and  $VMT$  shown in **Figure 3.3e** and **Figure 3.3f**, it is apparent that the increase of  $Q$  has a lower effect on costs, compared with the increase of  $W$ .

### 3.5.2 Optimal Percentage Flexibility

The percentage of the service area that is covered by the flexible region,  $f(\%)$ , can be calculated considering the results of implementing equation (13) and the dimensions of the service area, as shown in equation (15),

$$f(\%) = \frac{\int_0^L A(x)dx}{WL} 100\% \quad (15)$$

**Figure 3.4** shows  $f(\%)$  for different service areas, using inputs from **Table 3.1**. Transit agencies that do not implement hybrid services could also use such a graph as a guide for choosing either a fixed-route or flexible system.

## 3.6 Comparison Between System Costs for Fixed Route, Hybrid, and Route Deviation

**Table 3.2** compares the benefit of the optimized hybrid system with a fixed-route and a fully flexible service. The agency cost components considered in optimizing the hybrid service are the  $VHT$  and  $VMT$  costs. Three corridor lengths are considered,  $L \in \{3, 5, 10\}$  miles. The percent benefit,  $B^S(\%)$ , from implementing hybrid transit ( $HT$ ) is, per equation 16,

$$B^S(\%) = \frac{C^S - C^{HT}}{C^S} 100\% \quad (16)$$

where  $C^S$  represents the cost of system  $S$ , with  $S \in (FR, RD)$  for fixed route and route deviation, respectively. Fixed-route service has the lower agency costs among all three systems, so the benefit of hybrid service is negative. Route deviation has the highest agency costs, so the benefit of hybrid service is positive.

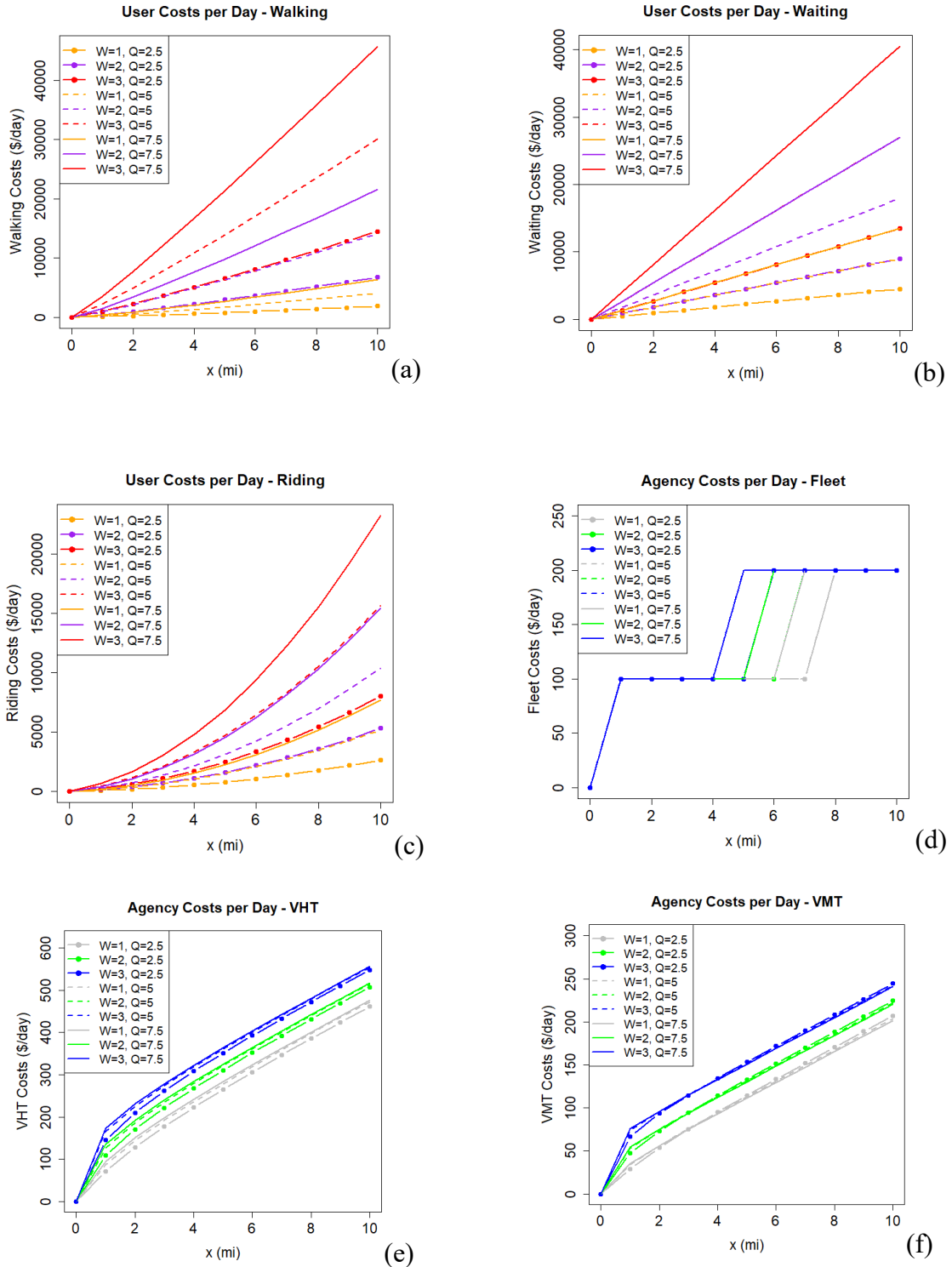
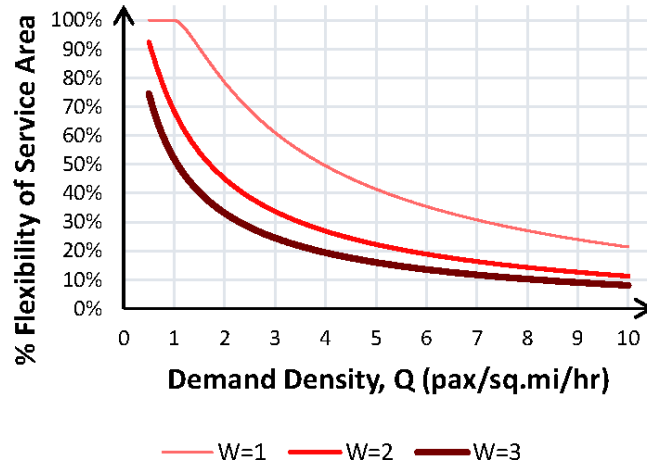
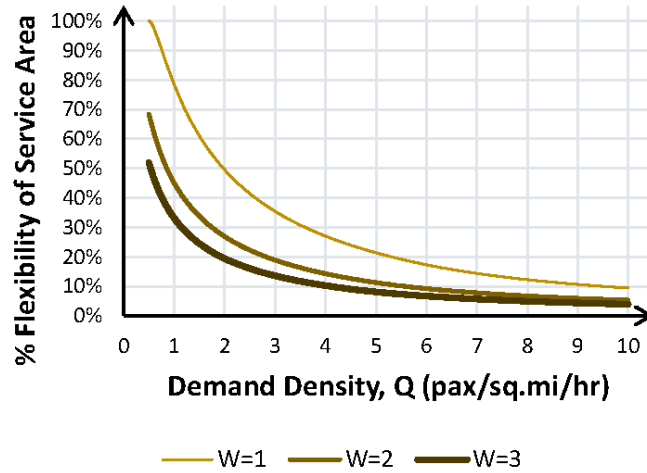


Figure 3.3: Daily costs of a) walking; b) waiting; c) riding; d) fleet size; e) VHT; f) VMT

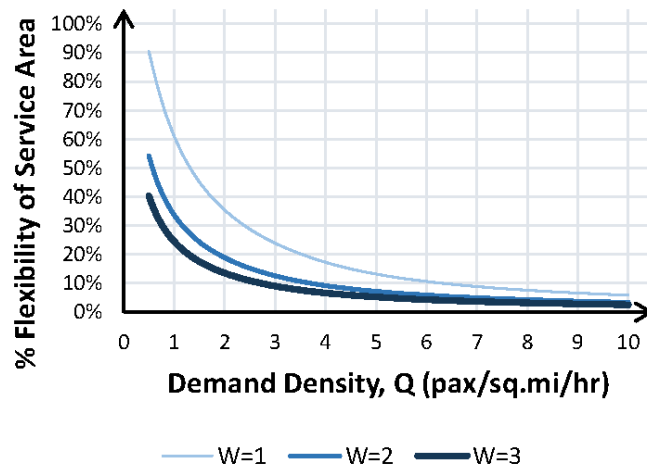




(a)



(b)



(c)

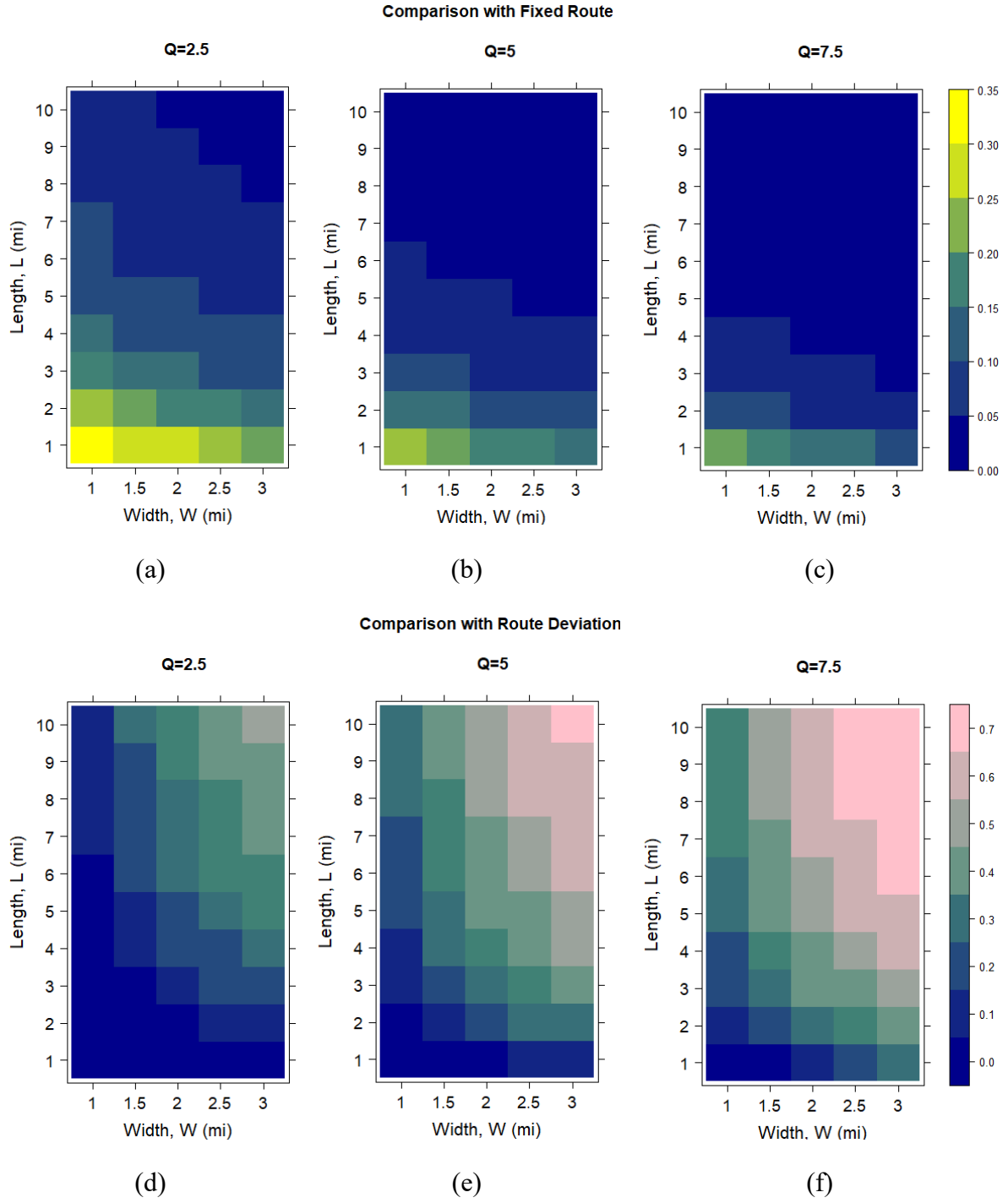
Figure 3.4: Optimal percentage flexibility of a service area with length  $L=10$  and headway = a) 0.5; b) 1; c) 1.5 hours/veh

**Table 3.2: Percentage benefit to agency cost from implementing optimized hybrid transit vs. fixed route (FR) and route deviation (RD)**

