Developing a Metric for the Cost of Green House Gas Abatement

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The authors introduce the levelized cost of carbon (LCC), a metric that can be used to evaluate MassDOT CO₂ abatement projects in terms of their cost-effectiveness. The study presents ways in which the metric can be used to rank projects. The data are summarized from a set of MassDOT projects, and an LCC of the projects is calculated. Findings show that the most cost-effective projects in this data set are dominated by Traffic Operations Projects, when co-benefits are ignored.
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Acknowledgements

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Disclaimer

The contents of this report reflect the views of the authors, who are responsible for the facts and the accuracy of the data presented herein. The contents do not necessarily reflect the official view or policies of the Massachusetts Department of Transportation or the Federal Highway Administration. This report does not constitute a standard, specification, or regulation.
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Executive Summary

This study of Developing a Metric for the Cost of Green House Gas Abatement Project was undertaken as part of the Massachusetts Department of Transportation (MassDOT) Research Program. This program is funded with Federal Highway Administration (FHWA) State Planning and Research (SPR) funds. Through this program, applied research is conducted on topics of importance to the Commonwealth of Massachusetts transportation agencies.

This study introduces a metric that can be used to evaluate MassDOT CO₂ abatement projects in terms of their cost-effectiveness. The metric, called the Levelized Cost of Carbon (LCC), correctly accounts for the time value of money in calculating cost-effectiveness. The authors show that the other commonly used metrics for cost-effectiveness are compatible with this metric under specific assumptions. It is noted that no cost-effectiveness metric is guaranteed to correctly prioritize projects. This problem, however, is theoretical; in practice, the LCC produces fairly robust rankings of projects.

The LCC accounts for the costs of CO₂ abatement projects. In some cases, these costs may reflect only agency out-of-pocket costs. In other cases, the costs reflect net social costs, including quantitative estimates of co-benefits, such as safety, other pollution reduction, and congestion reduction. If MassDOT is facing a decision situation in which it is required to reduce a set amount of emissions above and beyond the emissions reductions achieved by the business-as-usual projects, then considering only agency out-of-pocket costs is appropriate.

Data are summarized from a set of MassDOT projects, calculating the LCC of the projects. Findings show that the most cost-effective projects in this data set are dominated by Traffic Operations Projects, when co-benefits are ignored. Data are also summarized from non-Massachusetts transportation projects and from non-transportation projects. Most of these projects include quantified co-benefits and so are difficult to compare to the MassDOT dataset. However, this dataset may provide fruitful directions for future research.
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# List of Acronyms

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Expansion</th>
</tr>
</thead>
<tbody>
<tr>
<td>FHWA</td>
<td>Federal Highway Administration</td>
</tr>
<tr>
<td>GHG</td>
<td>Greenhouse gas emission</td>
</tr>
<tr>
<td>IRR</td>
<td>Internal rate of return</td>
</tr>
<tr>
<td>LCC</td>
<td>Levelized cost of carbon</td>
</tr>
<tr>
<td>MARR</td>
<td>Minimum acceptable rate of return</td>
</tr>
<tr>
<td>NPV</td>
<td>Net present value</td>
</tr>
<tr>
<td>SCC</td>
<td>Social cost of carbon</td>
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</tbody>
</table>
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1.0 Introduction

This study of Developing a Metric for the Cost of Green House Gas Abatement was undertaken as part of the Massachusetts Department of Transportation (MassDOT) Research Program. This program is funded with Federal Highway Administration (FHWA) State Planning and Research (SPR) funds. Through this program, applied research is conducted on topics of importance to the Commonwealth of Massachusetts transportation agencies.

1.1 Background

Many public agencies, including MassDOT, are faced with making decisions about projects aimed at reducing greenhouse gas emissions (GHGs). The resources available to agencies are limited, both in terms of the ability to invest in GHG projects and in terms of time and resources available for making decisions. This report investigates the theoretical and practical implications of using a cost-effectiveness metric to rank projects.

1.2 Objectives

Chapter 2 discusses alternate decision frameworks that may be used by public agencies. From an academic point of view, there is a “correct” way to evaluate such projects: quantify all costs and benefits and calculate the net present value (NPV) of the projects. All projects with a positive NPV improve social welfare and should be invested in; projects with the highest NPV should be prioritized. In practice, however, this process presents a number of challenges. Many agencies do not monetize all costs and benefits; the concept of monetizing benefits, in fact, is quite controversial and therefore often politically infeasible. Moreover, the benefit of reducing GHGs is itself quite uncertain and often politically infeasible to employ. Finally, many agencies face a slightly different problem: meeting a mandate to reduce emissions at lowest cost. The report discusses how this last problem is related to the goal of improving social welfare.

Despite the challenges of applying cost-benefit or cost-minimization frameworks, public agencies nevertheless face decisions about GHG projects in the face of budget constraints and would like to have some sense of which projects are more cost-effective than others. This study proposes a cost-effectiveness metric, which can be used to evaluate projects and, to some degree, to prioritize them. The report presents the challenges of using a cost-effectiveness metric to rank projects, including cases for which it is guaranteed to produce the highest social welfare and ways in which it can be used in other cases.

Finally, the report discusses how this metric can be interpreted in a cost-minimization framework and demonstrates that it can be used as a prioritization tool in this decision framework as well, again with caveats. The report discusses some of the complexities in the
accurate calculation of these metrics, such as including gasses beyond \( \text{CO}_2 \) and co-benefits of GHG-abatement projects.
2.0 Decision Frameworks

2.1 Social Welfare Perspective

The gold standard in decision making for public agencies is to maximize social welfare. In the case of GHG-reducing projects, this would account for all costs and benefits of the project, including the value of reducing GHG emissions, as well as the time value of money. In Chapter 3, this standard is applied to derive a cost-effectiveness metric. Equation (1) provides a general equation for the NPV of a project:

\[
NPV = \sum_{t=0}^{\tau} \frac{1}{(1+i)^t} (\tau e_t - c_t)
\]

where \(e_t\) is the emissions saved by the project and \(c_t\) is the costs of the project, with the subscript \(t\) referring to the time in each case. The net benefits are discounted in each year, using discount rate \(i\). The value of reducing emissions, \(\tau\), may depend on the context of the decision. For example, if the decision maker is a firm in a market with a cap and trade policy, then the value of reducing emissions would be equal to the emissions price. For a public agency, the value of reducing emissions may be equal to the social cost of carbon (SCC). If the NPV of a project is positive, then it increases social welfare and should be invested in.

