

Six Learning Areas and Student Difficulties

The *Addressing Accessibility in Mathematics* (AAM) group examined current research on student difficulties in mathematics, and analyzed the kinds of tasks students are asked to use in various middle school mathematics curricula. Based on this, AAM identified six areas in which students' strengths and needs strongly affect mathematics learning. The lists that follow detail the types of tasks commonly required in the six areas, along with examples of student difficulties. Note that some problems, such as multi-step problems, involve tasks from multiple areas.

The six areas addressed in this document¹:

- Conceptual Processing (page 2)
- Language (page 3)
- Visual-Spatial Processing (page 4)
- Organization (page 5)
- Memory (page 6)
- Attention (page 8)

¹ This list is not all-inclusive. Some other areas (such as psycho-social issues) also affect students' learning of mathematics

Conceptual Processing

Standards-based mathematics emphasizes the need to build a deep understanding of concepts. This involves making connections among mathematical ideas, facts, and skills, and reflecting upon and refining one's own understanding. In middle school, students begin to work with abstract concepts such as variables and linear functions, and make greater use of symbolic representations. Students who tend to think concretely may need additional support in order to move from concrete to abstract representations.

Type of Task	Examples of Student Difficulties
Use or manipulate symbols	<ul style="list-style-type: none"> • Does not always connect symbols with what they represent • Does not remember the meaning of symbols
Solve abstract problems	<ul style="list-style-type: none"> • Does not understand abstract problems easily • Tends to think concretely
Visualize and extend patterns	<ul style="list-style-type: none"> • Has difficulty visualizing and identifying patterns
Make generalizations	<ul style="list-style-type: none"> • Finds it difficult to make generalizations and to write rules • Tends to think concretely
Understand mathematical relationships and make connections	<ul style="list-style-type: none"> • Thinks of mathematics as disparate parts and doesn't see the connections
Learn, represent, and explain new concepts	<ul style="list-style-type: none"> • Tends to think concretely • Focuses on small parts and does not see big picture • Does not identify key points
Apply concepts to new situations	<ul style="list-style-type: none"> • Sees new problems as unfamiliar <p>Does not see a connection between new problems and those he or she has already solved</p>
Self monitor understanding and ask clarifying questions	<ul style="list-style-type: none"> • Lacks a metacognitive awareness of what he/she doesn't understand

Language

In mathematics, students need to describe strategies, explain their reasoning, justify solutions, and make persuasive arguments, both orally and in writing. They need to learn mathematical vocabulary and use it to express ideas with precision and clarity. In class and small group discussions, they need to build on the thinking of their classmates and to ask questions to help them understand another person's strategies.

Type of Task	Examples of Student Difficulties
Read directions and problems	<ul style="list-style-type: none"> • Has difficulty decoding words • Reads slowly • Finds comprehension challenging • Tends to misunderstand directions
Follow verbal directions	<ul style="list-style-type: none"> • Has difficulty with the auditory processing of verbal information • Does not understand verbal directions well
Write explanations of mathematical thinking	<ul style="list-style-type: none"> • Takes a long time to get started on writing tasks • Does not know how to organize ideas • Gets distracted rather than focusing on the writing task • Does not have necessary fine-motor skills for extended writing
Participate in Class Discussions	<ul style="list-style-type: none"> • Does not express ideas orally with ease • Does not listen well to other students' explanations and gets distracted easily
Give Oral Presentations	<ul style="list-style-type: none"> • Is not comfortable speaking in front of class • Speaks slowly

Visual-Spatial Processing

Representing mathematical ideas is key to understanding mathematics. Students use representations to solve problems, explore concepts, and communicate ideas. For example, students use different visual representations for percents, including number lines, fraction circles and hundred-grids. In algebra, students use visual patterns to determine rules, analyze graphical representations of functions, and create mathematical models. Some difficulties with such tasks are caused by a breakdown in the processing of visual information. Students may benefit from such strategies as color-coding systems to help them focus on key information, and from learning explicit strategies for interpreting visual representations.

Types of Tasks	Examples of Student Difficulties
Create and interpret visual representations	<ul style="list-style-type: none"> • Has difficulty representing mathematics concepts visually • Does not connect graphics to the concepts they represent <p>-----</p> <p>Finds it difficult to visualize and represent a three-dimensional model in two dimensions</p> <p>Finds it difficult to interpret a two-dimensional representation of a three-dimensional model</p>
Work with tables and graphs	<p>Has difficulty figuring out how to create tables or graphs or has difficulty physically creating them</p> <ul style="list-style-type: none"> • Has difficulty reading or interpreting graphs
Read text	<ul style="list-style-type: none"> • Cannot read standard-size text
Read handouts and book pages	<ul style="list-style-type: none"> • Finds crowded pages distracting • Has difficulty focusing on the important information • Finds extraneous material distracting <p>-----</p>
Copy or read information displayed on a blackboard, chart, or overhead	<ul style="list-style-type: none"> • Does not see board well • Does not know where to focus

Organization

Problem solving is integral to mathematical learning. The NCTM Problem Solving standard states that "students should have frequent opportunities to formulate, grapple with and solve complex problems that require a significant amount of effort." (NCTM, 2000) Complex problems make many organizational demands—students must figure out how to get started; carry out a sequence of steps; keep track of the information from prior steps; monitor their own progress and adjust strategies accordingly; and present solutions in an organized manner. Further, they must organize their time to ensure that they neither rush through tasks and make careless errors, nor spend too much time and yet not complete the task.