$Q = 2.5 \text{ pax/mi}^2/\text{hr}$									
Case	$W = 1 \text{ mi}$			$W = 2 \text{ mi}$			$W = 3 \text{ mi}$		
	$L = 3$	$L = 5$	$L = 10$	$L = 3$	$L = 5$	$L = 10$	$L = 3$	$L = 5$	$L = 10$
FR	-26.3%	-19.6%	-12.0%	-52.1%	-35.9%	-20.4%	-77.7%	-52.4%	-29.3%
RD	11.4%	18.0%	25.5%	44.3%	52.3%	60.0%	63.2%	70.1%	76.3%
$Q = 5 \text{ pax/mi}^2/\text{hr}$									
Case	$W = 1 \text{ mi}$			$W = 2 \text{ mi}$			$W = 3 \text{ mi}$		
	$L = 3$	$L = 5$	$L = 10$	$L = 3$	$L = 5$	$L = 10$	$L = 3$	$L = 5$	$L = 10$
FR	-28.8%	-19.8%	-11.2%	-53.5%	-35.5%	-19.5%	-78.7%	-51.9%	-28.4%
RD	31.7%	38.7%	45.6%	65.3%	71.1%	76.2%	79.0%	83.3%	87.0%
$Q = 7.5 \text{ pax/mi}^2/\text{hr}$									
Case	$W = 1 \text{ mi}$			$W = 2 \text{ mi}$			$W = 3 \text{ mi}$		
	$L = 3$	$L = 5$	$L = 10$	$L = 3$	$L = 5$	$L = 10$	$L = 3$	$L = 5$	$L = 10$
FR	-28.8%	-19.2%	-10.5%	-53.2%	-34.8%	-19.0%	-78.2%	-51.1%	-28.0%
RD	45.0%	51.3%	57.3%	74.9%	79.3%	83.1%	85.4%	88.5%	91.0%

The user benefits associated with the hybrid system compared to the fixed route are shown in **Figure 3.5**. The user costs of walking and riding affect the optimization process and are considered here. The user benefits range from 0 to 35% for all combinations of service areas and demand densities. Smaller service areas and lower demand densities lead to greater user benefits from the implementation of hybrid systems compared with fixed route. Comparing with full route deviation systems, the implementation of the hybrid transit has a user benefit of up to ~80%, with some cases having a small loss (e.g.,  $\leq 5\%$  for small areas and low demand densities). This loss is due to the effect of agency costs in the optimization process for the hybrid service.

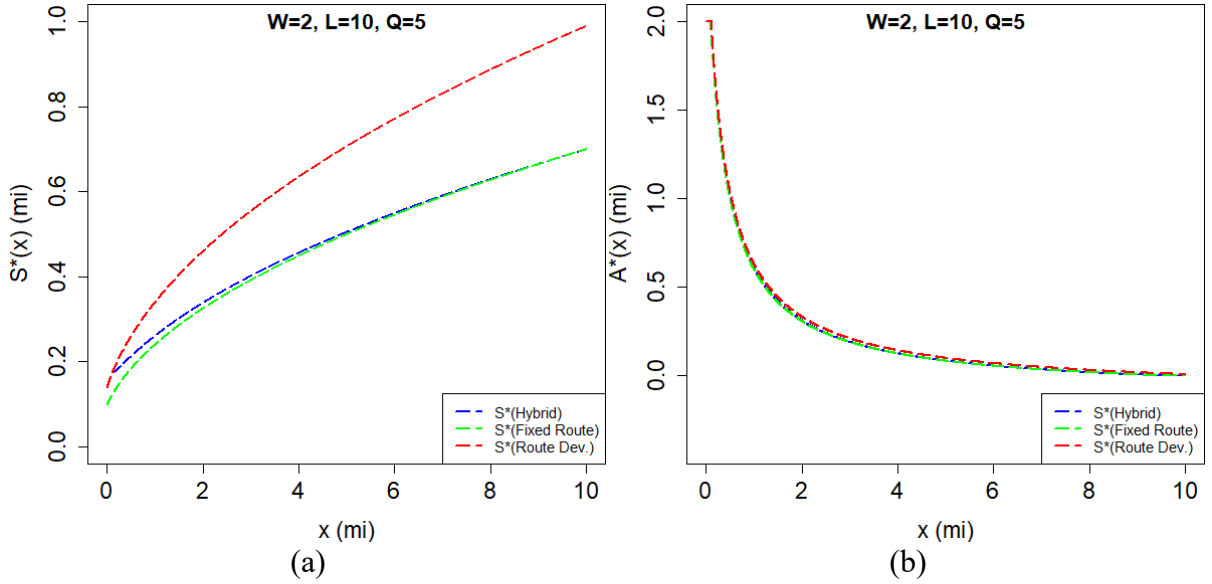
Given the coronavirus pandemic and the resulting decrease in transit ridership, it is noteworthy that for any one of the service areas studied here, there is a significant increase in users' benefits with a hybrid system as the demand density decreases. The hybrid system is also more beneficial for users than full route deviation systems, especially for  $W > 1$ . Finally, the agency loss associated with the hybrid system compared to conventional fixed route is slightly affected by falling demand. For these reasons, the proposed hybrid system has the potential for many beneficial applications in low-density communities or in areas where demand has dropped significantly due to the pandemic.



**Figure 3.5: Percentage user benefits from implementing hybrid transit instead of fixed route for a) Q=2.5; b) Q=5; c) Q=7.5 and route deviation for d) Q=2.5; e) Q=5; f) Q=7.5**

### 3.7 Optimization of Station Spacing for Fixed-Route and Route Deviation Systems

The following analysis concerns the effect of the size of the flexible region,  $A(x)$ , on the optimal fixed stop spacing and the costs of the system. Specifically, the focus is on the two extreme cases:  $A(x) = 0$ , which is a fixed-route system, and  $A(x) = W$ , which is a route deviation system. Although  $S^*(x)$ , as calculated in equation (12), is sensitive to the value of  $A(x)$  used,  $A^*(x)$  from equation (13) is not greatly affected whether  $S^*(x, A(x) = 0)$  or  $S^*(x, A(x) = W)$  is used. **Figure 3.6a** shows that the optimized fixed stop spacing for the hybrid transit,  $S_{HT}^*(x)$  lies between the optimized station spacings for fixed route,  $S_{FR}^*(x)$ , and route deviation,  $S_{RD}^*(x)$ . In this case,  $S_{HT}^*(x)$  overlaps  $S_{RD}^*(x)$  for locations  $x$  where  $A^*(x) = W$  and then moves towards  $S_{FR}^*(x)$ . Although the optimized spacings differ depending on what type of service is considered for their optimization, the optimized flexible regions that result from implementing each of the three optimal spacings are very similar, as shown in **Figure 3.6b**.



**Figure 3.6: Optimized decision variable of a) station spacing and b) flexible region for a corridor with  $W=2, L=10, Q=5$**

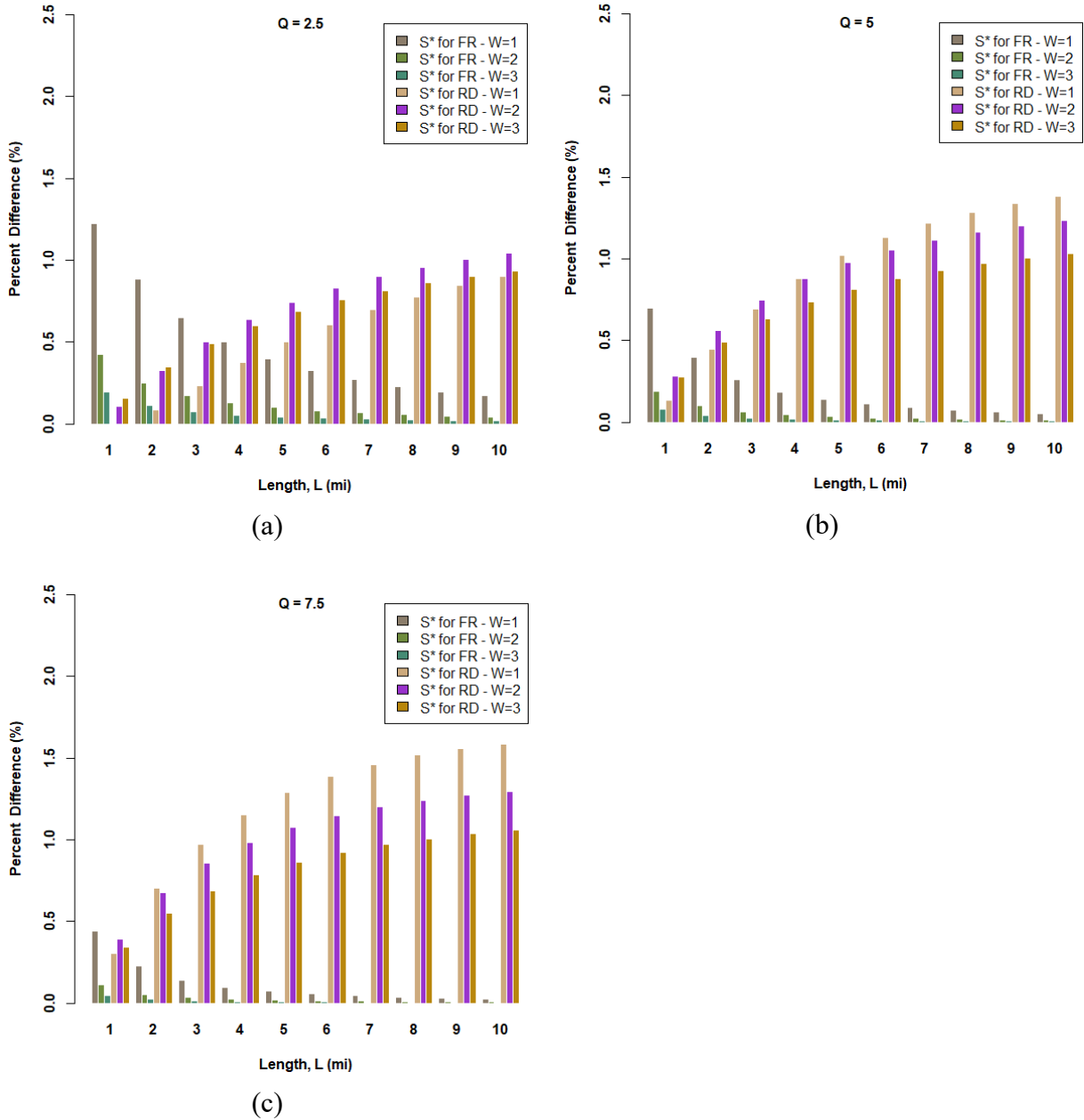
The difference in cost is more important than that difference in the design variables, because it is the generalized cost of the system that should be minimized. The percentage change in cost for implementing either the fixed-route or full-route deviation system relative to the optimized hybrid system is given by equation 17,

$$\Delta(\%) = \frac{C(S_T^*) - C(S_{HT}^*)}{C(S_{HT}^*)} 100\% \quad (17)$$

where  $C(S_T^*)$  is the cost of implementing the optimal station spacing for system  $T$ , with  $T \in (FR, RD)$ , and  $C(S_{HT}^*)$  the cost of implementing the optimal spacing for the hybrid service.

The costs considered in this analysis is the sum of costs that participate in the optimization process, namely walking, riding, VHT, and VMT costs.

This analysis shows that the effect of different optimized station spacings on the user and agency costs is always small; e.g., less than 2% for the cases presented in **Figure 3.7**. As a result, it is acceptable to approximate the joint optimization of  $A(x)$  and  $S(x)$  by implementing equations (12) and (13) independently. Although the optimized station spacing might differ based on what system is considered in its optimization, the optimal flexible region and the resulting operating costs are not severely impacted.



**Figure 3.7: Percentage difference between costs for a)  $Q=2.5$ ; b)  $Q=5$ ; c)  $Q=7.5$**

## 3.8 Sensitivity Analysis of Analytical Model

### 3.8.1 Effect of Headway, $H$

The headway of service has a significant effect on the system design and cost, because it determines the number of passengers served by each vehicle and the number of vehicles needed in the fleet. To facilitate the analysis in this study,  $H = 1$  hr was used as an exogenous value in accordance with many real-world flexible systems. The analysis now focuses on the effect of varying  $H \in (0.1, 2)$  hrs on the optimized design variables and the resulting costs. **Figure 3.8a** and **Figure 3.8b** show the effect of  $H$  on  $S^*(x)$  and  $A^*(x)$  for a service area with  $W = 2$  miles,  $L$  up to 10 miles, and  $Q = 5$  ( $pax/mi^2/hr$ ). Greater  $H$  leads to smaller flexible regions and shorter stop spacing as the system more closely resembles fixed route.

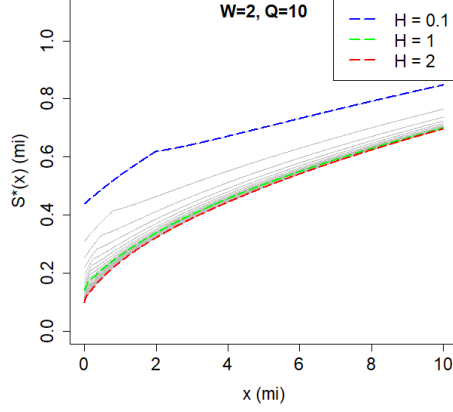
**Table 3.3** shows that as  $H$  increases, daily user costs increase significantly for any percentage of demand served curb-to-curb,  $\alpha$ . Lower  $H$  is associated with greater impact of  $\alpha$  on user costs. The costs included in **Table 3.3** refer to all types of user and agency costs in order to offer an overview of the overall cost magnitudes.