However, there are many challenges to using such an approach. One is the deep uncertainty about the value of reducing GHG emissions, including, in some cases, a political aversion to using something like the SCC. Another challenge is that public agencies often do not have access to an unlimited budget for projects; thus, they must choose only a subset of all possible projects that increase social welfare. Finally, in the case of GHG-reduction projects, many agencies are subject to targets for GHG reduction. Section 2.2 discusses this case, providing an alternate characterization of the GHG project selection problem.

2.2 Knapsack Problems

2.2.1. Cost Minimization

Often, decision makers are not faced with choosing the overall optimal projects. It is common in both the private and public sectors for higher-level decision makers to choose a target level of emission reductions. For example, the state of Massachusetts has GHG reduction targets for the years 2020 and 2050, signed into law by Governor Deval Patrick as the Global Warming Solution Act of 2008. Thus, one way of looking at an agency’s decision problem is to frame it as the need to achieve a given climate target at the lowest cost. This problem can be modeled as a cost-minimization problem, or what is often called a “knapsack” problem (1). The idea is that the decision maker has a knapsack equal to the size
of the emission reductions that must be achieved, and the knapsack must be filled at the least cost. Lower-level decision makers are faced with the problem of finding a set of projects that achieve the goal at the least cost. Equation (2) formulates a knapsack problem that is equivalent to the welfare maximization problem shown in equation (1). Let the decision variable \( x_k = 1 \) if project \( k \) is invested in, and 0 otherwise:

\[
\min \sum_k \sum_{t=0}^T \frac{1}{(1+i)^t} x_k e_t^k
\]

\[
s.t. \sum_k \sum_{t=0}^T \frac{1}{(1+i)^t} x_k e_t^k \geq E,
\]

where \( E \) is the goal for emission reductions. The top line shows that the decision maker is trying to choose projects in order to minimize the discounted cost; the second line shows that the discounted emissions need to satisfy a constraint. In order for this to be equivalent to social welfare, the emissions constraint is defined as follows:

\[
E \equiv \sum_{t=0}^T \frac{1}{(1+i)^t} e_t^*
\]

where \( e_t^* \) is the optimal level of emissions reductions from optimizing social welfare using equation (1). Given this formulation, the selected projects will be identical under the two frameworks.

Note, however, that in reality, the optimal values in equation (3) will often not be known. As long as the problem can be formulated as in equation (2), the LCC can be used as a heuristic.

There is a challenge in interpreting the meaning of some stated targets. Many targets, including that for the state of Massachusetts, are stated in terms of emissions (or emission reductions) in a given year. Obviously, the intent is not literally to reduce emissions only in the given year, but rather to achieve a fairly smooth, continuous, and persistent reduction in emissions that goes through the particular target point. The authors assume that the stated target can be translated into a total, discounted emissions reduction target such as \( E \) defined in equation (3).

**Cost minimization with additional constraints.** Often, regulations are written in terms of specific emissions goals in specific years. These can be added onto the problem as additional constraints, as in equation (4). This, however, is not ideal, as it can only increase the cost of implementation without changing the ultimate environmental outcomes. Many regulations also include flexibility: what is often called “banking and borrowing.” These flexibility rules allow an agency to bank emissions reductions if they are higher than those required for that year and apply them to future years, or borrow emissions reductions if they are lower than
those required for that year and make them up in a future year. This flexibility allows an agency to meet an overall emissions reduction goal in the most cost-effective way. With flexibility, the problem can be formulated as in equation (2). Without flexibility, it would be necessary to add constraints, such as the following:

\[
\sum_{k} x_{t} e_{t}^{k} \geq E_{t} \quad \forall t
\]

(4)

where some \(E_{t}\) may be equal to zero.

2.2.2. Fixed Budget

There is another possible way of looking at the problem. An agency may be allocated a fixed budget that is dedicated to reducing emissions. In this case, the knapsack problem would be formulated the other way around: the agency would try to reduce emissions as much as possible while spending the entire allocated budget, as shown in equation (5).

\[
\max \sum_{k} \sum_{i=0}^{T} \frac{1}{1+i} x_{i} e_{i}^{k} \\
\text{s.t.} \sum_{k} \sum_{i=0}^{T} \frac{1}{1+i} x_{i} c_{i}^{k} \leq C
\]

(5)

The authors note that in this formulation, the budget, \(C\), would typically be the financial budget faced by the agency; therefore, the “costs” included in any cost-effectiveness calculation would typically not include wider social costs or social benefits.
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3.0 Methodology

3.1 Derive LCC Using Social Welfare

The authors propose to use a levelized cost per ton of CO\textsubscript{2} avoided. This metric is useful because it can be compared to a carbon tax, the price of carbon in a cap and trade market, or the social cost of carbon (SCC), to determine if a project is worth doing. It is similar to the levelized cost of electricity, in that it correctly accounts for discounting to identify the break-even cost of carbon that makes a project worthwhile (2), (3).

It is well recognized that there is a time value to money, so that money spent now and money spent ten years from now are not equivalent. This needs to play a role in calculating the cost of carbon. For example, imagine two projects, both with a current cost of $1,000. One project reduces CO\textsubscript{2} by 100 tons ten years from now; the other project reduces CO\textsubscript{2} by 100 tons now. One could prefer the second project, since it could be delayed for ten years and produce the same outcome as the first project. Since there is time value to money, the second project delayed ten years is less costly than the first.

Using this logic, the authors developed the formula for the levelized cost per ton of CO\textsubscript{2} avoided. Consider a firm that must pay a fixed fee of \( \tau \) for every ton of CO\textsubscript{2} it emits. The firm is considering a project with annual costs of \( c_t \) and annual emissions reductions (in tons of CO\textsubscript{2}) of \( e_t \) for time periods \( t = 0, \ldots, T \); and has a discount rate, or minimum acceptable rate of return (MARR), of \( i \). Given this setup, each year, the firm would pay \( c_t \) and save \( \tau e_t \), so the value of the project in year \( t \) would be \( (\tau e_t - c_t) \). The net present value of this project is as shown in equation (1).

If \( NPV > 0 \), then this would be a good project, and the firm should invest; if \( NPV < 0 \), then it would not be a worthwhile project and should be avoided.