Type of Task	Examples of Student Difficulties
Solve multi-step or complex problems	<ul style="list-style-type: none"> • Has a hard time getting started • Does not know how to figure out a sequence of steps for solving the problem • Rushes through tasks or spends excessive time • Does not answer all of the questions or all parts of the investigation
Make a table, graph, chart, number-line, spinner, or map	<ul style="list-style-type: none"> • Gets confused by the multiple steps involved in making a table, graph, etc.
Collect and record data	<ul style="list-style-type: none"> • Records data in a disorganized manner that is difficult to analyze • Has difficulty organizing data into tables
Find information in prior student work	Does not organize class notes well and thus has trouble finding the needed information
Complete long-term assignments or projects	<ul style="list-style-type: none"> • Has difficulty organizing a large project • Works slowly or spends an excessive amount of time • Does not manage project resources well • Needs help breaking a large task into steps

Memory

Memory plays an important role in learning mathematics. For example, students use their memories to perform calculations and procedures, identify geometric figures, and create graphs that have all of the necessary parts.

Short-term memory can only hold small amounts of information for a brief amount of time (seconds). After information enters short-term memory, you need to decide what to do with it. Short-term memory difficulties can affect a student's ability to copy info from the board, to take notes, follow multi-step directions, follow a presentation, or add to a class discussion.

Working memory involves holding information in mind while you use it, such as performing mental calculations for a multi-step problem. Working memory holds information for seconds to hours and then may move some information into long-term memory. Students may lose track of steps when solving a multi-step problem and have difficulty remembering what they are reading.

Long-term memory involves storing information for long-term use so that it can be retrieved later. Students with long-term memory deficits may not easily store information in memory, or may have difficulty retrieving information. Long-term memory difficulties can affect students' abilities to remember math facts with accuracy and fluency, to use mathematical vocabulary and to make connections among concepts that they have learned in prior months or years.

Automaticity: rapid recall of information with little or no effort. This is linked to fluency.

Type of Task	Student Difficulties
Use math facts with accuracy and fluency	<ul style="list-style-type: none"> • Has difficulty memorizing or recalling basic facts • Retrieves incorrect facts • Lacks fluency: recall of facts is slow and inaccurate
Carry out algorithms and procedures	Does not remember sequence of steps in an algorithm or procedure
Perform mental calculations	Cannot keep the steps of a calculation in his or her working memory
Solve multi-step problems	Does not have needed information in his or her working memory to solve a problem Loses track of where they are in solving a multi-
Use previously-taught skills and concepts	Does not remember skills and concepts that were taught earlier in the year or in previous years
Use math vocabulary	• Has difficulty remembering math vocabulary
Show what they know on assessments such as quizzes and tests	<ul style="list-style-type: none"> • Performs poorly. • Tends to do better on class work than on assessments. • Works slowly due to difficulty recalling information
Follow multi-step directions	• Forgets directions and is unsure what to do
Read and understand word problems	• Forgets what he/she is reading
Write math explanations	<ul style="list-style-type: none"> • Difficulty remembering math vocabulary terms to use • Difficulty holding multiple pieces of information in mind in order to write a multi-step explanation
Take notes	• Notes are inaccurate and/or are missing information

Answer questions in class discussions	<ul style="list-style-type: none">• Responds slowly Anxious about making errors due to memory retrieval problems
Recall information about math concepts and procedures	<ul style="list-style-type: none">• Difficulty finding information in long-term memory• Retrieves incorrect information

Attention

In middle school, the increasingly complex math content and tasks lead to demands for longer attention spans from students. They need to complete multi-step investigations and long-term projects, pay attention to details, and complete tests and assessments, often within limited time. Students have to listen to directions and explanations, filter out extraneous information, participate in class discussions, and work effectively by themselves.

Type of Task	Examples of Student Difficulties
Attend to Directions and Presentations by Teacher and Other Students	<ul style="list-style-type: none"> • Inattentive • Tunes in and out Misses important directions and information. Make errors and has gaps in knowledge due to inattention <ul style="list-style-type: none"> • Difficulty sitting still
Work on Math Tasks	<ul style="list-style-type: none"> • Difficulty sustaining attention for long repetitive tasks. • Low productivity and difficulty completing class work and homework • Easily frustrated and has low persistence. • Responds to failure with decreased effort.
Solve Math Word Problems	<p>Makes frequent errors, especially when problems include irrelevant information</p> <ul style="list-style-type: none"> • Misses important details • Jumps in to solve problem without considering different actions. <p>Has difficulties when problems involve working memory, such as problems with mixed operations</p> <ul style="list-style-type: none"> • Gets lost in the middle of solving a problem <p>Difficulty categorizing problems by underlying structure. This can make it difficult to transfer prior knowledge to new problems.</p>
Perform Basic Skills Calculations	<ul style="list-style-type: none"> • Lower computational fluency than peers <p>Difficulty sustaining attention during repetitive tasks could contribute to difficulties automatizing basic computational skills</p> <ul style="list-style-type: none"> • Tends to make more errors on rote or overly familiar tasks.
Management of Time & Materials	<ul style="list-style-type: none"> • Difficulty with time management: working too quickly or too slowly • Difficulty finding materials and tendency to lose things at school • Difficulty finding completed homework (and thus not turning it in)