### 3.8.2 Effect if Flexible Service Demand

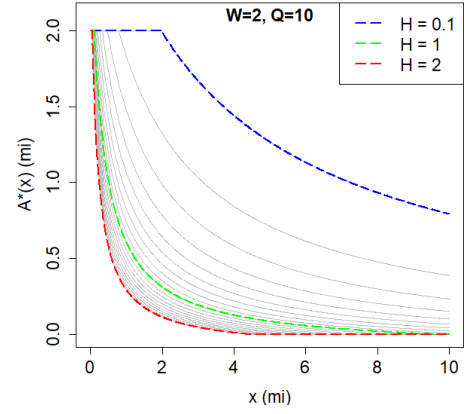
The percentage of demand receiving request stop service within the flexible region,  $\alpha$ , affects the distance and time traveled to serve the requested stops. **Figure 3.8c** shows that  $S^*(x)$  increases with  $\alpha$ . For the extreme case of  $\alpha = 1.00$ , the fixed spacing tends to infinity for  $x \leq 0.22$  miles,  $A^*(x) = W$  in this range so no passengers use fixed stops. Farther along the corridor,  $A^*(x)$  drops, increasing the number of passengers using fixed stops. Gray lines in **Figure 3.8c** and **Figure 3.8d** are associated with increments of  $\alpha$  from 0 to 1, with a step of 0.1. **Figure 3.8d** shows that the optimal flexible region is very insensitive to  $\alpha$ . Only when  $\alpha = 0.00$  does it have no impact on costs. Therefore, advanced knowledge of the percent of users served with request stops is not necessary for identifying  $A^*(x)$  is due to  $S^*(x)$ .

### 3.8.3 Effect of Cost Weights

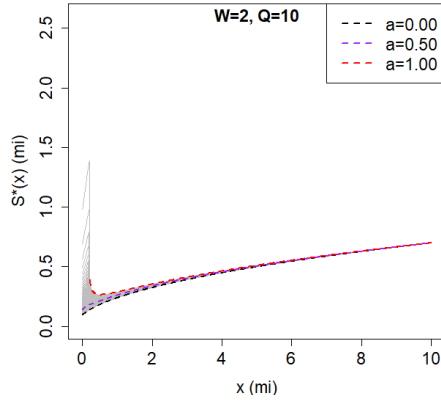
These weights can control the relative effect that agency costs and user costs have on the optimal values for the two decision variables. **Figure 3.8e** and **Figure 3.8f** show the effects of changing user cost weights from 0.1 to 1 with a step of 0.1 and 1 to 10 with a step of 1. When a cost weight is examined, the other is considered equal to one.



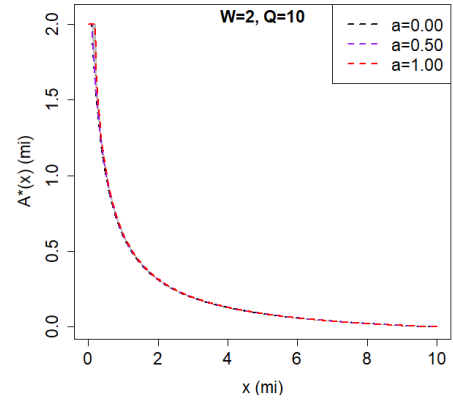
(a)



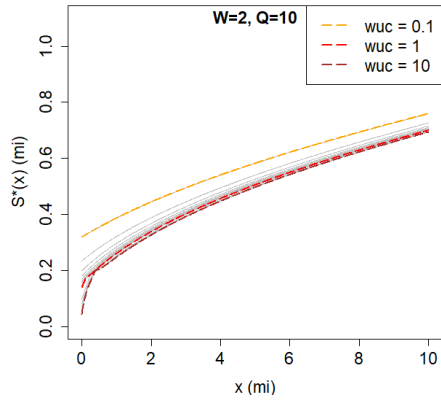
(b)



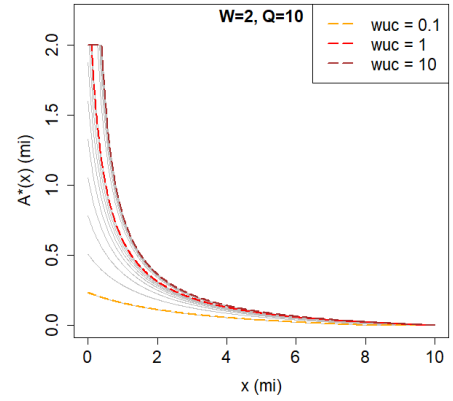
(c)



(d)



(e)



(f)

**Figure 3.8: Optimal decision variable of a)  $S^*(x)$  for various headways,  $H$ ; b)  $A^*(x)$  for various headways,  $H$ ; c)  $S^*(x)$  for various percentages,  $a$ ; d)  $A^*(x)$  for various percentages,  $a$ ; e)  $S^*(x)$  for various weights,  $wuc$ ; f)  $A^*(x)$  for various weights,  $wuc$**

**Table 3.3: User and agency costs per day for different headways and percentage of demand served curb-to-curb**

<i>H</i> = 0.5 hr/veh						
Case	User Costs (\$/day)			Agency Costs (\$/day)		
	$\alpha = 0.25$	$\alpha = 0.50$	$\alpha = 0.75$	$\alpha = 0.25$	$\alpha = 0.50$	$\alpha = 0.75$
<i>L</i> = 3 mi	7,599.9	7,260.9	6,908.1	722.3	832.0	947.8
<i>L</i> = 5 mi	13,984.5	13,591.2	13,183.4	968.8	1,087.6	1,212.7
<i>L</i> = 10 mi	33,509.7	33,066.5	32,608.2	1,738.5	1,864.4	1,996.6
<i>H</i> = 1.0 hr/veh						
Case	User Costs (\$/day)			Agency Costs (\$/day)		
	$\alpha = 0.25$	$\alpha = 0.50$	$\alpha = 0.75$	$\alpha = 0.25$	$\alpha = 0.50$	$\alpha = 0.75$
<i>L</i> = 3 mi	10,435.2	10,246.7	10,053.2	471.5	529.4	589.5
<i>L</i> = 5 mi	18,659.78	18,458.4	18,252.0	792.9	852.8	914.9
<i>L</i> = 10 mi	42,730.3	42,525.6	42,315.9	1,375.7	1,436.4	1,499.3
<i>H</i> = 1.5 hr/veh						
Case	User Costs (\$/day)			Agency Costs (\$/day)		
	$\alpha = 0.25$	$\alpha = 0.50$	$\alpha = 0.75$	$\alpha = 0.25$	$\alpha = 0.50$	$\alpha = 0.75$
<i>L</i> = 3 mi	13,192.0	13,064.4	12,934.2	483.6	522.5	562.6
<i>L</i> = 5 mi	23,226.6	23,096.3	22,963.4	764.0	803.4	844.1
<i>L</i> = 10 mi	51,800.2	51,669.9	51,537.1	1,452.2	1491.7	1,532.4

**Figure 3.8e** shows that station distance is decreased as user costs are taken on higher consideration. Intuitively, this could be attributed to walking costs, which are reduced as user costs have a higher impact on the total generalized costs. The change in  $S^*(x)$  is greater for  $0.1 < w_{UC} < 1$  and much lower for  $1 < w_{UC} < 10$ . At  $x \sim 0.20$  miles. The lines that correspond to  $w_{uc} > 1$  overlap, indicating that station spacing is independent of the user costs weight. This location is the point where the optimal flexible region boundaries reach their maximum value (i.e.,  $W = 2$  in this case). At some locations  $x$ , the optimal value for  $A^*(x)$  is bounded by the feasibility condition that  $A^*(x) \leq W$ , and the optimal spacing is estimated based on this bounded value of  $A^*(x)$ . These points are at  $x = 0.12$  for  $w_{UC} = 1$  and at  $x = 0.41$  for  $w_{UC} = 10$ . For  $w_{UC} = 0$ , the optimal value for station spacing goes to infinity. Similarly, for  $w_{uc} = 0$ , the optimal flexible region boundaries go to 0. **Figure 3.8f** shows the increase of flexible region boundaries as the weight of user costs increase. Again, for  $0.1 < w_{UC} < 1$ , the boundaries present a greater rate of increase compared to the respective changes in  $1 < w_{UC} < 10$ .

Regarding the effect of the agency cost weight,  $w_{AC}$ , it is observed that an increase in  $w_{AC}$  leads to an increase in the station spacing and decrease in the flexible region boundaries. At the same locations  $x$  as in the case of  $w_{UC}$ , there are overlaps between the lines of station spacing when  $A^*(x)$  is bounded by  $W$ . The overlap in this case occurs when the agency costs are undervalued. Overall, undervaluing the agency costs has a smaller effect on the two optimized decision variables than overvaluing it. Undervaluing the weight of agency costs



has a greater effect on the two decision variables than overvaluing them, even if the agency costs are overvalued by 10 times (i.e.,  $w_{AC} = 10$ ).

### 3.9 Simulation Evaluation

The simulation process is developed using the R programming language and aims to evaluate the assumptions made during the analytical model development. The output of the simulation algorithm is the scheduling of vehicles in terms of times of arrival at the fixed and curb-to-curb stops, as well as the costs that result from their operation. The demand is generated considering a Poisson distribution. The generated values include location coordinates and requested time. The next step is to identify which of the generated trip requests lies within the flexible region borders. From the eligible trips, a percentage of  $\alpha\%$  is randomly chosen to be served curb-to-curb. The percentage of trips served curb-to-curb is assumed to be a constant number throughout the day. The algorithm serves curb-to-curb passengers following a first-come, first-served pattern, and the vehicles do not backtrack.

The performance of the simulation algorithm is demonstrated in this subsection through considering the case study of  $W = 2\text{ mi}$ ,  $L = 10\text{ mi}$  and demand density  $E(Q) = 10\text{ pax/mi}^2/\text{hr}$ . Other model parameters for the case study are the same used for the analytical analysis, as shown in **Table 3.1**. In this case, it is assumed that  $\alpha = 50\%$ , so half of the requested trips are served as curb-to-curb trips if located within the flexible service area. For this service area and demand rate, the optimized stop spacing and flexible service region would be as shown in **Figure 3.9**. **Figure 3.10** shows the demand generated in each direction of operation for four realizations. **Figure 3.11** shows the subset of demand that is served curb-to-curb. The remaining demand must walk to access the transit service at one of the fixed stops.

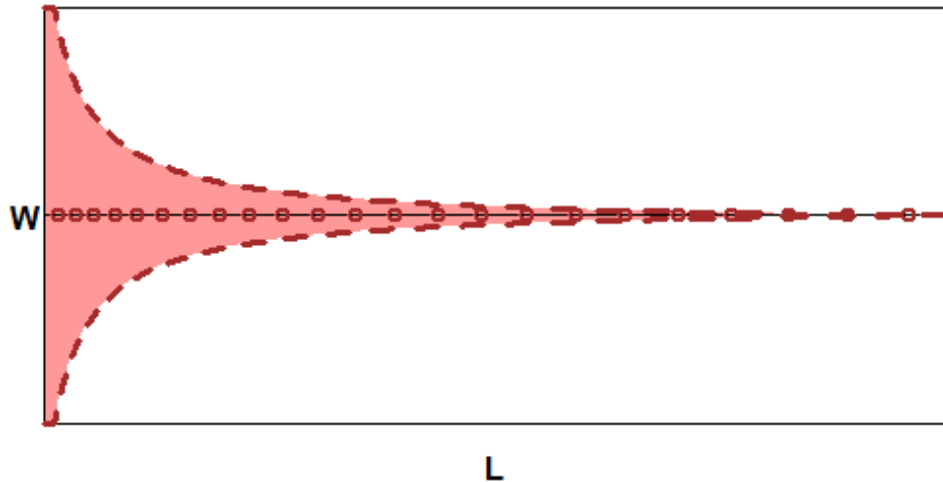
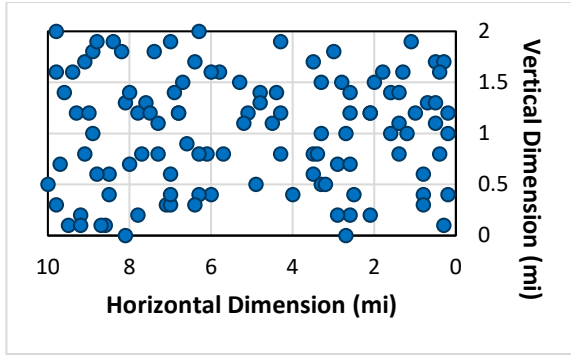
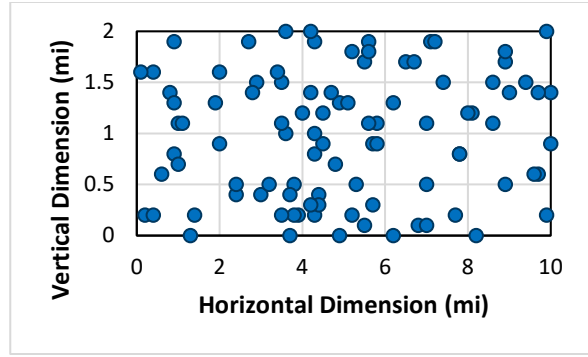