Note that one can work backward from this equation and solve for the value of \( \tau \), for which equation (1) is exactly equal to zero. If this solved-for value is lower than the value of reducing a ton of carbon, then the project would be good and the firm should invest; the firm would save more in emissions fees than it pays for the project. One can interpret this solved-for value as the levelized cost per ton of CO\textsubscript{2}.

Setting the NPV equal to zero and solving for the resulting value of \( \tau \), the authors find:

\[
\tau = \frac{\sum_{t=0}^{T} \frac{1}{(1+i)^t} c_t}{\sum_{t=0}^{T} \frac{1}{(1+i)^t} e_t}
\]

(6)
Equation (6) shows the levelized cost per ton of CO$_2$ for a project is equal to the NPV of the cost of the project divided by the discounted emissions saved by the project. Note, however, this does not imply that the authors are discounting emissions in the future. It is simply a result of discounting the monetary costs and benefits through time.

Equation (5) was derived under the assumption that the value of reducing a ton of CO$_2$, $\tau$, is fixed across time. There is some controversy about this point, but it appears that the SCC (and, therefore, carbon taxes) may change through time, most likely increasing. The authors would adjust the LCC to account for that, by assuming that the value of reducing a ton of carbon will change at a fixed rate, which they call $\gamma$. The NPV of the project in which the value of reducing a ton of carbon is changing at a fixed rate of $\gamma$ is shown in equation (7).

$$NPV = \sum_{t=0}^{T} \frac{1}{(1+i)^t} \left( (1+\gamma)^t \tau e_t - c_t \right)$$

(7)

Solving for $\tau$ by setting equation (7) to zero results in a levelized cost of carbon metric with the value of reducing a ton of carbon changing at a fixed rate through time.

$$\tau = \frac{\sum_{t=0}^{T} \frac{1}{(1+i)^t} c_t}{\sum_{t=0}^{T} \frac{(1+\gamma)^t}{(1+i)^t} e_t}$$

(8)

This indicates that if $\gamma > 0$ (the value of reducing carbon is increasing in time), the LCC will be lower than when $\gamma = 0$.

Equation (8) can be rewritten as

$$\tau = \frac{\sum_{t=0}^{T} \frac{1}{(1+i)^t} c_t}{\sum_{t=0}^{T} \frac{1}{(1+\beta)^t} e_t}$$

(9)

with $\beta \equiv \left( \frac{1+i}{1+\gamma} \right) - 1$. Hence, the cost is discounted at rate $i$ and the emissions are discounted at rate $\beta$. When $\gamma$ is positive, $\beta$ is smaller than $i$, indicating that as $\gamma$ increases, relatively more weight is put on future emissions reductions.
The authors note here that while the LCC is derived from the social welfare decision problem, it is relevant to the knapsack problems as well. This is discussed in more detail in Chapter 4.

### 3.2 Comparison to Metrics in the Literature

The majority of the literature uses one of two different metrics, typically called “cumulative lifetime cost-effectiveness” and “annualized lifetime cost-effectiveness,” respectively. In a survey of 33 papers, Kok, Annema, and Wee (4) found that 42% used the first, while 18% used the second; the remaining 27% used more ad hoc methods, which they categorized as “anticipated market penetration effect.” These ad hoc methods are scenario-dependent and therefore less relevant for the purpose of this study.

The cumulative lifetime cost-effectiveness, $\tau_{\text{cum}}$, is defined as follows:

$$
\tau_{\text{cum}} = \frac{\sum_{t=0}^{T} \frac{1}{(1+i)^t} c_t}{\sum_{t=0}^{T} e_t}
$$  

(10)

The annualized lifetime cost-effectiveness, in which the emissions reduction, represented here as $e$ with no subscript, is assumed to be constant each year, is defined as

$$
\tau_{\text{ann}} = \left[ \frac{\sum_{t=0}^{T} \frac{1}{(1+i)^t} c_t}{(1+i)^T - 1} \right] \left[ \frac{i(1+i)^T}{(1+i)^T - 1} - 1 \right]
$$  

(11)

In equation (11), the first bracket represents the NPV of the costs, and the second bracket is the factor that converts an NPV to an equivalent annual value.

The metric presented in this report encompasses both of these metrics. If the rate of increase in the value of reducing a ton of CO$_2$, $\gamma$, is exactly equal to the discount rate $i$, then the LCC is equal to the cumulative lifetime cost-effectiveness metric. If the rate of increase in the value of reducing a ton of CO$_2$, $\gamma$, is zero, and emissions reductions are assumed to be equal each year, then the LCC is equal to the annualized lifetime effect. The analytical proof of this second point is provided in Appendix 7.1.

In order to illustrate the differences between the LCC and the existing metrics in the literature, the authors defined three simple projects, each of which reduces the same total amount of CO$_2$ over a period of ten years. However, the projects have different emissions profiles through time, with one increasing, one decreasing, and one constant, as shown in the second, third, and fourth rows of Table 3.1. In all the projects, the initial investment cost is
$3,500 million, the discount rate is 5%, and there is an annual cost savings of $400 million. The application of these metrics to a real project is presented in the last row of Table 3.1.

### Table 3.1: Comparison of different cost-effectiveness metrics

<table>
<thead>
<tr>
<th>Projects</th>
<th>Emissions Reduction (MMtCO₂)</th>
<th>Total Emissions Reduction (MMtCO₂)</th>
<th>Cost Effectiveness Metric ($/tCO₂)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Year</td>
<td>Cumulative</td>
<td>Annual</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>Increasing</td>
<td>0.01</td>
<td>0.09</td>
<td>0.19</td>
</tr>
<tr>
<td>Constant</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Decreasing</td>
<td>0.19</td>
<td>0.11</td>
<td>0.01</td>
</tr>
<tr>
<td>Transportation &amp; Land Use (SC)</td>
<td>0</td>
<td>0.8</td>
<td>3.8</td>
</tr>
</tbody>
</table>

Whereas the two other metrics value each of these projects the same way, the LCC differentiates by accounting for the time value of money. The cumulative lifetime and annualized lifetime have an equal cost per ton of CO₂ value in all three projects defined in Table 3.1, as they do not account for differences in the emission reduction path. Note that the cumulative lifetime metric will always be less than the annualized lifetime metric. The LCC values were calculated using fixed values for the change in the value of reducing a ton of CO₂ of 0, 0.02, and 0.05 respectively. When \( \gamma \) is equal to the discount rate \( i \), here 0.05, the LCC and cumulative metric have the same values. The key point is that cumulative and annualized lifetime methods ignore the emissions path, which does not make sense if there is a time value to money.

