MASSACHUSETTS
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Mathematics Lesson Plan

Getting Into Gear

Classroom:	Teachers:	Subject: Mathematics	Dates:
Grade: 6	Unit:		
State Standard(s):	Massachusetts Curriculum Framework for Mathematics 2017 - 6.RP.2.a: Make tables of equivalent ratios relating quantities with whole-number measurements. Find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.		
Objective:	Students will gain understanding of 1) how bicycle gears work, 2) the relationships between the sizes of front and rear gears, and 3) calculating gear ratios as related to speeds of the bicycle wheels.		
Key Vocabulary Terms:	gear tooth torque		
Lesson Outline: Students will observe different bicycle gear ratios in action, either through an educational video or (if available) by observing how an actual geared bicycle works.			Student groupings:
Accommodations and Modifications:	Bicycle (optional but recommended). In lieu of a real bicycle, a video (linked below) from the Corporation for Public Broadcasting- "Bianca gets into Gear"- illustrates the relationship between bicycle gears.		

Activity

1. Read the following to your students:

> *"Have you ever ridden a bicycle with multiple gears? In this activity, we will consider how mathematics can help us understand how gears work. This video from Cyberchase is about a girl named Bianca who buys a new bicycle and learns about gear ratios."*

2. Play the video here

    a. *Alternative if a bike is available:* Put the bike into the lowest gear (the chain will be on the *smallest* gear in the front and the *largest* gear in the back), lift the rear wheel off the ground from the seat and rotate the pedal one full revolution. There will not be much resistance, and the bike will not move but the rear wheel should spin a little, have the kids count out loud the number of seconds the bike wheel turns. Then put the bike into the highest gear (*largest* gear in front, *smallest* gear in back) and rotate the pedal one full revolution, there will be more resistance and the rear wheel should spin a lot. Have the kids count out loud the second until the wheel stops.

3. Ask the kids: *"How does the number of teeth on a gear affect the speed of the wheels?"*

Ideally, we are looking for students to note a distinction between the front gear and back gear. Students should take away that it is not the absolute number of teeth on either gear, but the *ratio* of teeth from one gear to another that is important.

At this point, the key takeaway from the discussion can be written on the board:

Front Gear: **more** teeth = **more** wheel spin, **less** teeth = **less** wheel spin
Back Gear: **more** teeth = **less** wheel spin, **less** teeth = **more** wheel spin

> *"A ratio is a relationship between two numbers indicating how many times the first number contains the second. For example, if a front gear contains eight teeth and the rear gear contains six teeth, then the ratio of front teeth to rear teeth is eight to six, notated as, 8:6, which is equivalent to the ratio 4:3."*

4. Ask the students to complete page 2 of the handout.

5. Explain conversion of ratios into decimals:

"A ratio of 30:12 can be reduced to 5:2. This can also be expressed in decimal notation at 2.5 by dividing 5 by 2."

"Complete the second activity by dividing the larger number in the ratio by the smaller number and putting all the decimals in the fourth column. We will call this these decimals the 'speed rank.' The largest numbers will give us faster speed on our bike. Which gear ratios will make the rear wheel go the fastest?"