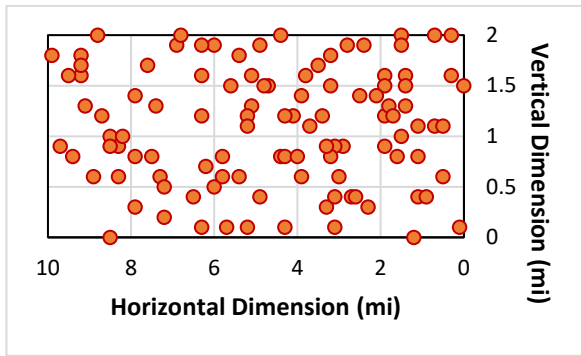
Figure 3.9: Optimized flexible region and station spacing



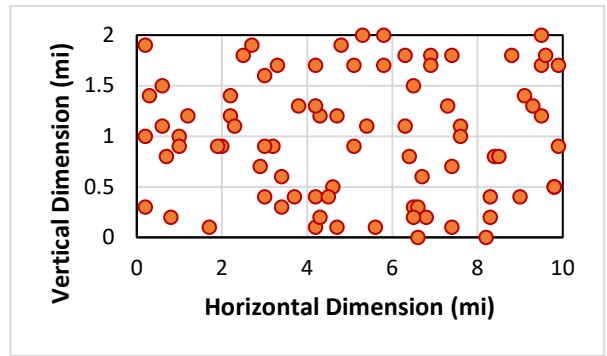
(a) 1<sup>st</sup> Operational Hour – Direction 1



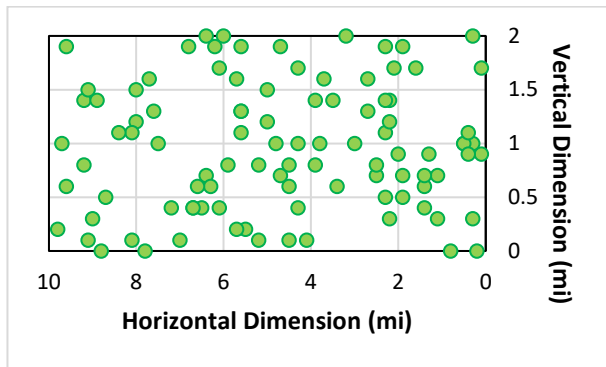
(b) 1<sup>st</sup> Operational Hour – Direction 2



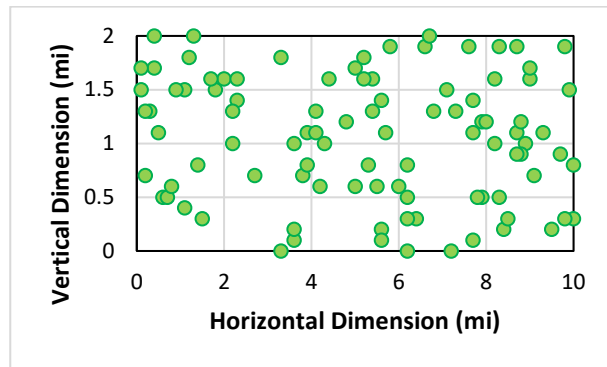
(c) 2<sup>nd</sup> Operational Hour – Direction 1



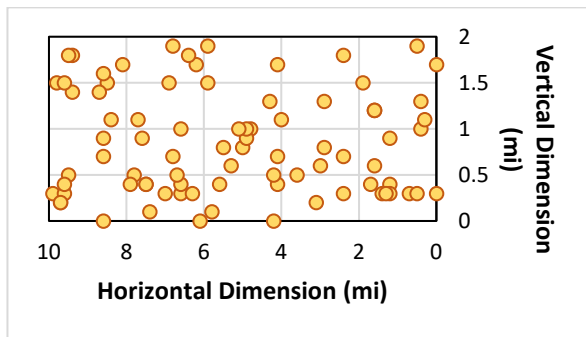
(d) 2<sup>nd</sup> Operational Hour – Direction 2



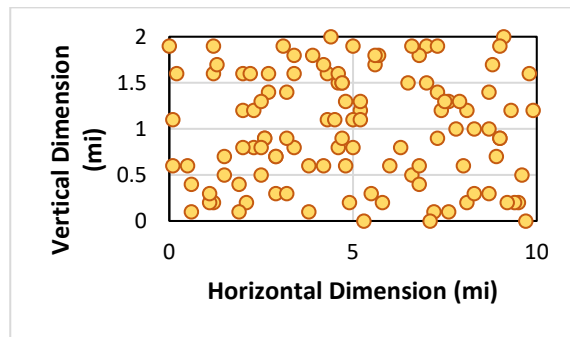
(e) 3<sup>rd</sup> Operational Hour – Direction 1



(f) 3<sup>rd</sup> Operational Hour – Direction 2

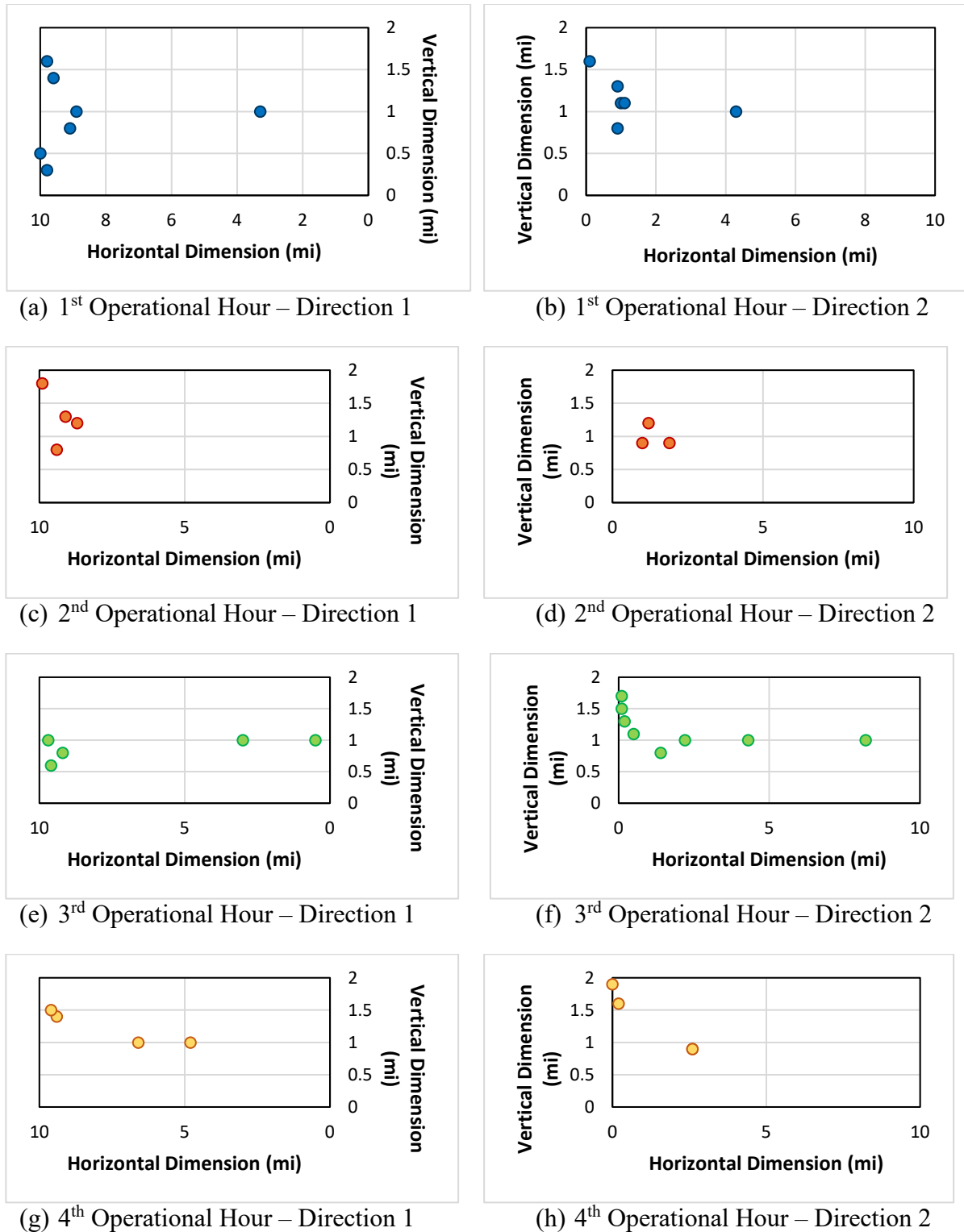


(g) 4<sup>th</sup> Operational Hour – Direction 1



(h) 4<sup>th</sup> Operational Hour – Direction 2

**Figure 3.10: Distribution of simulated demand per hour per direction**



**Figure 3.11: Distribution of simulated curb-to-curb demand per hour per direction**

The flexible service area for this case study is a relatively small part of the total area, so only a few of the trips in each realization would be served curb-to-curb. Although the distribution

of all random points appears consistent across the whole region (**Figure 3.10**), the variability is more apparent for the low number of eligible curb-to-curb trips (**Figure 3.11**). This distinction is important, because larger demand tends to lead to more consistent system performance, whereas the variability associated with low demand can make operations quite different from hour to hour. The analytical models provide estimates of performance in the average case, but the simulation provides estimates for the performance of the system in each realization, which reflects the effect of varying demand.

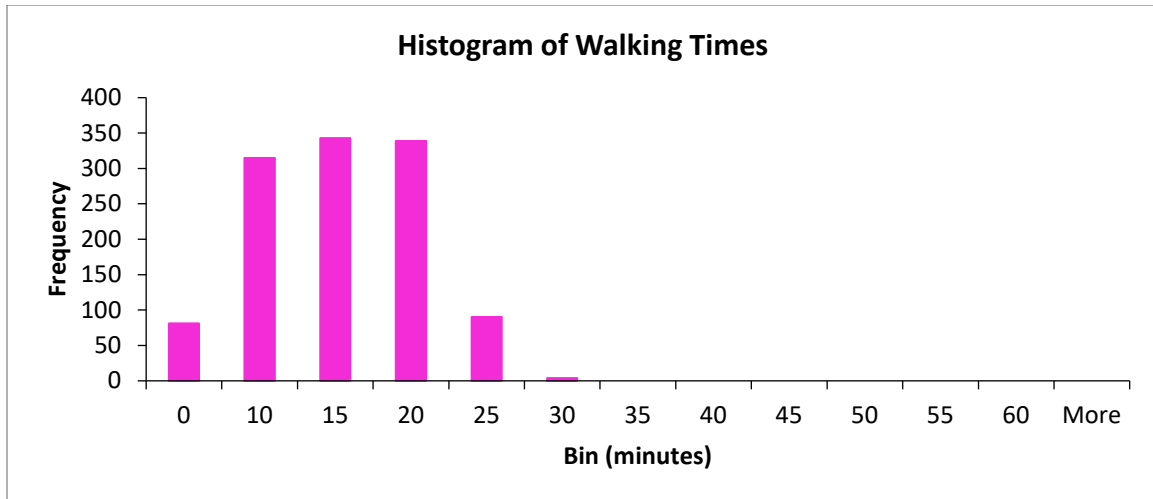
The experience of each system user is quantified as part of the simulation. The distributions of the times that users spend walking, waiting, and riding in the vehicle provide indications of how much the user experience varies from one hour to another. **Figure 3.12** shows histograms of the three components of the user cost: walking time, waiting time, and riding time. For each component of the user cost, there is a distribution of experienced times, with some users experiencing very low costs and others much higher depending on the location of the trip origin/destination and whether they are eligible for curb-to-curb service.

The results from the simulation can then be compared with the results of the analytical model in order to assess whether the analytical model provides robust estimates of the average system performance. **Table 3.4** shows a comparison of performance measure estimates from the analytical model and the average from 50 simulations. The column of accuracy (%) refers to how accurate is the analytical model when comparing with the simulated case study, assuming that the simulation more accurately approaches reality.