In the fifth row of Table 3.1, the metrics are applied to the data of an aggregate project labeled “Transportation and Land Use” in the final report of the South Carolina Climate, Energy, and Commerce Committee (5). This aggregate project runs from 2007 to 2020. The report does not present the full emissions path, but it does specify emissions in 2012 and 2020. The emissions path was estimated by assuming that emissions increase linearly between 2007 and 2020, resulting in total emission reductions of approximately 29.3 MMtCO₂. The NPV of the cost of this project is reported to be $2.582 billion. Note that the different methods result in different amounts, and that both the cumulative and the annual metrics appear to overestimate the cost-effectiveness of this project. This is because the emissions reductions increase over time.
3.3 Applying the LCC Theory

3.3.1. Social Welfare Framework

Yes or No Decision: Compare with SCC or Tax
In order to use the LCC, one must compare it to some kind of benefit of reducing carbon. If a firm must pay a fee of \( \tau^* \), then the benefit to the firm of reducing carbon is equal to \( \tau^* \) per ton of carbon. A project would be acceptable if the LCC were less than the fee \( \tau^* \). In the absence of a carbon fee or price of some kind, government entities would need to consider the societal value of reducing carbon. The most commonly used concept to measure this is the SCC, which captures the impacts of climate change over time. Theoretically, it is the economic value of the discounted damages through time caused by a ton of CO\(_2\) emitted (6).

If the LCC of a project is less than or equal to the SCC, then that project is cost-effective; if it is greater, then the project’s costs are higher than its benefits and it should not be adopted.

However, while the SCC is useful conceptually, there are difficulties in calculating an actual value (2), (7). This is addressed further in the next section. Moreover, the costs, \( c_t \), would have to account for the entire net social cost of the project; they would need to account for all social benefits, aside from carbon benefits. For example, if the project under consideration were a new roundabout, the estimated costs would need to account for any costs or benefits associated with changes in congestion, safety, and other pollutants.

Prioritizing Projects: Challenges
While the LCC is valid to determine whether a particular project is cost-effective or not, it faces a weakness when used to compare projects. Namely, there are cases in which a project with a higher LCC will nevertheless be the preferred project with a higher NPV. To be precise, let \( PV_k(\tau) \) represent the value of project \( k \) given SCC of \( \tau \):

\[
PV_k(\tau) = \sum_{t=0}^{\infty} \frac{1}{(1+i)^t} \left( \tau e_i^k - c_i^k \right)
\]

(12)

where the superscript \( k \) indicates the emissions and costs for project \( k \). Let the LCC for project \( k \) be referred to as \( \tau_k \) and calculated according to equation (5). The problem is that there exist projects A and B, and SCC of \( \tau^* \) such that:

\[
\tau_A < \tau_B \\
But \\
PV_A(\tau^*) < PV_B(\tau^*)
\]

(13)

That is, if the LCC were used, one would think that project A was better than project B; however, given an SCC of \( \tau^* \), project B is in fact better.
For example, consider the four projects described in Table 3.2. They are listed in order of increasing cost and increasing emissions saved. If the LCC was simply used to prioritize them, then project 1 would appear to be the best project, followed by projects 3, 4, and 2, in that order.

Table 3.2: Four illustrative projects

<table>
<thead>
<tr>
<th>Project</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>NPV of Cost ($)</td>
<td>50</td>
<td>100</td>
<td>150</td>
<td>200</td>
</tr>
<tr>
<td>NPV of Emissions Saved (tons)</td>
<td>4</td>
<td>6</td>
<td>10.1</td>
<td>13</td>
</tr>
<tr>
<td>LCC ($/tCO₂)</td>
<td>12.5</td>
<td>16.7</td>
<td>14.9</td>
<td>15.4</td>
</tr>
</tbody>
</table>

However, Figure 3.1, which shows the NPV of each of these projects using specific values for the SCC, tells another story. Project 1 (blue line) has the highest NPV when the SCC is below 16.5; between 16.5 and 17, project 3 (gray line) is highest; and project 4 (orange line) has the highest NPV thereafter.

Figure 3.1: NPV of projects as function of SCC

What is going on? To understand why the LCC does not always predict the project with the highest NPV, the authors use an analogy, the well-known internal rate of return (IRR). The IRR is used by many firms to evaluate projects, yet it cannot correctly be used to compare projects. This is because the IRR implicitly (and incorrectly) assumes that any additional funds are being reinvested at the IRR, rather than the MARR. Similarly, LCC (or any other cost per unit of carbon) implicitly (and incorrectly) assumes that any additional tons of carbon that can be reduced are valued at the LCC rather than at the SCC.
This problem is fundamental. There is no way to ensure that one project is preferred to another without considering the appropriate SCC. There is no possible metric that can accurately prioritize projects in the absence of the SCC. Chapter 5 discusses some possible approaches to using the LCC to prioritize projects in the absence of an agreed-upon value for reducing carbon emissions.

3.3.2. Knapsack Framework

This section discusses how the LCC can be used within a knapsack framework. There is a well-known “heuristic” for solving knapsack problems: choose projects in terms of cost-effectiveness until the constraint has been satisfied. This heuristic would imply that projects could be chosen by comparing LCCs, in either of the frameworks discussed previously in Section 2.2. This method is a heuristic, meaning that it is a method that gives a potentially reasonable answer, but it is typically not guaranteed to give the optimal solution. It is, however, well known to give quite good solutions most of the time. Moreover, the solutions get better as the size of the problem grows with respect to the size of the individual projects. Specifically, define the error to be the fractional increase in cost over the optimal solution,

\[ \varepsilon = \frac{c - c^*}{c^*}, \]

where \( c \) is the cost of the heuristic solution and \( c^* \) is the cost of the optimal solution. Then, the worst-case error has the following characteristic (8), as shown in equation (14):

\[ \varepsilon \leq \frac{E_{\max}}{E_{\max} + E} \]

where \( E_{\max} \) is the discounted emissions of the largest candidate project and \( E \) is the emissions reduction goal in equation (3). Equation (14) shows that the worst-case error gets smaller as \( E \) increases with respect to \( E_{\max} \).