Getting into Gear

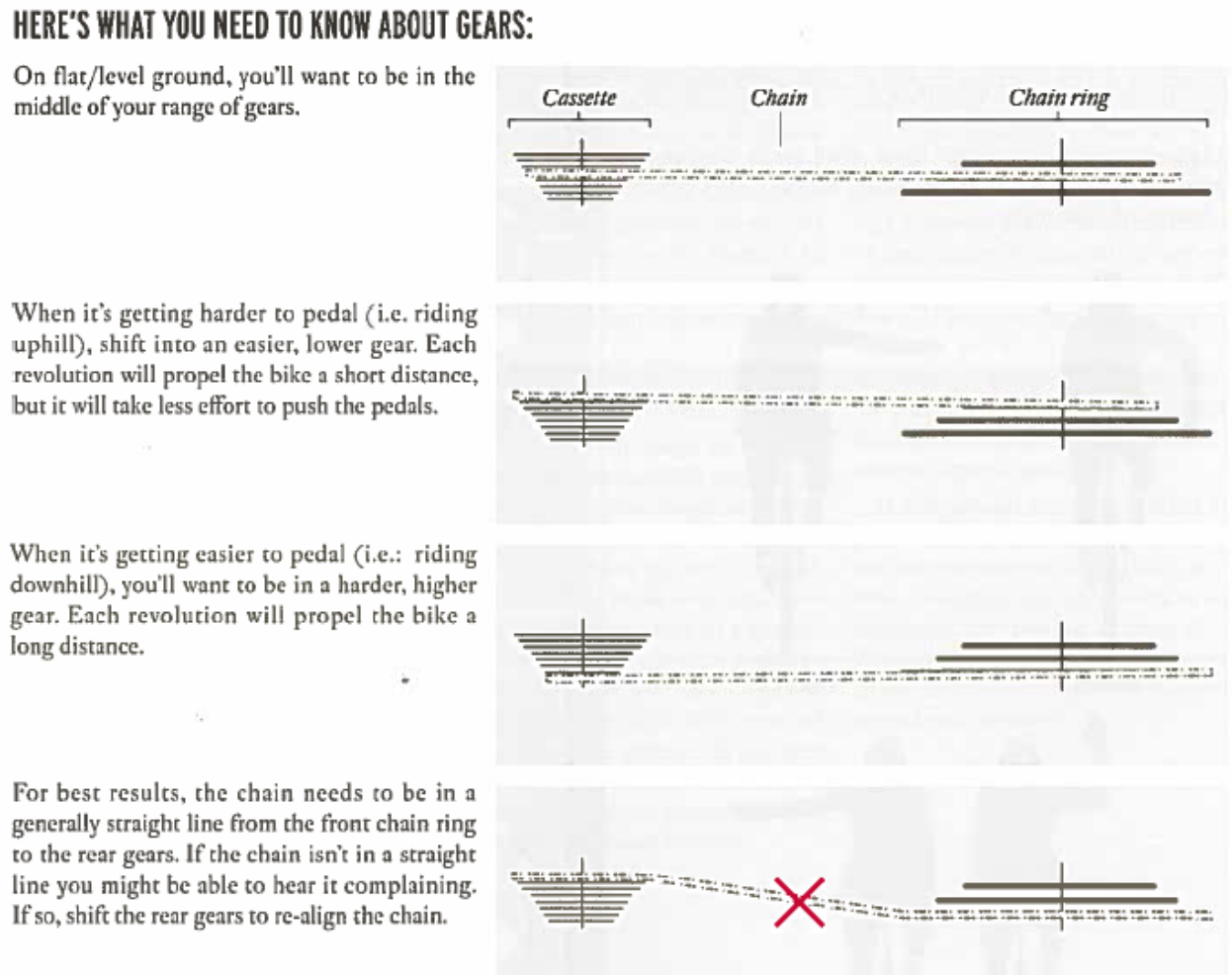

Figure 1. Gear Ratios, from League of American Bicyclists

- The front gears (also called the "chain rings") are attached to the pedals.
- The rear gears (also called the "cassette") are attached to the back wheel.
- The two sets of gears are connected by the bicycle's chain. All the gear teeth are the same size.

Activity #1

Read the following information and then complete the table.

Assume that the bicycle has 2 gears in the front, one of which has 30 teeth and the other 24 teeth. The bicycle also has 5 gears in the back, with 8, 12, 16, 20, and 24 teeth. List all the pairs of possible ratios of gears in the following format: (# of teeth on the front gear, # of teeth on the rear gear). For instance, one ratio would be (30, 12), indicating the 30-tooth front gear connected by the chain to the 12-tooth rear gear.

Enter the values in Table 1 below. The first example is entered already.

Table 1

Number of Teeth in Front Gear	Number of Teeth in Rear Gear
30	**12**

Activity #2

Read the following information and then answer the questions.

If a front gear has 24 teeth and a rear gear has 12 teeth, the gear ratio is equal to 24:12. Therefore, this 24:12 ratio represents a gear ratio of 2:1.

A manufacturer has created a new bike with 2 front gears (56 teeth and 42 teeth), and rear gears of 14, 21, 28, 35, 42, and 56 teeth.

1. How many gear combinations are possible for this new bicycle? Justify your answer.

2. In Table 2, complete the first four columns of the table below with all of the possible gear combinations.

3. Based on your Table 2 data, identify gear combinations that have the same gear ratio.

4. Based on Table 2, identify the 'speed ranking' for each gear combination.

Table 2

Number of Teeth in Front Gear	Number of Teeth in Rear Gear	Ratio of # Front Teeth to # Rear Teeth	Gear Ratio	Speed Ranking

Answer Key

Activity #1

Read the following information and then complete the table.

Assume that the bicycle has 2 gears in the front, one of which has 30 teeth and the other 24 teeth. The bicycle also has 5 gears in the back, with 8, 12, 16, 20, and 24 teeth. List all the pairs of possible ratios of gears in the following format: (# of teeth on the front gear, # of teeth on the rear gear). For instance, one ratio would be (30, 12), indicating the 30-tooth front gear connected by the chain to the 12-tooth rear gear.

Enter the values in Table 1 below. The first example is entered already.

Table 1

Number of Teeth in Front Gear	Number of Teeth in Rear Gear
30	**12**
30	8
30	16
30	20
30	24
24	8
24	*12*
24	*16*
24	*20*
24	*24*

Activity #2

Read the following information and then answer the questions.

If a front gear has 24 teeth and a rear gear has 12 teeth, the gear ratio is equal to 24:12. Therefore, this 24:12 ratio represents a gear ratio of 2:1.

A manufacturer has created a new bike with 2 front gears (56 teeth and 42 teeth), and rear gears of 14, 21, 28, 35, 42, and 56 teeth.

1. How many gear combinations are possible for this new bicycle? Justify your answer.

56:14 56:21 56:28 56:35 56:42 56:56
42:14 42:21 42:28 42:35 42:42 42:56

12 gears

2. In Table 2, complete the first four columns of the table below with all of the possible gear combinations.

3. Identify gear combinations that have the same gear ratio.

56:56 42:42 1:1
56:28 42:21 2:1

4. Based on Table 2, identify the 'speed ranking' for each gear combination.

Table 2

Number of Teeth in Front Gear	Number of Teeth in Rear Gear	Ratio of # Front Teeth to # Rear Teeth	Gear Ratio	Speed Ranking
56	14	56:14	4:1	4
56	21	56:21	8:3	2.67
56	28	56:28	2:1	2
56	35	56:35	8:5	1.6
56	42	56:42	5:3	1.67
56	56	56:56	1:1	1
42	14	42:14	3:1	3
42	21	42:21	2:1	2
42	28	42:28	3:2	1.5
42	35	42:35	6:5	1.2
42	42	42:42	1:1	1
42	56	42:56	3:4	.75

Key Vocabulary Terms

1. **gear:** a toothed wheel designed to transmit torque

2. **tooth:** a projecting part on a tool or other instrument, especially one of a series that function together

3. **torque:** a twisting force that tends to cause rotation