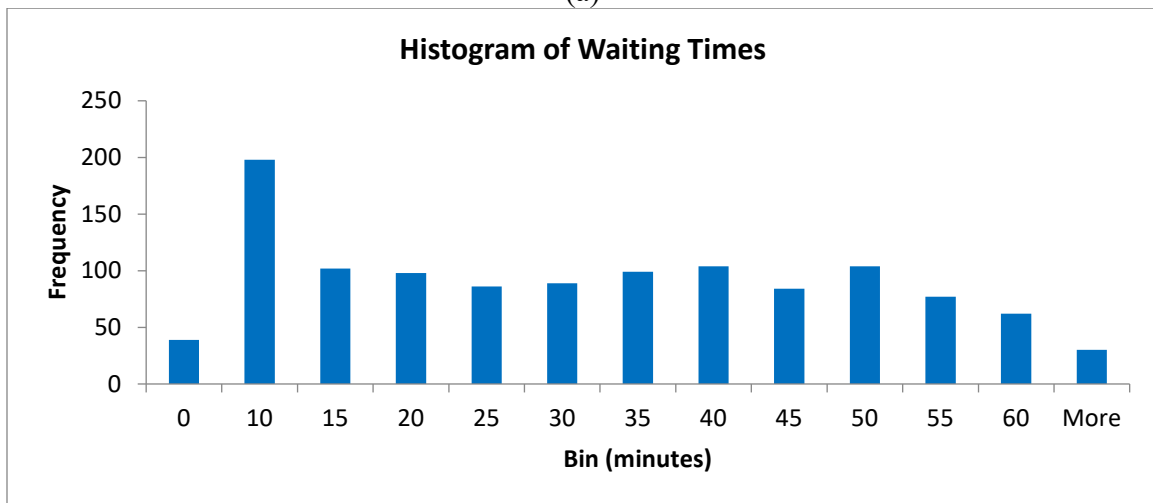
**Figure 3.13** presents costs resulting from analytical models accompanied by error bars based on the confidence interval from running  $N = 50$  simulations. The t-value considered is  $t = 2.01$  and the confidence intervals,  $CI$ , are calculated as shown in equation 18,

$$CI = (\bar{X} - t \frac{s.d.}{\sqrt{N}}, \bar{X} + t \frac{s.d.}{\sqrt{N}}) \quad (18)$$

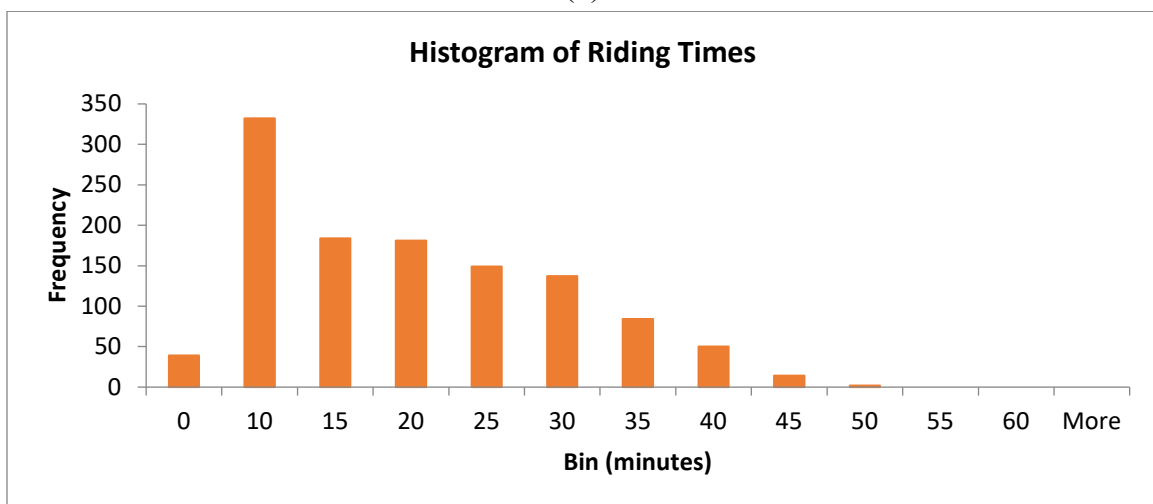
where  $\bar{X}$  represents the mean value and  $s.d.$  the standard deviation of each cost component for 50 simulation runs. The case study considered here is  $W = 2$  miles,  $L = 10$  miles and  $Q = 5$  ( $pax/mi^2/hr$ ). Input values are as in **Table 3.1**, with  $O = 8$  hours. The system's flexible region and station spacing in each run are optimized for the expected demand density per direction,  $Q = 5$  ( $pax/mi^2/hr$ ) using equations (12) and (13). The simulation, however, allows in each run the demand density per direction to be determined by the randomly generated demand. Finally, for this case study, it is confirmed that the analytical model's results are always statistically equivalent to the simulation, since the analytical values are always within the simulation confidence intervals. In the case of agency costs, the error bars are not visible, since the confidence intervals are narrow.



(a)



(b)

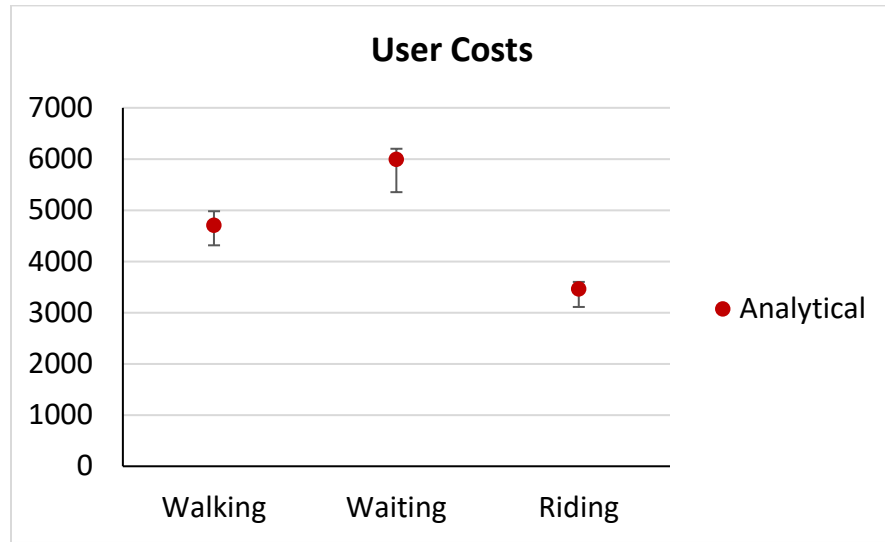


(c)

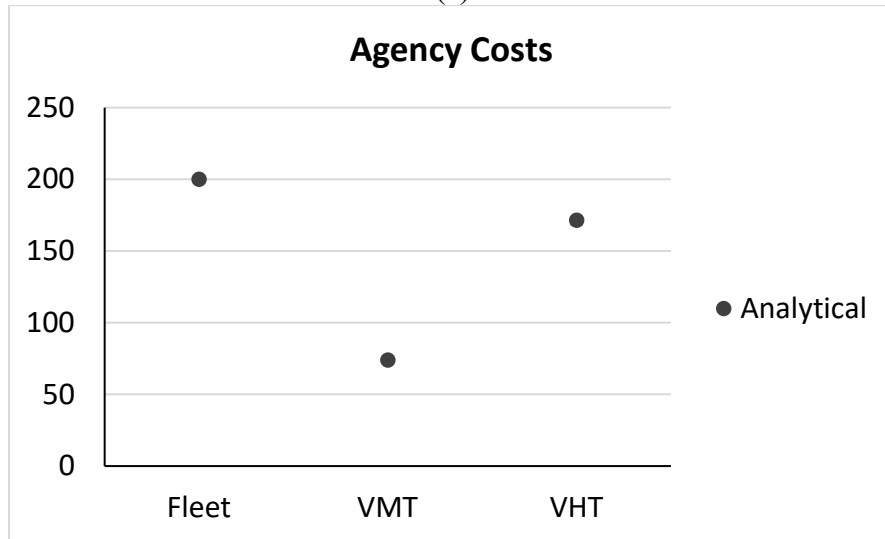
**Figure 3.12: Histograms of a) walking times; b) waiting times; c) riding times experienced by each user**

**Table 3.4: Comparison of average system performance estimates**

<b>Cost (\$)</b>	<b>Analytical</b>	<b>Simulation</b>	<b>Accuracy (%)</b>
<b>Walking</b>	4,709	4,720	99.77
<b>Waiting</b>	6,000	5,600	92.86
<b>Riding</b>	3,466	3,257	93.58
<b>Fleet</b>	200	200	100.00
<b>Vehicle-Hours</b>	172	170	99.14
<b>Vehicle-Miles</b>	74	74	99.99



(a)



(b)

**Figure 3.13: Validation of analytical costs of a) users and b) agency**

### 3.10 Summary

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Flexible transit systems are widely implemented in real-world service areas, but according to existing literature, there are still areas of improvement of current services, in terms of both operation and design. This study focuses on a hybrid transit system, with elements of both fixed-route and route deviation systems. The main outputs of this study include two formulas for optimizing the flexible region boundaries and the station spacing for a hybrid transit service in any given service area. It is highlighted that if agencies prefer to use either fixed-route or full flexible-route deviation services, the proposed formulas can serve as a guidance in deciding which service to choose. The numerical analysis performed here adopts input values based on existing flexible service areas and reveals the behavior of the modeling approach under various case scenarios. The analytical method's performance is evaluated considering a simulation approach developed in R programming language. The hybrid transit has significant user benefits over full route deviation services.

Regarding agency costs for the three systems considered here, fixed route is associated with the lowest agency costs, followed by the optimized hybrid system and the full route deviation flexible system. An important finding is that a service area could switch from fixed route or full deviation to hybrid service within a day, adjusting to any level of demand and maintaining the same station spacing and infrastructure without negative impacts on the operational costs. The benefits from the analyzed hybrid system as transit demand decreases is promising for the implementation of such systems during and after the coronavirus pandemic.

## 4.0 Results: Microtransit Pilots in Massachusetts

In an effort to provide more innovative and effective transit service across the Commonwealth of Massachusetts, MassDOT has funded a series of microtransit pilot projects with Regional Transit Authorities (RTAs). These pilots are intended as demonstration projects to test whether flexible microtransit services are viable in different types of communities around Massachusetts (see **Figure 4.1**). The idea is that microtransit, which can take any of the forms of flexible transit service presented in this report, can provide a customizable solution to fill the gaps in traditional fixed-route transit networks.

MassDOT's RTA Discretionary Grant Program funded six microtransit pilot programs across the Commonwealth of Massachusetts:

1. Cape Cod Regional Transit Authority (CCRTA) SmartDART.
2. Franklin Regional Transit Authority (FRTA) Access.
3. Greater Attleboro Taunton Regional Transit Authority (GATRA) Go Coastline/Go Connect.
4. Montachusett Regional Transit Authority (MART) Subscription Service.
5. MetroWest Regional Transit Authority (MWRTA) CatchConnect.
6. Worcester Regional Transit Authority (WRTA) VIA partnership.

Two microtransit programs are funded by MassDOT's Community Transit Grant Program:

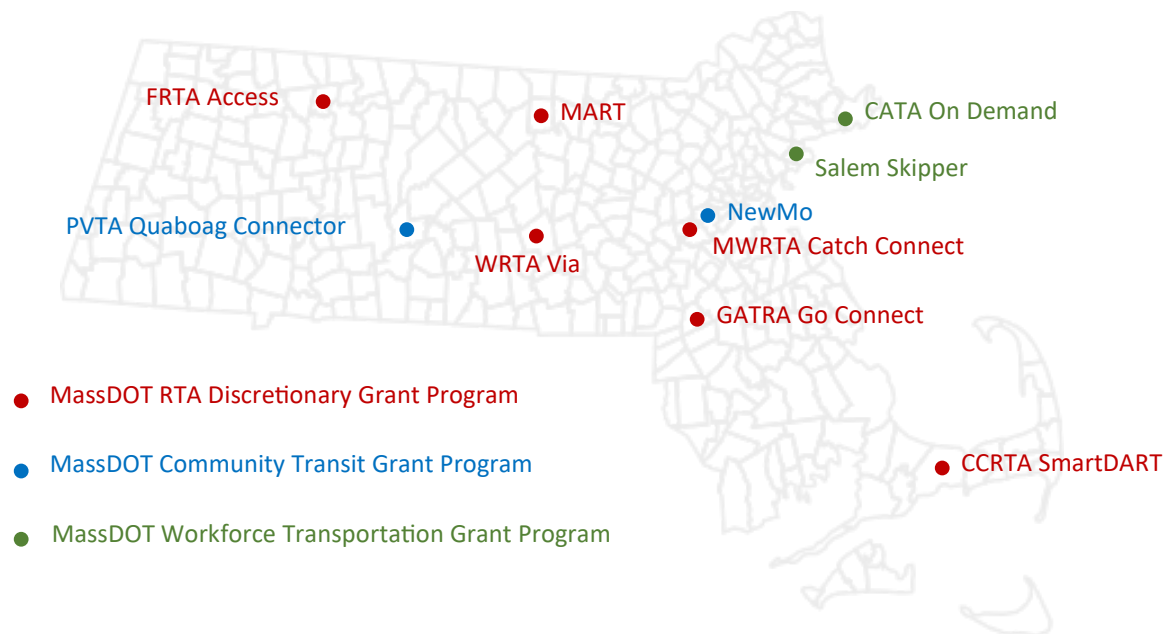
7. NewMo is a microtransit service in Newton, MA.
8. PVRTA Quaboag Region Microtransit is a proposed program in Western Massachusetts.