Sometimes, the heuristic of choosing projects in order of their cost-effectiveness results in the exact answer. If projects are fully scalable, then in a knapsack framework, one would only ever choose the project with the lowest LCC and scale it to meet the constraint. This will always have the lowest cost. If the projects cannot be scaled up but can be scaled down, then this is a “fractional” problem, and it can be solved exactly by choosing projects in order of the LCC (9). However, like in the social welfare problem, if projects cannot be scaled, then a lower LCC is not generally a guarantee that a project is preferred. For example, consider the problem in Table 3.2. If one has a constraint to reduce emissions by 20 tons, then the optimal solution is to choose projects 1, 2, and 3, for a total cost of $300 and a total emissions reduction of 20.1. One could choose project 2, even though its LCC is higher than that of project 4, because it helps to hit a specified goal at a lower cost. Finally, additional constraints, for example requiring specific emissions reductions in a specific year, will make the solution less cost-effective. In this case, even if all projects are scalable, prioritizing in order of the LCC is not guaranteed to provide the optimal solution.
3.4 Recommendations for Using LCC

This section presents recommendations for how and when the LCC can be used for comparing and prioritizing projects. In addition, the authors present a methodology for communicating with decision makers, namely a break-even SCC, defined as follows. Figure 3.2 illustrates this discussion.

Figure 3.2: Application of LCC with respect to decision types

First, as Figure 3.2 shows, regardless of the decision framework, if projects are scalable, or if they have the same discounted emission reductions, then the LCC will always compare correctly, regardless of the SCC. Scalable means that a project can be linearly scaled to reduce any amount of emission, then projects can be scaled to the same size, and the LCC works, for both the social welfare and knapsack frameworks. Third, if a project is both smaller (in terms of NPV of emissions) and has a higher LCC, then it will never be preferred in a social welfare framework (see Appendix 7.2 for proof). An example of this can be seen in Table 3.2 and Figure 3.1: project 2 will never be preferred to projects 3 and 4 if projects are scalable.

This last result suggests that a Pareto analysis can be done using LCC and total discounted emissions. In considering any group of alternatives that are evaluated under multiple metrics, one can identify alternatives that are “Pareto-dominated.” An alternative is Pareto-dominated if it is worse under all metrics than another alternative. In this case, a project is Pareto-
dominated if it has a higher LCC and lower discounted emission than another project. The set of projects that are non-dominated is called the “Pareto frontier.”

For this analysis, a break-even point between any two adjacent projects on the Pareto frontier can be calculated. The break-even SCC is the SCC at which decision makers are just indifferent between two projects. This point is obtained by finding the SCC that causes $PV_1 = PV_2$. Let project $i$ and $i + 1$ be adjacent on the Pareto frontier, with project $i + 1$ reducing more emissions. Then the break-even point is:

$$
\frac{C_{i+1} - C_i}{E_{i+1} - E_i}
$$

This can be used to communicate to decision makers. For example, Figure 3.3 shows projects 1, 3, and 4 from Table 3.2 on the Pareto frontier, and it reports the break-even SCC values between the projects. This figure illustrate the LCC of the projects versus the discounted emissions reduction The line connecting projects is a theoretical illustration of Pareto frontier: projects that lie above the line are dominated; if a new project were conceived of that was below the line, it would dominate at least some of the current projects. Regarding the break-even points, if, in this example, a decision maker feels confident that the SCC is larger than $18/t\text{CO}_2$, then he or she can confidently prioritize project 4.

![Figure 3.3: Pareto frontier of projects in Table 3.2](image)

In Appendix 7.3, a plausible range of values for the SCC, ranging between $11$ and $207$ per metric ton of CO₂-eq, is shown. If a project is preferred under this entire range, then the LCC can be used with confidence in the short term.

In summary, as depicted in Figure 3.2, the LCC can be applied for both investments in social welfare and fixed constraints. The LCC can be used as an approximation under any of the decision frameworks described in this chapter. It will provide the optimal prioritization in
cases where the projects are scalable, while it is an approximation in other cases and may result in choosing some projects that are suboptimal.

Note that none of the methods suggested here, save for full cost-benefit analysis, requires knowledge of the SCC or any other value of the benefit of reducing carbon emissions. The Pareto analysis does require the ability to define a plausible range for the SCC.

### 3.5 Other Considerations

This section discusses some other considerations when using the LCC or any other related cost-effectiveness metric. Two of the most important considerations are (1) other GHGs besides CO$_2$; and (2) the definition of costs.

In transportation, CO$_2$ is by far the most important GHG, so the authors have focused on that. But in some applications, other gasses are equally or more important. Incorporating multiple gasses adds new complexities, since different gasses have different lifetimes and different warming effects (10). The most widely adopted methodology is to calculate emissions in terms of CO$_2$ equivalences.

Of more central importance to MassDOT is the issue of how to calculate the costs, $c_t$, of GHG-reducing projects. First, costs can be borne by different parties. When considering social welfare, all costs should be included, regardless of who bears them. But in practice, many agencies only consider costs to themselves and not to other parties. Second, ongoing costs should be included, not just initial investment costs. Again, in practice, some agencies don't do a good job estimating ongoing costs and focus primarily on upfront investment costs. Finally, the costs in equation (1) should be net social costs: the social cost minus social benefits. Benefits may be direct financial benefits, such as a reduction in energy or maintenance costs from adopting electric vehicles. But they would also include all the social benefits of a project. For example, replacing an intersection with a roundabout might have benefits in terms of congestion reduction and safety. To be accurate, such benefits should be monetized and subtracted from the costs when calculating the net costs of a GHG-reduction project. This is extremely difficult to do in practice. It provides a significant challenge in comparing the cost-effectiveness of different projects, since some agencies include estimates of these social welfare benefits in their cost-effectiveness calculations and others do not. Projects that consider the social welfare benefits (what can be called co-benefits, in the case of GHG-reduction projects) often show a negative cost-effectiveness. What this means is that the project is social-welfare maximizing, even in the absence of climate change. These projects should be done regardless of their climate change impacts; the fact that they also reduce emissions simply makes them more attractive. Of most interest are projects that are not quite beneficial on their own, but the consideration of GHG benefits pushes them over the top to become worthwhile projects.

On the other hand, an agency, such as MassDOT, may be faced with a decision situation in which it must meet a specific emissions cap and it must minimize agency out-of-pocket costs. In this case, the LCC can be calculated using only agency out-of-pocket costs, and
ignoring co-benefits. This metric can then be used to find a way of satisfying the emissions cap at least cost to the agency. In the case where the LCC is very high, the agency can use this metric to determine how much it would be willing to pay to other agencies or organizations in return for emissions reductions.
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4.0 Application of Framework to Project Data

In this section, the authors present an application of the LCC framework to a set of data on transportation and non-transportation projects. The data sets are described, followed by a brief description of how the LCC was calculated on these data sets. Results are then presented, illustrating the range of LCCs and how the LCC is related to different project types.

4.1 Data

The data can be categorized into three types: transportation projects from Massachusetts; transportation projects from other states; and non-transportation projects from multiple states.

The data on the transportation projects from Massachusetts were provided by MassDOT. This data set includes 295 projects. The most relevant data in this set include project type, project description, total project cost, GHG reduction amount (metric ton/year), a cost per ton calculated by DOT, and the lifetime for each project. These projects types include four categories: Complete Streets Projects; Bike and Pedestrian Projects; Traffic Operational Projects; and Transit. All projects reported a single value for the emissions reduction per year; the authors assume that this is intended to be a constant amount across all years. The data are summarized in Table 4.1; the last column contains the calculated LCC. Note that all data is in metric tons, shown as “tonnes.”

<table>
<thead>
<tr>
<th>Project Type</th>
<th>Total number of projects</th>
<th>Median total project cost</th>
<th>Median project GHG reduction amount (tonnes/yr)</th>
<th>Median Reported Cost Effectiveness ($/tCO₂)</th>
<th>Lifetime</th>
<th>Median LCC ($/tCO₂)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traffic Operational Improvement</td>
<td>94</td>
<td>$3,079,424</td>
<td>76</td>
<td>864</td>
<td>50</td>
<td>1,829</td>
</tr>
<tr>
<td>Transit</td>
<td>92</td>
<td>$487,500</td>
<td>8</td>
<td>8,190</td>
<td>12</td>
<td>9,519</td>
</tr>
<tr>
<td>Complete Streets</td>
<td>37</td>
<td>$5,442,985</td>
<td>15</td>
<td>3,793</td>
<td>50</td>
<td>10,809</td>
</tr>
<tr>
<td>Pedestrian and Bicycle Improvement</td>
<td>73</td>
<td>$3,310,644</td>
<td>7</td>
<td>8,267</td>
<td>50</td>
<td>17,494</td>
</tr>
</tbody>
</table>
The data on non-Massachusetts transportation projects include 37 projects, which are from Arkansas (11), California (12), Colorado (14), Florida (15), Iowa (16), New Mexico (17), and South Carolina (5). Of these, 7 projects include data on annualized costs; 13 projects include the total (non-discounted) cost; and 17 provide the NPV of costs. In terms of emissions data, 24 projects report a flat emissions reduction per year; the rest include the total emissions reduction plus a specific value for two specific years within the lifetime of the project. Thirty-five of these non-Massachusetts transportation projects explicitly state that co-benefits are considered in the cost-effectiveness calculation. Ten of the projects, which are all from California, state that co-benefits are considered either quantitatively or qualitatively. The remaining 2 projects do not clearly reference co-benefits; they simply state that there are net savings from investing in these projects.

The data on non-transportation projects include 118 projects, from Arkansas (11), Colorado (14), Florida (15), Iowa (16), New Mexico (17), and South Carolina (5). Each project includes the NPV of costs, the total emissions reductions, and emissions reduction in two specific years within the project’s lifetime. The reported cost-effectiveness ranges from -$140 to $835/tCO$_2$ with mean of $14 and median of $1. In comparison, the authors calculated the LCC to be in the range of -$262 to $1407/tCO$_2$, with a mean of $22 and median of $1.40. The Arkansas, Colorado, Iowa, Montana, and South Carolina reports explicitly state that they include potential co-benefits. The Florida report does not clearly state if it has quantified the co-benefits; however, its projects show negative costs, implying that co-benefits may be included. The New Mexico and Montana reports mention that numerous co-benefits would result from implementation of the recommended policies, but they do not explicitly state if they have quantified co-benefits.

4.2 Method

In order to estimate the LCC for each project, the authors made sets of assumptions, which vary by the type of data available. The following describes the three methods used, depending on the data available.

All the Massachusetts projects and 14% of non-Massachusetts projects provide data on annual emissions reduction. In this case, it was assumed that the emissions reduction is constant during the lifetime of the project. Hence, LCC is calculated by dividing the annualized costs by the annual emissions reduction, using equation (11).

Ten percent of non-Massachusetts projects provide data on the total emissions reduction. In this case, it was also assumed that the emissions reduction is constant during the lifetime of the project. Hence, LCC is calculated using equation (11).

The rest of the non-Massachusetts projects include the NPV of costs, the total emissions reduction, and the amount of emissions reduction for two specific years. The authors assumed a linear emission path between the two given years. In most cases (73%), the total emissions from the estimate is within 10% of the reported emissions. In the remaining cases,
the estimate does not line up well with reported emissions; thus, these projects were dropped from this analysis. The LCC is calculated using equation (6).

4.3 Results

This section starts with a caveat. The authors have reported values based on the data described above, some of which contain co-benefits and some of which do not. Thus, these results must be interpreted in this context. In the cases where co-benefits are not given, such as for the Massachusetts projects, the results would refer to projects that would be done only for CO\textsubscript{2} benefits, ignoring all other benefits to society.

In 70% of the Massachusetts transportation projects, the value of the LCC differed significantly (by more than 10%) from the reported cost-effectiveness. This difference is because the reported values did not include discounting, and many of the projects have lifetimes of 50 years. For the non-Massachusetts transportation projects, which mainly used the annual method, the reported values were close to the calculated LCC, except for two projects that appeared to be in error. The non-transportation projects consistently have a calculated LCC that is higher than the reported cost-effectiveness, but the values are not significantly different. As expected, when the report used the annual value, the LCC was equal; when the report used NPV, the LCC was higher.