Three microtransit programs are funded by MassDOT's Workforce Transportation Grant Program:

9. Cape Ann Transit Authority (CATA) On Demand.
10. Greater Attleboro Taunton Regional Transit Authority (GATRA) Go Connect.
11. Salem Skipper.

Starting in 2019, microtransit pilots started operating services in areas where fixed-route transit had been underperforming or where there had been no transit service before. This chapter provides three types of analysis. First, the demand and operating characteristics are compared with the results of the modeling analysis to identify whether any of the service areas could benefit from an alternative form of flexible transit. Second, the characteristics of each RTA's microtransit pilot are summarized. Finally, patterns are identified to provide a set of lessons learned and guidance for implementation of microtransit in Massachusetts.





**Figure 4.1: Microtransit pilot programs funded by MassDOT**

## 4.1 Comparison with Flexible Transit Models

The microtransit pilots that were funded and launched as part of MassDOT’s Discretionary RTA Grant Program were all intended to start small in order to identify whether a microtransit service is viable for low-density suburban or rural areas. Each of the microtransit pilots implemented a service that operates as a fully flexible demand-responsive service in which all users are served curb-to-curb.

Shortly after the pilot began to launch, the Commonwealth of Massachusetts was struck by the coronavirus pandemic, which brought about extensive lockdown orders starting in March 2020. With the pandemic raging, several pilots were halted, but even when the transit systems have been operating, it has been in an environment of suppressed public transportation demand. As a result, demand levels for the microtransit pilots cannot be construed as representing normal conditions. Nevertheless, even low levels of demand provide some useful data and insights.

The first analysis question related to the pilot programs is how the services compare to the characteristics of flexible transit service presented in the theoretical models. Across the pilot programs, demand did not exceed about 50 trips per day. Expressing this total daily demand in terms of demand density by dividing by the size of the service area and the hours of daily service, the demand density is typically just one or two trips/mi<sup>2</sup>/hr. When the flexible transit system is optimized, low-demand density corresponds to a large corresponding flexible service region where passengers may be served curb-to-curb (see **Figure 3.4b**). Therefore, for all of the microtransit pilots, the analytical models would indicate that a fully flexible

demand-responsive system in which all customers are served curb-to-curb would be desirable. This is consistent with the type of flexible transit operation selected for each pilot.

## **4.2 Microtransit Pilot Implementations**

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Each of the microtransit pilots was an implementation of a fully demand-responsive service that provides curb-to-curb service for customers within the designated zone. There are some differences between each of the pilots, because each agency deployed a microtransit service to fit the context and needs of its region. A description of each RTA's pilot is provided as follows.

### ***Cape Cod Regional Transit Authority (CCRTA) SmartDART***

- Service Area Currently in Hyannis/Barnstable; plans to introduce service in Yarmouth, Dennis, and Falmouth
- Pricing \$3 fixed price (1.5 times the fixed-route fare of \$2)
- Demand 745 trips by 237 unique users from September 8, 2020 to January 1, 2021 (6–7 trips per day)
- Platform In-house app

CCRTA introduced microtransit in a small area, including Hyannis and Barnstable, with the goal of serving short-distance trips that complement the fixed-route network rather than competing with the seven existing fixed routes. The goal is to expand the reach of transit service by addressing the first/last mile problem of getting passengers to and from activity centers that are also served by the fixed-route bus service. Free transfers are allowed between the microtransit and fixed-route buses in order to encourage transfers and increase the appeal of the whole transit system. The initial pilot focused on serving trips to and from the Hyannis shopping plazas and housing complexes. In Barnstable, the total transit ridership, including fixed route, increased by 2.5%, which suggests that the service is having the intended impact of increasing transit use.

SmartDART makes use of an app that was initially developed in-house for MWRTA, and this allows the agency to retain control over the data and functionality of the service. Customers are able to make cashless payments through the app, which eliminates the need for a farebox in the microtransit vehicles. In addition to the app, customers can use a call-in service to request a trip by phone.

### ***Franklin Regional Transit Authority (FRTA) Access***

- Service Area 4 zones (1: Greenfield, Montague, Deerfield, Whately, Gill, Leyden, Erving; 2: Orange, New Salem, Warwick, Wendell, Erving; 3: Downtown Shelburne Falls, Charlemont Park & Ride; 4: Athol)
- Pricing \$3 within a zone (same as ADA and double the fixed-route fare), \$4 between zones
- Demand >1,000 trips per month (45–50 trips per day)
- Platform Ecolane

Franklin County is a rural county in Western Massachusetts, and the microtransit pilot serves a large and dispersed region that has been divided into four zones. Despite the very low density of demand, many people in Franklin County lack transportation alternatives. There are not enough Uber or Lyft drivers working in the area to make either of those ridesourcing services a reliable travel option. There is also only one taxicab company, located in Greenfield.

Initially the service required trips to be reserved using the Ecolane app either the same day or the day before travel, but this was found to suppress demand from customers who want to be able to plan a trip many days in advance. This is especially important for customers who want to be certain they will have a ride for critical purposes like a medical appointment or a job interview. Now, the service also allows customers to book using a computer and to request trips up to a week in advance. As a result, demand rose to over 1,000 trips per month. In order to manage the limited capacity of the system, eligible individuals (older adults, veterans, nursing home residents) are able to reserve trips in advance. Any remaining capacity is available to the general public for on-demand reservations made in real time through the app.

The manager of the pilot program said that the pandemic did have an impact on suppressing demand and slowing the growth of the service, but this also gave the agency time to assess how the system was working. The pilot started with a focus in the more urban parts of the region, with the idea that it is beneficial to start small and expand service later. Currently, the service only operates on weekdays, 6:30 a.m.–7:00 p.m. Weekend service is something that customers are asking for.

***Greater Attleboro Taunton Regional Transit Authority (GATRA) Go Connect***

- Service Area Go Connect: Foxborough, Mansfield, Plainville;  
Go Coastline: South Plymouth; Go United: Foxborough, Franklin,  
Norfolk, Wrentham
- Pricing \$1.50
- Demand For Go Connect: October 2019–March 2020 average 20 trips/day;  
April 2020–September 2020 average 30 trips/day
- Platform Transloc

GATRA introduced Go Connect as a microtransit pilot in October 2019 in order to serve rural areas that cannot be efficiently served by fixed-route transit. With the shutdowns that occurred during the pandemic, Go Connect was a service that continued to operate, and it turned out to serve a critical role for public mobility during this period. In fact, ridership increased during the pandemic months from an average of 20 rides per day to 30 rides per day. Although this fell short of GATRA's pre-pandemic goal of 65 rides per day, this level of utilization is viewed as a success in light of the dramatic drops in transit utilization experienced in other communities.

For rural and low-density suburban communities, microtransit has proven to be a viable service option for GATRA, and the agency has expanded operations. Go Coastline was launched in South Plymouth in October 2020, and Go United was launched in Foxborough,

Franklin, Norfolk, and Wrentham in December 2020. This expansion of microtransit services to additional communities reflects the value that GATRA sees in providing on-demand mobility to customers in communities that would not otherwise be served by public transit.

***Montachusett Regional Transit Authority (MART)***

- Service Area City of Fitchburg
- Pricing \$4 per day (more than the cost of ADA paratransit, but low enough to be competitive)
- Demand 1287 trips over four months (10–12 trips/day)
- Platform Global Scheduling Engine

MART introduced a microtransit pilot in the City of Fitchburg as a way to serve MassHealth subscription riders more efficiently. The deployment of the system was centered around using the Global Scheduling Engine to build routes around the recurrent subscription trips. Once the system was up and running for subscription trips (usually for work), the service was integrated with on-demand service for the general public, who book trips through a call center or using an app. The system makes use of two vans to provide service.

***MetroWest Regional Transit Authority (MWRTA) Catch Connect***

- Service Area Town of Wellesley
- Pricing Fares suspended during the pandemic, looking to move to an account-based system
- Demand 4,500 trips over 6 months (30 trips/day)
- Platform In-house app

MWRTA introduced a microtransit pilot in Wellesley in order to replace underperforming fixed-route services in the town. The goal of this program is to improve the quality of service and reduce costs and emissions by right-sizing the transit service to the community. The system utilizes an app that was developed in-house in order for the agency to retain control of the data and functionality. This was developed in partnership with CCRTA, so the effort to develop the app yielded benefits for another RTA, as well.

During the coronavirus pandemic, fares were suspended; however, the app has functionality to collect fares with an account-based system. The idea is that customers can deposit funds into an account with MWRTA and then withdraw from the account each time they use the system. This method of fare collection links trip data with customer information to allow the agency to gain more information about demand patterns that can be used to shape the transit services provided. With the initial success in Wellesley, MWRTA has planned to expand the microtransit service into Framingham and to add service on Sundays.

***Pioneer Valley Transit Authority (PVRTA) Quaboag Connector***

A proposed microtransit service is being planned but has not yet been launched in Western Massachusetts by the PVRTA. The service area would cover nine rural communities around and including Ware and Palmer, which also have a high percentage of low-income residents. This is a dispersed region that cannot be efficiently served with fixed-route transit, so a microtransit service provides an opportunity to provide needed mobility to residents who lack

other choices. The proposed system will utilize Ecolane software for app-based trip requests, vehicle scheduling, and data reporting. If the implementation in the Quaboag region is successful, the PVRTA has also identified the towns of Agawam, Hadley, and Easthampton as potential candidates for microtransit service to improve transit access in low-density communities.

### **4.3 Emerging Patterns and Lessons Learned**

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The experiences of RTAs in Massachusetts that implemented microtransit pilots reveal several patterns that appear to be common across the different regions. Many of these are practices and perspectives that may serve as guidance for other RTAs looking to introduce microtransit services in other areas.

1. ***Microtransit is used to serve rural and low-density suburban areas.*** In all cases, RTAs introduced microtransit pilots to serve dispersed demand in low-density areas that are either rural or suburban in nature. These communities do not have sufficient demand to support fixed-route transit service, so microtransit either replaced underperforming fixed routes or provided new transit service where none had been provided before.
2. ***Microtransit is most efficient for short trips within small service areas.*** Most of the microtransit implementations were limited to small service regions that constitute only a part of the RTA's full service area. In cases where the microtransit program was considered successful and worth expanding, additional zones were added (e.g., CCRTA and GATRA) rather than expanding the single microtransit service to a larger region. This scale makes trips easier to schedule, leads to fewer miles of empty vehicle repositioning, and supports the function of microtransit as a local complement to a larger fixed-route network.
3. ***Start small and expand.*** All RTAs advocated for starting with a small microtransit service that is limited in geographic coverage and hours of operations. This allows the agency to respond to the challenges of initial implementation and fix problems before expanding the service more broadly.
4. ***Microtransit fares are set above those for fixed-route services.*** Generally, the fares that RTAs chose to set for microtransit services were 1.5 to 2 times the fixed-route fare. This level is comparable to the fares charged for ADA paratransit. The rationale is that fares should be high enough that existing transit riders do not switch away from fixed-route services but low enough to incentivize demand and not place a burden on people who rely on transit for mobility.
5. ***Microtransit provided valuable service during the coronavirus pandemic.*** With the need to provide safe mobility to the public during the pandemic, the low vehicle occupancies associated with microtransit are compatible with public health constraints of the pandemic. Several RTAs viewed the microtransit service as an

effective way to provide critical mobility service during the pandemic, despite low levels of ridership overall.

6. ***Marketing and outreach is a challenge.*** A common experience across RTAs was the challenge in communicating to the public how to use the new microtransit service. Successful outreach involved extensive efforts to coordinate with community organizations, senior centers, and existing transit riders about the new opportunities associated with on-demand microtransit. Otherwise, the service is generally too small in scale for people to notice that a microtransit system has been introduced in a community.

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## 5.0 Results: Data Requirements

Many data are required for the implementation and operation of flexible transit systems. This section contains three main parts. First, the data requirements for planning and design of a flexible transit service are presented. Second, the data requirements for operating a flexible transit service in real time are described, along with a comparison of the benefits and drawbacks of using an in-house-developed app versus an off-the-shelf commercial product for organizing service provision. Third, the data requirements for ongoing monitoring of the system are discussed, with specific attention to the types of metrics that reflect quality of service for users.

### 5.1 Data for Planning and System Design

There are many reasons to consider implementation of a flexible transit service, such as a demand-responsive microtransit system, including improving quality of service for users and reducing operating costs. Effective planning for a new service requires data in order to identify the most appropriate form of transit service and to optimize its design for the context in which it will operate. As presented in Chapter 3.0, a flexible transit system can be designed and its performance predicted based on several model inputs. These inputs are the required data for planning and system design. Broadly speaking, the data inputs for planning-level models can be classified in three categories: service area, demand, and mode characteristics. The technical listing of data inputs was provided in **Table 3.1**. In the following subsections, a more qualitative description is provided of each data input and why it is important.

#### 5.1.1 Service Area Characteristics

**Service Area Size** [ $\text{mi}^2$ ]. For a demand-responsive microtransit service, it is important to know the size of the geographic area over which service is provided. Larger service areas capture more potential demand and allow customers to make longer trips. From a user perspective, a large service area provides access for more people to reach more destinations. However, systems are more costly to operate over larger areas, because the vehicles tend to travel longer distances, including costly deadhead travel when no passengers are onboard.

**Service Corridor Length** [mi] **and Width** [mi]. For more structured transit services, it is useful to think of the service area as a corridor with length,  $L$ , and width,  $W$ . For a fixed-route transit service, the corridor length represents the length of the route, and the width is the maximum distance that customers are expected to travel to access the route. More flexible or hybrid services, such as those modeled in Chapter 3.0, may traverse the region along the length direction and then deviate within the width of the region to pick up and drop off passengers closer to their preferred origins and destinations. The service area size, defined above, is simply the product of length and width,  $L \times W$ . Large regions may be better served by dividing the service area into



multiple corridors. A simple way to do this is to choose a dimension to serve as the length and then divide the other dimension until the width is acceptable for the assumed access mode (e.g., small width for walking, larger is acceptable for driving).

**Operational Hours [hrs].** In addition to the spatial coverage of the transit service, it is important to define the temporal coverage in terms of hours of the day and days of the week. Like the area, longer operating hours allow the service to capture more demand and provide greater accessibility to users. Longer hours also increase costs, as the agency must pay for drivers, fuel, and maintenance for the additional hours that vehicles are in automation.

### 5.1.2 Demand Characteristics

**Density of Demand [pax/mi<sup>2</sup>/hr].** One of the most important determining factors for selecting and designing a transit service is the density of demand for the service. A basic measure of demand is the density of trip origins and destinations per area per time. Where the density of demand is high, there are more opportunities for passengers to ride in vehicles together, making shared rides or conventional transit more effective. Where the density of demand is low, conventional fixed-route transit is not very efficient, because either people must walk a long distance to access the service or wait a long time for the vehicle to serve them, or the cost of operating a system with low vehicle occupancy is very high. A challenging aspect of demand is that the demand density typically varies by location and time of day, so a region that has low average demand may have a short period of intense transit demand. Therefore, a region that may be most cost-effectively served by fixed-route transit during the morning and evening rush hours may be better served by flexible service during evenings and weekends.

**Distribution of Demand.** In addition to the magnitude of travel demand, it can be very important to understand how travel patterns compare across users. In communities with a centralized town center or small number of main trip attractors, the demand pattern may be characterized as a many-to-one system in which transit vehicles gather passengers and bring them to a common destination and then pick up passengers at a common origin and distribute them back to their destinations. In less-centralized environments, the origins and destinations may be distributed across the region without any significant clustering, in which case it may be characterized as a many-to-many system. More dispersed demand patterns tend to be better suited for demand-responsive microtransit, because very high-demand densities are required to sustain a network of fixed-route service, which is more typical of dense urban environments.

**Percentage of Eligible Curb-to-Curb Demand [%].** As a policy, agencies should consider which customers will be served with a curb-to-curb service that carries customers from their preferred origin address to preferred destination address. The ADA requires transit agencies to provide curb-to-curb paratransit service for customers with a disability. Many agencies that choose to operate a microtransit service also offer curb-to-curb service to seniors or even the general public. A flexible transit service that is intended to provide curb-to-curb service for the general public would use  $\alpha = 100\%$ .

**Passenger Value of Time** [\$/hr]. It is often useful to translate the time that passengers spend using the transit system into an equivalent monetary cost that can be compared directly with the costs to the agency. The value of time is difficult to measure in a precise way because it varies from person to person, by trip purpose, and by time of day. Nevertheless, it can be useful to consider different values of time for different parts of the travel time experience, to account for the fact that time spent in some activities is more onerous than others. For example, it may be appropriate to use separate values for the time spent walking to and from transit stops, the time spent waiting for transit service, and the time spent riding in the vehicle. Even this is simplistic, because passengers are likely to value time spent waiting at home for a curbside pickup differently than time waiting at a transit stop on the roadside.

### 5.1.3 Mode Characteristics

Critical determinants of the performance of a flexible transit service are the characteristics of the access mode, transit vehicle, and the costs associated with vehicle operations.

**Access Mode Speed** [mi/hr]. The speed at which users can travel to access transit has a big impact on the travel time associated with moving to reach a transit stop. A basic assumption for the access mode would be that customers travel by walking, but walking speeds are slow and restrict the area that can be served in a transit corridor. Faster modes, such as bicycles, scooters, or cars, allow users to travel longer distances to reach a transit stop. The access mode is only relevant for passengers who use fixed stops. In a fully demand-responsive system with  $\alpha = 100\%$ , this value is irrelevant, because all trips would be served curb-to-curb.

**Transit Vehicle Speed** [mi/hr]. The speed at which transit vehicles can travel determines how long it takes for the vehicle to traverse distance. For a flexible service, this may include a significant amount of time associated with deviations to serve customers curb-to-curb or for a flexible routing. An appropriate baseline estimate for transit speed is to assume that vehicle travels at the same speed as traffic when not stopping to pick up or drop off passengers.

**Dwell Time at Stops** [sec/stop]. In addition to the travel time associated with moving through the network, transit vehicles spend time additional time for each stop. This time is associated with losses due to deceleration and acceleration, as well as the time that the vehicle is stopped while passengers board and alight the vehicle. The dwell time may differ by the type of stop, especially if more passengers are likely to board and alight at the route's terminal or at fixed-stop locations. On the other hand, a curb-to-curb stop may involve additional waiting if vehicles must wait for passengers to come out to the vehicle upon its arrival.

**Cost of Vehicles** [\$/veh]. Each transit vehicle that is used to provide service must be either purchased or leased. While a full-size city bus can cost several hundred thousand dollars, smaller vehicles that are designed to serve a few passengers at a time can be much less costly.

**Cost per Vehicle Hour Traveled** [\$/veh/hr]. Operations of vehicles are associated with costs that accrue per time in operation. The largest component of this cost is the wages for the driver.

**Cost per Vehicle Distance Traveled** [\$/veh/mi]. Many costs associated with the operations and maintenance of the transit vehicles are associated with the vehicle distance traveled. Resources like fuel, brake pads, tires, and oil tend to be consumed in proportion with the total distance operated.

## **5.2 Data for Flexible Transit Operations**

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Moving from planning to implementation requires a different set of data requirements. Rather than systemwide averages and general characteristics, the implementation of flexible transit service requires detailed data about individual trips, vehicle capacity and location, and the road network. Furthermore, the data requirements for a system that uses advanced reservations (e.g., typical ADA paratransit service) or that works in real time (e.g., app-based dispatch) differ as well in terms of the types of data that are needed to manage operations.

### **5.2.1 Data Requirements for Flexible Transit Operations**

Unlike a fixed-route transit system, in which stop locations, routes, and schedules are planned in advance, flexible transit services are adjusted to the locations and times that customers wish to travel. At a minimum, the transit agency needs to collect data for each requested trip.

**Trip Origin/Destination Location and Time.** The essential data required for each flexible transit are the details of the requested trip. The transit operator needs to know the locations of the origin and destination and the date and time when the person wishes to travel.

**Vehicle Fleet Size.** The transit agency needs to know how many vehicles in the fleet are available for operation at any day and time. This is not only the number of vehicles that the agency owns or leases but also the hours that each vehicle can be staffed with a driver in order to serve passengers.

**Vehicle Capacity.** The maximum number of passengers that each vehicle can carry is essential for planning flexible transit routes that can serve all of the intended customers. When demands are very low, it is common for demand-responsive transit to serve one passenger at a time, but with increased demand comes the opportunity to improve operating efficiency by grouping passengers into vehicles simultaneously.

**Vehicle Location Tracking (real time).** During operations, the locations of vehicles need to be tracked in real time so that a dispatcher or automatic dispatching software can monitor the progress of each vehicle along the planned route and make changes or assign additional trips as needed. For systems that rely on advanced reservations,

location tracking provides a way to identify disruptions and delays, which can be communicated to customers awaiting service. For system with real-time booking, the tracking data is critical for assigning trips to vehicles as they are requested.

Conventional demand-responsive transit services, such as most ADA paratransit systems, require customers to make reservations at least 24 hours in advance of their planned travel. Customers typically call a reservation center to tell an operator the relevant data about the trip they would like to make, and then the operator finds an available vehicle to assign to the requested trip. With such advance notice, vehicle routes can be optimized the day before service, because complete information has already been collected about demand. A dispatcher only needs to monitor the system in real time in order to handle disruptions in service. Although advance reservation systems have been in place for decades, a big drawback is that users can only make trips with careful advanced planning.

Modern demand-responsive or microtransit systems increasingly use smartphone or web-based apps to collect passenger requests and assign trips to available vehicles in real time. This makes microtransit work more like a taxi or ridesourcing service (e.g., Uber, Lyft) in which customers send data about their intended trip at the time that they wish to start traveling. Then, an available vehicle from the fleet is assigned to the serve the demand as soon after the request as possible. In this case, real-time vehicle location data is critical for the assignment process, because the dispatching software needs to account for the distance that a vehicle must travel to serve a customer.

Optional data for flexible transit operations include the following:

**Fare Payment Information.** If the transit agency charges fares that vary with distance traveled or time of day, or can accept payment by app, then this fare payment information needs to be communicated between the agency and the passenger. For example, a customer requesting a trip should expect to know how much the trip will cost before entering the vehicle. If payment can be made through an app, then it would make sense to set up credit card payment information so that the appropriate fares can be collected.

**Subscription or Repeating Trips.** Many customers make regular trips by transit, for example to commute to and from work. Rather than making separate reservations for each of these recurring trips, most agencies are able to book repeating trips as a subscription. Aside from simplifying the trip reservation process for the customer, the additional information about recurring trips allows the transit agency to plan future operations with at least partial information about the demand for the system.

### 5.2.2 Commercial Software for Flexible Transit

There are a number of companies that produce commercial software products that provide an app-based platform for customers to request trips and transit agencies to route and dispatch vehicles. Of the six microtransit pilots funded through MassDOT's RTA Discretionary Grant Program, four RTAs use off-the-shelf commercial products to run their systems:

- Franklin Regional Transit Authority (FRTA) uses Ecolane.
- Greater Attleboro Taunton Regional Transit Authority (GATRA) uses Transloc.
- Montachusett Regional Transit Authority (MART) uses a Global Scheduling Engine, which is connected with data for MassHealth and other subscription service clients.
- Worcester Regional Transit Authority (WRTA) uses Via.

The commercial products from Ecolane, Transloc, and Via have similar functionalities that include a smartphone app that facilitates communication between users and the agency. A smartphone app allows users to request rides from anywhere at any time by transmitting their relevant trip data. The transit agency can also purchase services from these companies to facilitate real-time scheduling and dispatch of transit vehicles.

A benefit of commercial, off-the-shelf products is that they provide a comprehensive set of tools that can (at least in theory) be implemented quickly, automatically log required performance measures, and can even manage fare collection data. Commercial software products have two main downsides. First, use of the software for dispatch and communications requires some support that typically requires ongoing payments for service. Second, commercial products generally have proprietary software that is not transparent and cannot be easily adapted to other service strategies. This second point is an important one, because the needs of each transit agency can vary in ways that cannot be easily accommodated by a single one-size-fits-all product.