Figure 4.1 presents the marginal abatement cost curve, including all the Massachusetts transportation projects in the data set. Fig 4.2 presents the same information, but focused only on projects with an LCC less than $200/t\text{CO}_{2}. These charts can be interpreted as follows. If the value of reducing carbon emissions is set to $200/t\text{CO}_{2}, then it would be optimal to invest in all but the last project in Fig 4.2. This would lead to annual emissions reductions of 7,493 tonnes.
Figure 4.1: Marginal abatement cost of Massachusetts projects

Figure 4.2: Marginal abatement cost of Massachusetts projects with LCC <$200/tCO₂

Table 4.2 lists the Massachusetts projects with LCC less than $200 in order of LCC. Note that most of these projects are of the type TOP (Traffic Operations Projects). This does seem an indication that these projects are the most favorable for pure CO₂ reductions, as they make up about one-third of all the projects in the data set, but 80% of the projects in Table 4.2.
Note that the Cape Bike Shuttle would be considered relatively less cost-effective if using the reported cost-effectiveness metric: it would have come farther down the list.

**Table 4.2: List of Mass. transportation projects with LCC<$200/tCO₂**

<table>
<thead>
<tr>
<th>Project Title</th>
<th>Category</th>
<th>Total Project Cost ($)</th>
<th>Emissions Reduction (tonnes/yr)</th>
<th>Reported Cost Effectiveness ($/tCO₂)</th>
<th>LCC ($/tCO₂)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oak Bluffs</td>
<td>Traffic Operations Projects</td>
<td>412,370</td>
<td>263</td>
<td>34</td>
<td>73</td>
</tr>
<tr>
<td>West Bridgewater</td>
<td>Traffic Operations Projects</td>
<td>2,805,960</td>
<td>1745</td>
<td>35</td>
<td>75</td>
</tr>
<tr>
<td>Northampton</td>
<td>Traffic Operations Projects</td>
<td>2,106,590</td>
<td>1140</td>
<td>41</td>
<td>86</td>
</tr>
<tr>
<td>Avon</td>
<td>Traffic Operations Projects</td>
<td>3,888,000</td>
<td>1886</td>
<td>45</td>
<td>96</td>
</tr>
<tr>
<td>Worcester</td>
<td>Traffic Operations Projects</td>
<td>2,902,792</td>
<td>1116</td>
<td>57</td>
<td>121</td>
</tr>
<tr>
<td>Easton Signalization &amp; Geometric Improvements</td>
<td>Traffic Operations Projects</td>
<td>1,044,228</td>
<td>359</td>
<td>64</td>
<td>135</td>
</tr>
<tr>
<td>Cape Bike Shuttle</td>
<td>Transit</td>
<td>87,610</td>
<td>68</td>
<td><strong>118</strong></td>
<td><strong>137</strong></td>
</tr>
<tr>
<td>Easton Intersection Improvements</td>
<td>Traffic Operations Projects</td>
<td>1,062,986</td>
<td>359</td>
<td>65</td>
<td>138</td>
</tr>
<tr>
<td>Boylston Street</td>
<td>Complete Street Projects</td>
<td>8,214,319</td>
<td>1959</td>
<td>92</td>
<td>195</td>
</tr>
</tbody>
</table>

Figure 4.3 shows each of the ten projects in Table 4.2. Each project is placed so that its LCC is on the Y axis and its emissions saved is on the X axis. If projects were chosen in order of LCC, then projects would be chosen from the bottom up. Blue dots indicates the individual project. The circled projects are those that are non-dominated: there is no project that is both more cost-effective and has greater savings. The projects that are not circled are each dominated by one of the circled projects. The values on the right show the tradeoffs between some of the non-dominated projects. Specifically, this is the value of the SCC that would
justify prioritizing the circled project to the right over the project to the left. Since these values are much higher than current estimates for the SCC, it is robust to simply prioritize these projects in order of the LCC.

![Figure 4.3: Pareto analysis of projects in Table 4.2](image)

Table 4.3 summarizes the 15 (out of 37) non-Massachusetts projects with LCC less than $200/tCO₂. Note that most of these projects account for co-benefits, and so these results cannot be compared to the Massachusetts results. Most of these projects are related to Vehicles and Vehicle Improvement, and Transit. Note, these projects come from different reports using potentially very different methodologies.

<table>
<thead>
<tr>
<th>Table 4.3: Summary of non-Mass. transportation projects</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Total number of projects</strong></td>
</tr>
<tr>
<td>Vehicles and Vehicle Improvement</td>
</tr>
<tr>
<td>Transportation System Management</td>
</tr>
<tr>
<td>Transit</td>
</tr>
</tbody>
</table>
5.0 Conclusions

5.1 Summary

In this study, the authors have introduced a new metric for calculating the cost-effectiveness of GHG-reduction projects, called the levelized cost of carbon (LCC). It accounts correctly for the time value of money and also for a possibly changing value of reducing GHGs through time. This definition is shown to encompass the most common definitions in the literature. The LCC can be compared to the SCC, or to any other appropriate value, to determine whether a project should be invested in or not.

The study then points out that it is not generally valid to use this metric—or any other cost-effectiveness metric, for that matter—to rank projects. A project that is more cost-effective in terms of its cost per ton of CO₂ is not necessarily the project that maximizes social welfare. This is because, if the value of reducing emissions is large enough, then it is better to choose a larger, if slightly less cost-effective, project over a smaller, more cost-effective project.

The assumptions are outlined under which it is valid to use LCC to compare projects, and a methodology is provided for communicating the comparison of projects for which LCC cannot be used on its own. If projects can be scaled down linearly, then the LCC provides a correct way to compare projects. More generally, while it is not guaranteed to produce the optimal solution in theory, in practice it often produces very good solutions.

The application of the methodology to data on projects inside and outside of Massachusetts confirms that using the LCC to prioritize projects may in fact be quite reasonable in practice. The authors found that if Massachusetts faces a constraint on either costs or on emissions reduction, the maximum error from using the LCC is less than 0.07%. The authors also found that among the ten most cost-effective projects, it was robust to prioritize these in terms of the LCC. It is noted that a single category of project, Traffic Operations Projects, seems to be most cost-effective.

The data analysis comes with an important caveat, however. The Massachusetts data does not include information on broader social costs and benefits. Therefore, this analysis is only valid if MassDOT is considering projects only as GHG-reduction projects and ignoring all other benefits, such as congestion reduction, safety, or other reduction of pollutants.