### 5.2.3 Development of an In-House Scheduling App

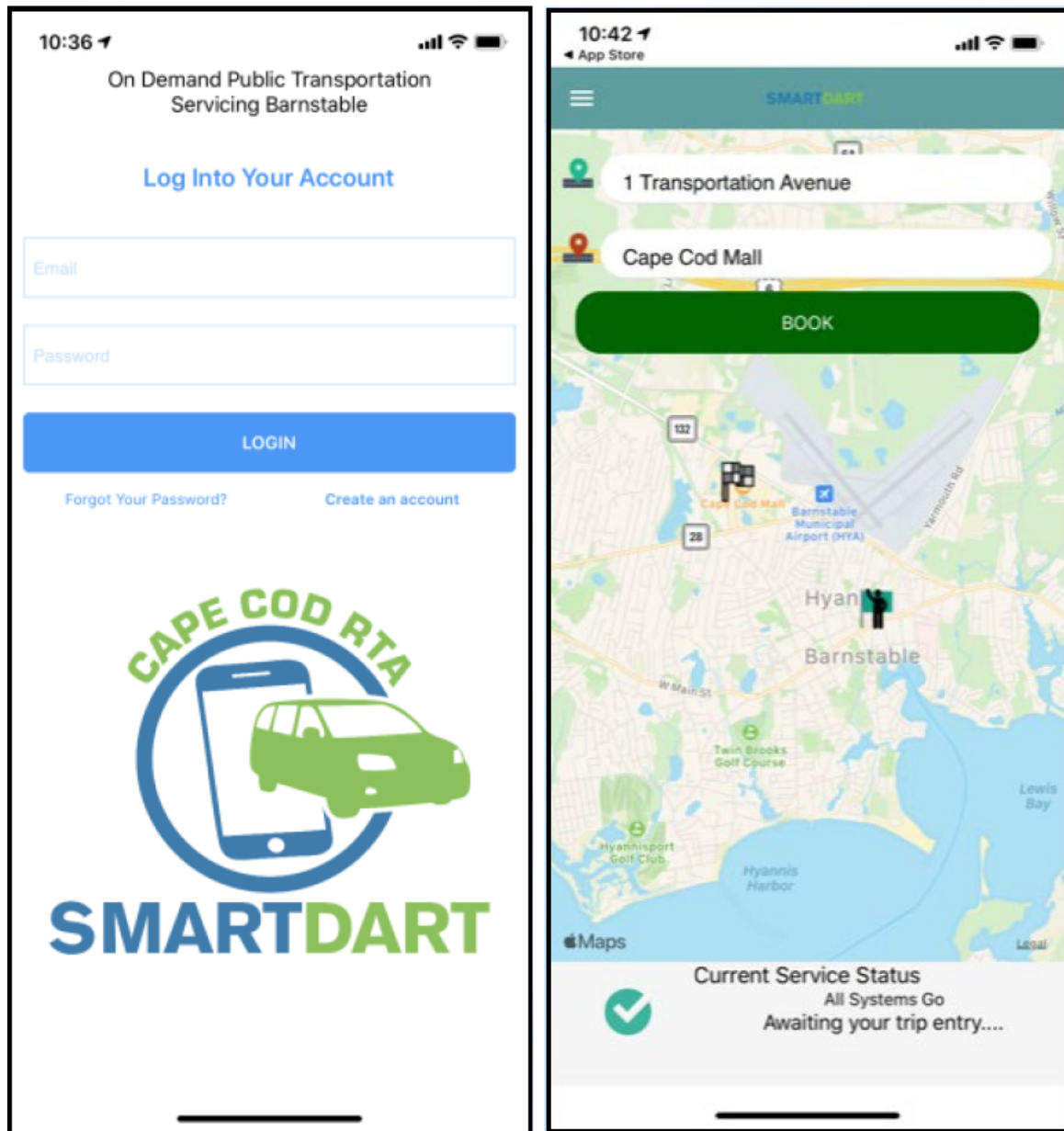
The alternative to purchasing a commercial product for booking and scheduling demand-responsive trips is to develop an app and supporting software in-house. Development of an app requires some domain-specific skills and knowledge, but it can be customized to collect exactly the data of interest to the agency and to allow deployment of a service without any limitations from commercial software providers.

Of the six microtransit pilots funded through MassDOT's RTA Discretionary Grant Program, two use an app that was developed in-house. MetroWest Regional Transit Authority (MWRTA) uses an app developed by Daniel Fitch, which he developed for the agency, and Cape Cod Regional Transit Authority (CCRTA) has adopted the same app. The MWRTA system is called CatchConnect, and the CCRTA system is called SmartDART. Screenshots of the app, as implemented for CCRTA's SmartDART system, are shown in **Figure 5.1**.

Several functionalities are integrated into the app to facilitate the necessary transfer of data for operation of the microtransit service (D. Fitch, personal interview, March 3, 2021). These functions include:

- **Trip Booking.** Customers log into the app and provide information about their location and their requested destination. The app includes Google autocomplete to assist with identifying common destinations and pinpointing the address of each customer's requested destination.

- **Vehicle Tracking.** The app tracks the locations of microtransit vehicles in real time and uses the tracking information to identify the available vehicle to which each requested trip is assigned. The vehicle tracking also allows the app to calculate estimated time of arrival for each vehicle to assigned pickup and drop-off points.
- **Automated Dispatch.** As trip requests are made, the app assigns each trip to an available vehicle. With the low demand rates and public health precautions during the coronavirus pandemic, vehicles are serving one customer at a time, but the app can schedule shared rides when the opportunity exists. The app also has the ability to queue requested trips when there is insufficient capacity to serve the trip immediately upon request.
- **Routing.** Once trips have been assigned to each vehicle, the app is use to identify shortest paths between points and to communicate directions to the microtransit vehicle driver.
- **Ongoing System Monitoring.** In addition to the data and computation required for real-time operations, the app logs data that is useful for ongoing monitoring and necessary for reporting. Information such as requested trip records and actual pickup and drop-off times are recorded, which can also be used to calculate the waiting time that each customer experienced from the time they requested service to the time they were assigned a vehicle for pickup, then the waiting time until the vehicle arrived for pickup, and finally the travel time experienced in the vehicle until drop-off. Other data related to the clients, such as records of new microtransit clients, the total number of clients, and the revenue collected, are also relevant useful data that are collected.



**Figure 5.1: Screenshots of app used by CCRTA SmartDART for customers to request microtransit service**

Although it took longer to develop than purchasing a commercial platform, the development and use of the in-house app allows MWRTA and CCRTA to retain complete control of the data and functionality of the system. The rationale for developing an in-house app for booking, operating, and monitoring are the following (D. Fitch, personal interview, March 3, 2021):

- **Maintain Flexibility.** By developing the app specifically for MWRTA and then CCRTA, the tool remains in the control of the agencies and can be changed at any time to suit their needs. Although commercial providers to offer support, the functionality of their software products is not usually changed easily.

- **Integrate with Existing Services.** Since the microtransit service operates alongside existing fixed-route services, the app allows for integration of transit location and schedule information across multiple services, such as fixed-route services. This is particularly useful for customers who are not intricately familiar with the existing transit system, because an integrated way-finding service can help users understand what travel options suit them best for their particular circumstances.
- **Maintain Control of Operations.** Adoption of a commercial software tool for booking trips and dispatching vehicles can lock a service operation into tools that are within the provider's control. This can have costly consequences down the road if it is not possible to make certain changes to operating strategies because of the way that the commercial products limit the data that are collected and the ways that vehicles can be scheduled and routed.
- **More Flexible Budgeting.** Development of an app can be supported with capital funds, which are easier to procure than operating funds, which would be necessary for paying ongoing fees associated with a commercial product. This helps to free up the operating budget for the costs that are more directly related to operations, allowing limited funds to be stretched further.

### **5.3 Data for Ongoing Monitoring**

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Finally, it is important to collect data for ongoing system monitoring, some of which is required for reporting to the Federal Transit Administration (FTA). The value of ongoing monitoring data is that the data allow an agency to track performance of a flexible transit system relative to its own benchmarks. It may be that the most useful comparison is with a fixed-route system that the flexible transit service replaces, or it may be that the performance of a given system is tracked over time.

The following are examples of performance measures that are relevant to monitoring flexible transit services from the supply side.

**Trips per Revenue Hour.** The number of trips that are served per revenue hour of vehicle operation is an indicator of how intensively each vehicle is utilized. With greater density of demand, there are increased opportunities for vehicles to serve multiple customers together.

**Trips per Revenue Mile.** Another metric of productivity is the number of passenger trips that are served per distance that each vehicle travels. This metric provides a combined indication of the distances that passengers are traveling and that vehicles travel empty to serve the next passenger pickup.

It is also useful to track performance metrics that reflect the experience of system users. Examples of relevant performance metrics for transit users include the following.



**Average Wait Time.** This is the average time that customers must wait to be picked up from the time that they request service. In an advance reservation system, this is the difference between the preferred pickup time that customers request and the time that they are actually picked up. For a real-time system, this is the time from when customers submit a service request to when a vehicle picks them up.

**Average In-Vehicle Travel Time.** The time that passengers riding within the vehicle is a measure that reflect the circuitry of the transit service. A fixed-route system, for example, typically requires passengers to wait for many intermediate stops while a transit vehicle serves other customers. In a flexible transit system, there may be delays associated with deviating the route to pick up or drop off other customers. In cases that customers are served one at a time, this travel time would likely converge toward the travel time by car.

**Average Vehicle Occupancy.** This is the number of transit passengers that are onboard at any given time. While increased occupancy is an indication that the vehicles are being used more intensively, it also reflects greater crowding. The benefits of serving more passengers per vehicle are already measured in the supply metrics listed previously. The potential disbenefit of more circuitous routes associated with deviations and stops to serve other customer is measured in the average in-vehicle travel time. What remains is a measure that reflects the discomfort of crowding. In light of the coronavirus pandemic, this may also be interpreted as a measure of public health risk.

**Average Fare Paid.** For services that charge different fares depending on the type of customer or the distance traveled, the fares that are paid is a metric that represents the transfer of funds from users to the agency.

## 6.0 Conclusions

Flexible transit can take a variety of forms, but the principle of adjusting transit operations to serve demand provides opportunities to provide more efficient and higher-quality transit service. This study involved the development of models to systematically compare different structures of flexible transit service to identify the characteristics of areas in which flexible transit outperforms fixed-route service. This was followed by an analysis of microtransit pilots that were implemented by RTAs across Massachusetts, each with some unique characteristics but all operating as fully demand-responsive services. Finally, an analysis of the data requirements for planning, implementing, and monitoring a flexible transit or microtransit service was conducted to summarize the data needs and identify lessons learned that can be of use for future deployments.

### 6.1 Literature Review and Model Development

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The literature review and model development show that there are many flexible transit service models that can reduce costs of transit service when demand is too low to support fixed-route service. This is especially true of areas where the demand is dispersed and at a low enough density that conventional buses cannot operate enough routes or at short enough headways to be an attractive choice for travel. The threshold demand that justifies flexible transit or microtransit depends on the size of the regional service, the distribution of demand in that region, and the characteristics of the travel mode (e.g., vehicle speed, loss time for stops, vehicle capacity, and vehicle comfort). Although the specific tipping point depends on these characteristics, there is a general pattern that demand density below 10 trips/mi<sup>2</sup>/hr indicates an area in which flexible transit can provide mobility to users at a lower generalized cost (travel time and agency operating cost) than conventional fixed-route transit.

### 6.2 Microtransit Pilots

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The implementations of microtransit pilots across Massachusetts involved six RTAs. The pilots were generally viewed as successful, with several RTAs actively working to expand service. In each case, RTAs justified the introduction of a new demand-responsive microtransit service that either replaced underperforming fixed-route service or introduced service to communities with low demand density (e.g., suburban, exurban). Most of the pilots started with relatively small service areas and hours of operation limited to weekdays. The plans for expansion include increasing the size of the geographic area, adding more service areas, and adding more hours of service. For example, GATRA is now operating microtransit in three service areas.

## **6.3 Impacts of Coronavirus Pandemic**

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The coronavirus pandemic suppressed demand for travel, but microtransit continue to serve a critical need in some communities. The lockdown orders that were implemented in March 2020 to protect public health reduced demand for certain types of travel and led many people to avoid public transit altogether. The lower levels of transit demand that coincided with launching the microtransit pilots allowed RTAs to work out the details of apps, scheduling, and marketing with reduced pressure from customers.

Some RTAs continued to operate microtransit through the pandemic as a means to provide ongoing mobility for essential travel. The viability of microtransit during the pandemic showed that it is a resilient transit solution, especially for rural and suburban communities. The nature of flexible transit services are that they adapt to the demand that exists, which allows for services to be scaled back when demand drops and scale back up as it returns. Furthermore, the nature of demand-responsive transit service in low-density communities is that vehicles are typically carrying few passengers at a time, which is beneficial from a public health perspective.

## **6.4 Data Requirements**

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Data are required for planning, operating, and monitoring flexible transit implementations. At the planning stage, the necessary data are related to the service area, demand, and mode characteristics, all of which impact the cost and efficiency of flexible transit service compared to fixed-route service.

For flexible transit service, in which trip requests and vehicle routing is handled in real time, a system is needed to assign trips to available vehicles. This is commonly done using smartphone apps that allow customers to request service and directions to be communicated to drivers. Although there are several commercial products available to support flexible transit operations, these systems typically require ongoing payment for service and support. Data access and app functionality are also restricted to the provider's policies. An alternative is to develop an app in-house, as is used by MWRTA and CCRTA. In-house development requires staff with expertise in programming and app development, and the process can be more time-consuming, but the result is a scheduling tool that is fully customizable and gives the agency full control of data and policies for operations. Therein lies a trade-off that each agency must consider for its own context and needs.

Ongoing evaluation of flexible transit services requires agencies to monitor data that reflects supply side measures and the user experience. Supply side measures reflect the efficiency of the transit operations for moving people. Examples include trips per revenue-hour and trips per revenue-mile of service. Comparisons between regions are not particularly meaningful, because differences in the geography of a region or density of demand can have big implications on the trips served per unit of vehicle operation. It is useful, however, to track performance of a system over time as a way to measure progress, especially as demand for a

service changes over time. Another important set of metrics is related to the experience of users, including wait time, access distance or time, in-vehicle riding time, average vehicle occupancy, and fare paid. These reflect the quality of service to users and provide a useful measure of the attractiveness of flexible transit compared to fixed-route transit or other modes.

Overall, the study shows potential for flexible transit solutions to serve customers with more cost-effective and faster service in a variety of contexts, especially in rural and suburban communities. The microtransit pilots that were funded by MassDOT were impacted by the coronavirus pandemic but in all cases showed improvements in transit access for the communities served. The plans to expand microtransit services in several communities are evidence of successful implementation. The pilots also provided valuable experience and lessons learned for other RTAs in Massachusetts to build on for future microtransit and flexible transit implementations.

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