5.2 Implementation

Three potential decision situations are discussed as follows, in which the LCC concept can be implemented at MassDOT, in the service of contributing to achieving a cap on emissions in the transportation sector.
First, MassDOT may face a cap on emissions from MassDOT-owned sources. Developing marginal abatement cost curves for MassDOT-owned sources, similar to the figures in Chapter 4 of this report, would allow MassDOT to choose the most cost-effective measures for reducing its own emissions. Furthermore, to the degree that reducing emissions from MassDOT-owned sources is very costly, MassDOT can make an informed decision on whether to buy emission reduction permits from other agencies.

Second, MassDOT may contribute to achieving an overall transportation sector cap through MassDOT investments. This report indicates that the current set of MassDOT investments are quite expensive (ignoring co-benefits), with most projects having an LCC of over $200/tCO₂. Thus, MassDOT may want to investigate different types of projects—those aimed more specifically at reducing CO₂. In particular, using this methodology, MassDOT may be able to identify types of projects that are scalable and have much lower LCCs. Of most interest are projects that could be tested at the pilot level within Massachusetts.

Third, it may be most efficient to achieve an overall transportation sector cap in the presence of policy instrument flexibility. The LCC can be used to compare a wide range of investments, regulations, policies, and pricing, to identify those instruments that are most cost-effective. In this case, the LCC would need to be calculated using the full net social costs of the instruments. Different instruments may vary significantly on how they allocate costs and benefits across different stakeholders.
6.0 References


7.0 Appendices

7.1 Analytic Proof

As mentioned in Section 3.2, in the special case where emission reductions are constant in every year, say equal to \( e \), and the value of reducing a ton of CO\(_2\), \( \gamma \), is zero, then the cost per ton of CO\(_2\) can be calculated by dividing the annual worth of the cost by \( e \). Hence, the metric in the special case of constant emissions per year is equal to the annualized lifetime effect.

The analytical proof with the assumption: \( \gamma = 0 \) is as follows:

\[
\tau_{\text{ann}} = \frac{[I-c] \left[ \frac{c (i+1)^{\tau}}{(i+1)^{\tau} - 1} \right]}{e},
\]

\[
\tau = \frac{\sum_{t=0}^{T} \frac{1}{(i+1)^t} c_t}{\sum_{t=0}^{T} \frac{1}{(i+1)^t} e_t} = \frac{C_0 + \sum_{t=1}^{T} \frac{1}{(i+1)^t} c_t}{\sum_{t=0}^{T} \frac{1}{(i+1)^t} e_t},
\]

Numerator:

\[
\sum_{t=0}^{T} \frac{1}{(i+1)^t} = \alpha + \alpha^2 + \ldots + \alpha^T,
\]

\[
\sum_{t=1}^{T} \alpha^t = \alpha \frac{1 - \alpha^T}{1 - \alpha} = \left( \frac{1}{1+i} \right) \left( \frac{1 - \left( \frac{1}{1+i} \right)^T}{1 - \left( \frac{1}{1+i} \right)} \right) = \frac{(1+i)^T - 1}{i(1+i)^T},
\]
Denominator:

\[
\sum_{r=0}^{T} \frac{1}{(i+1)^r} = \alpha + \alpha^2 + \ldots + \alpha^T,
\]

\[
\sum_{r=0}^{T} \alpha^r = \alpha \frac{1-\alpha^{T+1}}{1-\alpha} = \left( \frac{1}{1+i} \right) \left( \frac{1 - \left( \frac{1}{1+i} \right)^{T+1}}{1 - \left( \frac{1}{1+i} \right)} \right) = \frac{(1+i)^{T+1} - 1}{i(1+i)^T},
\]

\[
\tau_{LCC} = \frac{C_0 \frac{i(1+i)^T}{(1+i)^T - 1} + C \frac{i(1+i)^T}{(1+i)^T - 1}}{e}
\]

which is identical to \( \tau_{ann} \).
7.2 Proof of Claim in Section 3.4

Here the authors prove that if a project is both smaller (in terms of NPV of emissions) and has a higher LCC, then it will never be preferred in a social welfare framework under the condition that it has a positive NPV. First, three terms are defined:

\[ E_k \equiv \sum_{t=0}^{T} \frac{1}{(1+i)^t} e_t^k \]

\[ C_k \equiv \sum_{t=0}^{T} \frac{1}{(1+i)^t} c_t^k \]

And \( LCC_k = \frac{E_k}{C_k} \)

Proposition: Assume \( NPV_k > 0 \). If \( E_k < E_{k'} \) and \( LCC_k > LCC_{k'} \), then \( NPV_{k'} > NPV_k \) for all values of the SCC, \( \tau \).

Proof: Note that by definition of NPV, the following inequalities exist:

\[ NPV_k < NPV_{k'} \iff \tau E_1 - C_1 < \tau E_2 - C_2 \iff \tau (E_1 - E_2) - (C_1 - C_2) < 0 \]

If \( \tau \) is less than \( LCC_1 \), then project 1 is not cost-effective under any circumstances. If \( \tau = LCC_1 \), then by the second line above, \( NPV_1 = 0 \) and \( NPV_2 > 0 \). The third line above shows that this inequality is decreasing in \( \tau \), since \( E_k - E_{k'} < 0 \). Therefore, for all \( \tau > LCC_1 \), \( NPV_1 \) will be less than \( NPV_2 \).
7.3 Value of SCC

There is not a clear agreement on the value of the SCC. Table 7.1 shows some plausible values.

Table 7.1: Social cost of carbon estimates

<table>
<thead>
<tr>
<th></th>
<th>Low Case (5 percent discount rate, mean)</th>
<th>Medium Case (3 percent discount rate, mean)</th>
<th>High Case (2.5 percent discount rate, mean)</th>
<th>Tail Case (3 percent discount rate, 95th percentile)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2010</td>
<td>10</td>
<td>31</td>
<td>49</td>
<td>84</td>
</tr>
<tr>
<td>2015</td>
<td>11</td>
<td>36</td>
<td>54</td>
<td>102</td>
</tr>
<tr>
<td>2020</td>
<td>11</td>
<td>40</td>
<td>61</td>
<td>121</td>
</tr>
<tr>
<td>2025</td>
<td>13</td>
<td>45</td>
<td>66</td>
<td>135</td>
</tr>
<tr>
<td>2030</td>
<td>15</td>
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<tr>
<td>2050</td>
<td>25</td>
<td>67</td>
<td>92</td>
<td>207</td>
</tr>
</tbody>
</table>

Note: 2013 IWG SCC estimates (2009 $ per short tonne)